1. What is a BAR Magnet?

2. GAUSS' LAW FOR $\vec{B}$ - FIELD

Any object which has a non-zero magnetic moment ($\vec{\mu}$) can be designated as a BAR MAGNET because it will experience a torque $\vec{\tau} = \left[\vec{\mu} \times \vec{B}\right]$

When placed in a $\vec{B}$-field.

CURRENT CARRYING LOOP IS A "BAR" MAGNET.

To begin, let us recall that a current carrying loop of area $\vec{A}$ experiences a torque $\vec{\tau} = IA\hat{n} \times \vec{B}$. Because it has a magnetic moment $\vec{\mu} = IA\hat{n}$. Incidentally, it also has potential energy $u_B = -\vec{\mu} \cdot \vec{B}$. So that $\hat{n}$ wants to line up along $\vec{B}$.

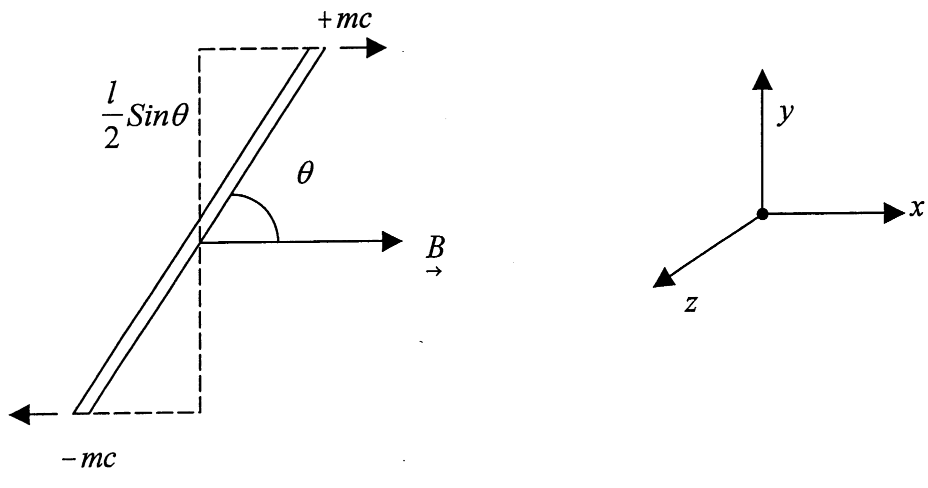
Note, the similarity to an Electric Dipole of moment $\vec{p} = q\vec{l}$ placed in $\vec{E}$.

Torque is $\vec{\tau} = [\vec{p} \times \vec{E}]$

Potential Energy is [redacted] $u_E = -\vec{p} \cdot \vec{E}$

Next, let us take a compass or a store bought bar magnet. If you suspend it in Earth's magnetic field, the compass or bar will line up along the Earth's $\vec{B}$-field. Again, this happens because of the torque on the compass (B.M.)

To understand this let us pretend that the B.M. can be imagined as consisting of magnetic "charges" $\pm m_c$ separated by l.



And we find that it experiences a Torque: $\underset{\rightarrow}{\tau} = -mclB\,Sin\theta\,\hat{z}$ or $\underset{\rightarrow}{\tau} = \left[\underset{\rightarrow}{\mu} \times \underset{\rightarrow}{B}\right]$ with $\underset{\rightarrow}{\mu} = m_c \underset{\rightarrow}{l}$

Question: Is the pretense justified?
Answer: A FIRM NO!
Why: If you break the B.M. in ~~the~~ to two parts you will not separate $+mc$ and $-mc$. You will get two bar magnets. Keep breaking and you get more and more bar magnets.
1→ 2→ 3→ 4→ 8→ ....→ Single atom→ ELECTRON

Ultimately, you will discover that a SINGLE Electron is a complete bar magnet because it has a magnetic dipole moment

$$\mu_e = 9.27 \times 10^{-24} \frac{N-m}{T}$$

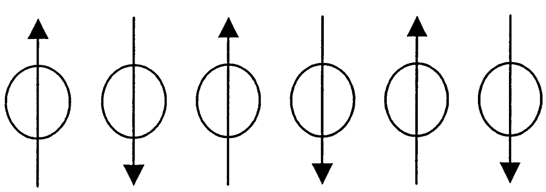
Which is called a Bohr magneton (μ_B). Thanks to quantum mechanics we now know that in addition to charge e ($= 1.6 \times 10^{-19} C$) and mass m_e($= 9 \times 10^{-31} kg$) an electron has an intrinsic property called spin s (=1/2) which endows it with a magnetic moment

$$\mu_e = \mu_B = \frac{2e\hbar}{2m_e} s$$

where $\hbar \cong 10^{-34}\ J-s$. So, if an electron is placed in a 1 Tesla field and its moment is $\perp$ to $\underset{\rightarrow}{B}$ it will experience a torque of 9.27×10^{-2} N-m

