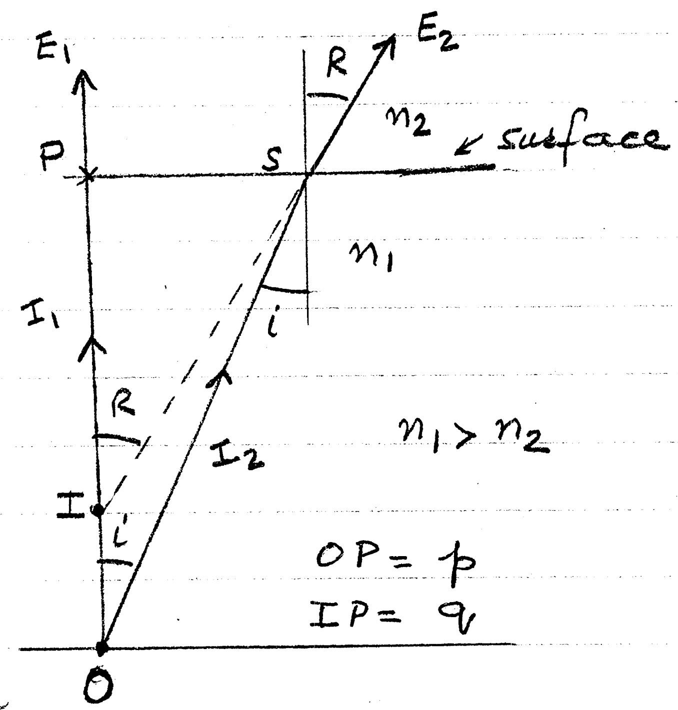
FORMATION OF IMAGES – REFRACTION AT A SINGLE SURFACE [SIGN CONVENTION: ALONG LIGHT +ive, AGAINST LIGHT –ive]

I: <u>Apparent</u> <u>depth</u> <u>of</u> <u>Water</u> <u>in</u> <u>a</u> <u>pool</u>.

Supposing you are standing at the edge of a swimming and look straight down. If the actual depth of water is d meters what value do you perceive?



We can solve this problem by putting a point object O at the bottom and locate its image formed by the water as the light refracts through its surface. Look at the picture

OPTICAL SYSTEM so all distances are measured from P.

Take two rays starting from O:

I_1 makes angle of incidence zero and gives rise to E_1

I_2 makes angle of incidence i and causes E_2 satisfying

$$n_2 \sin R = n_1 \sin i$$

Since you are looking straight down all angles are small.

The virtual IMAGE at

I [q is –ive].

is located by intersection of E_1 and E_2

(extended backwards).

Next, from the picture we see

$$\tan R = \frac{SP}{IP} \qquad 1)$$

$$\tan i = \frac{SP}{OP}. \qquad (2).$$

Divide ② by ①

$$\frac{IP}{OP} = \frac{\tan i}{\tan R}$$

$$\simeq \frac{\sin i}{\sin R} \qquad \begin{bmatrix} i \ll 1 \\ R \ll 1 \end{bmatrix}.$$

$$= \frac{n_2}{n_1}$$

Clearly IP = apparent depth
OP = Rl. depth

$$\frac{d_{app}}{d} = \frac{n_2}{n_1}$$

for water $n = 1.33$
for air $n = 1.$

$$\text{So} \quad \frac{d_{app}}{d} = \frac{3}{4}.$$

So if water is 80 cm deep, to a person at the edge it will appear to be only 60 cm [small children should be warned before they jump in and suddenly find that they are too short].

THIN LENSES

A lens is a piece of transparent material of refractive index n placed within a second material of refractive index n'. (i) We will take the second material as air so $n' = 1$. (ii) We will consider only those lenses whose surfaces are spherical. (iii) We will assume that the thickness (t) of the lens is much smaller than the radii of its surfaces

SIGN CONVENTION (REPEAT)

Here, we will be dealing with the phenomenon of refraction only. The "sign" convention will be that distances measured along the direction of light travel will be labelled positive, those opposite to that direction will be taken as negative.

LENS MAKER'S FORMULA

We begin by stating, without proof, the lens maker's formula for thin lenses

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_F} - \frac{1}{R_B}\right] \qquad (1)$$

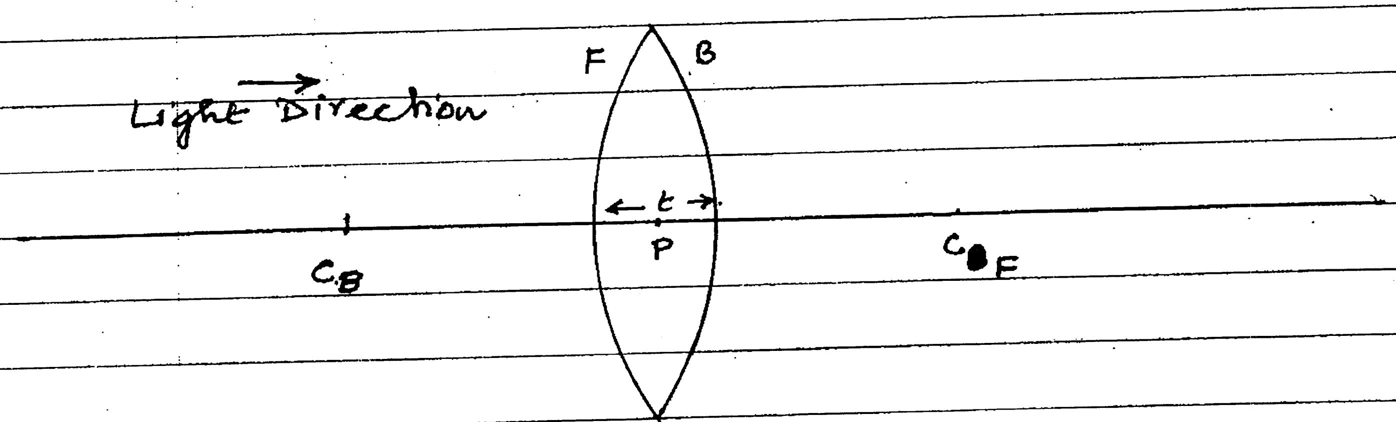
where f = Focal length

R_F = radius of front surface (facing the light ray)

R_B = radius of back surface.

CONVEX (CONVERGENT) LENS.

Light Direction →



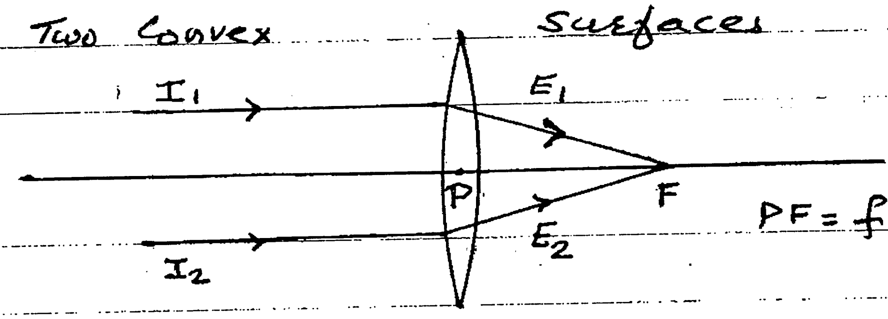
C_F, C_B are the centers of the front and back surfaces, respectively.

$R_F = C_F P$ is positive $(t \ll R_F, R_B)$

$R_B = C_B P$ is negative

Hence, from Eq (1) it follows that f is positive. That is, the focal point F lies to the right of the lens. Result is that a parallel beam of light falling on such a lens will converge to a point after it passes through the lens.