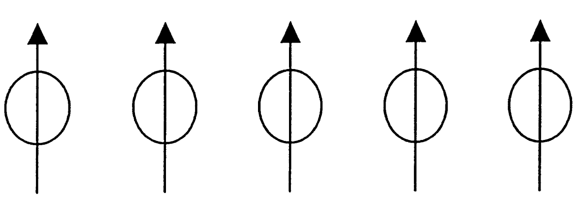
Indeed, even nuclei have a magnetic moment but since mass of proton is about 2000 m_e, nuclear moments are much smaller.

So now that we have the elementary building blocks let us start "constructing" a bar magnet beginning with electrons. The next constituent is the atom. Here, to get a non-zero μ_A we need unpaired Electrons. That is, if the electrons are arranged in pairs so that they look like



then $\mu_A \equiv 0$

and that atom is no good to make a B.M. Iron, on the other hand, is a "good" atom. Fe^{3+} has five unpaired electrons.



$\mu_{F_e} = 5\mu_B$

at the simplest level.
Next, put these atoms into a solid. At high temperatures, the thermal energies make these atomic moments wobble rapidly so that the time average

$$\langle \mu_A \rangle \equiv 0$$

and all we get is a paramagnet. There is no net moment, so no bar magnet.

However, [recall the dielectric in an $\underset{\rightarrow}{E}$ -field] if we put the material in an applied magnetic $\underset{\rightarrow}{B}$ it will cause all the moments to line up and the solid will acquire a magnetic moment μ_{Sol}: We can define the Magnetization

$$M = \frac{\mu_{Sol}}{\cancel{\mu_0}\ \text{Volume}}$$

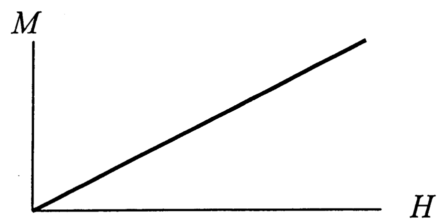
It has units of $\frac{A - m^2}{m^3}$ or $\frac{A}{m}$ which are the same as units of $\frac{B}{\mu_0}$

Simple Experiment: Take a Solenoid. Pass a current I. You will get $B_0 = \mu_0 n I = \mu_0 H$

This can be applied to our paramagnet. We will find that the $\underset{\rightarrow}{B}$ -field enhances to

$$\begin{aligned} B &= \mu_0 nI + \mu_0 M \\ &= \mu_0(H + M) = \mu_0 H(1 + \chi_m) \end{aligned}$$

Which defines the magnetic susceptibility $\chi_m = \frac{M}{H}$



In the paramagnet, χ_m is a constant, independent of H. M is linear in H. However, χ_m is a function of temperature.

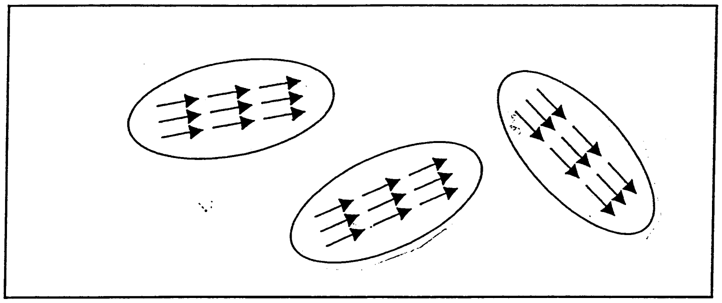
On reducing T, χ_m increases according to $\chi_m = \frac{c}{T}$ and this will hold if the atomic moments do not "talk" to one another and we will never get a bar magnet.

Fortunately, again thanks to quantum mechanics, in some materials (Fe and Ni are most familiar) the atomic moments interact with one another and produce an internal field proportional to M. In such systems one can write $M = \chi_m[H + \lambda M]$ where λ is a constant.

Again $M = \frac{c}{T}(H + \lambda M)$ and we should find $\chi'_m = \frac{M}{H} = \frac{c}{T - \lambda c}$ and we note that if T is reduced until $T = \lambda c$, χ'_m will blow up. That is, the material will have a spontaneous magnetic moment M. We have succeeded in making a **FERROMAGNET.** Remember that in class we did an experiment to show that Ni becomes a ferromagnet at $T \leq 630K$

At lower temperatures the system can be imagined as consisting of "domains" each of which contains billions of atoms with their μ_A's aligned. These domains

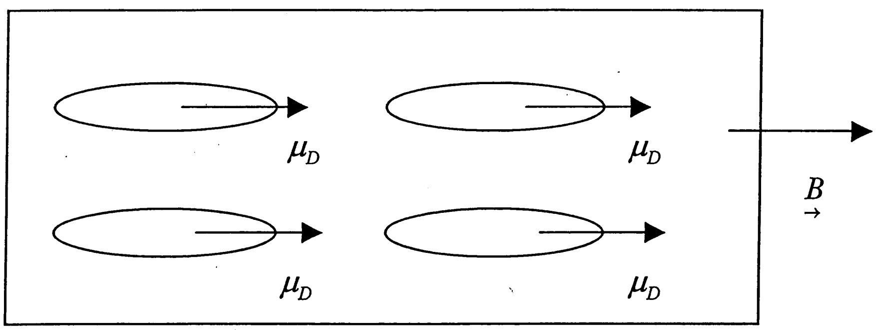
Ferromagnet



(Only few domains are drawn)

have giant magnetic moments μ_D and the response to an applied field is greatly enhanced [Experiment in class: nickel was attracted strongly when it was cold]

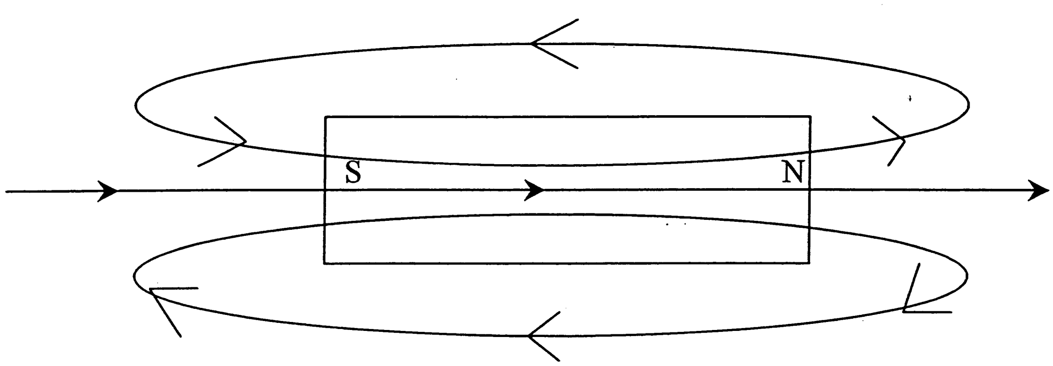
The last step in constructing a B.M. is to recognize that we need a material in which there is a preferred direction. That is, the μ_D 's prefer to lie along some axis. Let us assume that for our bar this is along $\hat{x}$. If we apply a $\underset{\rightarrow}{B} \| \hat{x}$ all the μ_D 's will align along $\hat{x}$ and we get



and every domain has its μ_D along direction where it prefers to stay. So next if we remove $\underset{\rightarrow}{B}$, the μ_D 's will stay put, the Bar has a "permanent" magnetic moment. Indeed, we ~~first~~ have "constructed" a

BAR MAGNET

Which has a $\underset{\rightarrow}{B}$ field looking like



2. <u>GAUSS'S LAW FOR $\vec{B}$</u>

<u>FACT</u>: A single electron is a complete magnet with a dipole moment μ_B.
<u>CONSEQUENCE</u>: Elementary source of $\vec{B}$ is a <u>DIPOLE</u> with effectively "zero" size. "sources" and "sinks" are coincident. When we use lines to map $\vec{B}$-fields the lines must close <u>on Themselves</u>. There is no "beginning" and no "end" to a $\vec{B}$-field line.
<u>PROFOUND IMPLICATION</u>: Total flux of $\vec{B}$ through any closed surface must be always equal to zero:

$$\boxed{\Sigma_c \vec{B} \bullet \Delta\vec{A} \equiv 0}$$

Every line that comes into enclosed volume must go out as $\vec{B}$-field lines <u>do not</u> stop or start anywhere.

<u>NOTE:</u> The law tells you that total <u>flux</u> of $\vec{B}$ is zero. It says nothing about $\vec{B}$.

Also, surface must be closed!

<u>EXAMPLE</u> SUPPOSE THAT THE "SURFACE" IS THE SIX FACES OF A CUBE. WE ARE TOLD THAT THROUGH FIVE FACES THERE IS A FLUX OF $(0.5)T\text{-}m^2$ PER FACE. THEN TH'S LAW SAYS THAT THROUGH THE SIXTH FACE THERE MUST BE A FLUX OF $-(2.5)T\text{-}m^2$ SO THAT $[5 \times 0.5 - 2.5] \equiv 0$!