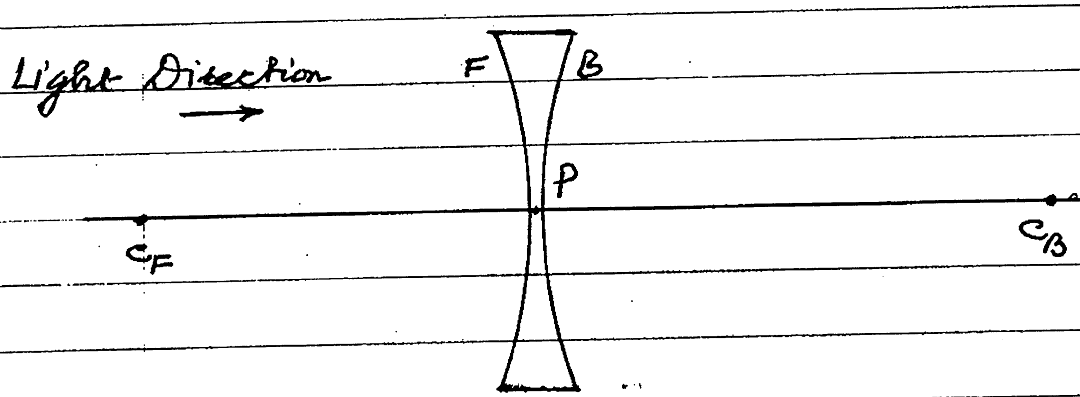
Namely, Two Convex surfaces



$PF = f$

make a CONVERGENT LENS

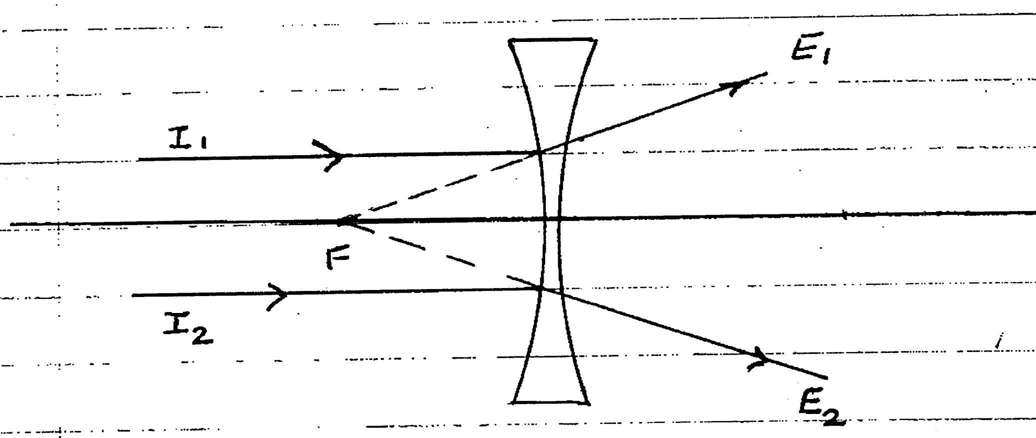
CONCAVE (DIVERGENT) LENS



$R_F = C_F P$ is negative

$R_B = C_B P$ is positive

Hence from Eq (1) f is -ive and therefore the focal point F lies to the left of the lens (on dark side). Result is that a parallel beam of light falling on such a lense will appear to diverge from F after it passes through the lens. Namely, two Concave ~~fo~~ Surfaces make



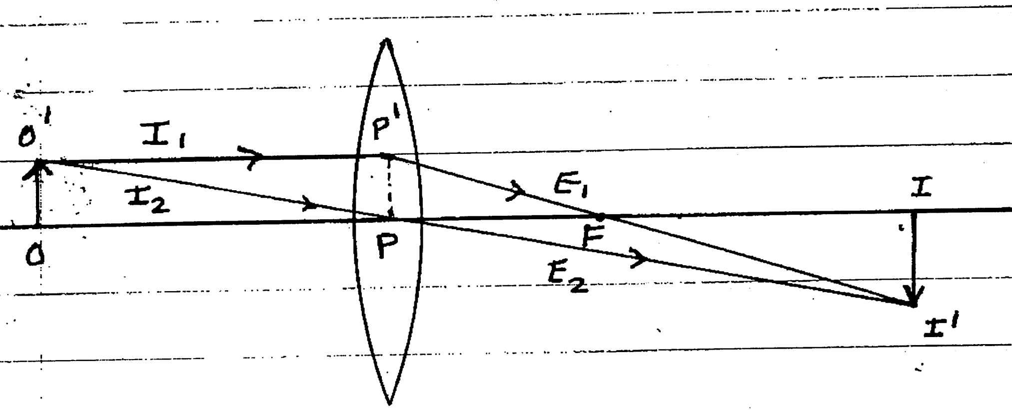
A DIVERGENT LENS.

IMAGE FORMATION — CONVERGENT LENS

We will use the same method as before to locate the image: Take two rays starting from O, go through the lens, look for point of intersection to find I.

Again p = distance of object from lens
q = distance of image from lens

magnification $m = -\frac{q}{p}$



Please note that I_2 goes through central portion of the lens which is more like a parallel plate of thickness t. In this case we have shown ~~that~~ that the ray shifts sideways by

$$\delta = t \sin i \left[1 - \frac{n_1}{n_2} \frac{\cos i}{\cos R} \right]$$

Since t is small and, as before, we deal only with paraxial rays ~~so~~ i is small therefore δ is negligible. That is, I_2 goes straight through.

Next, $\angle(OO'P) = \angle(II'P)$

Therefore $$\frac{II'}{OO'} = \frac{q}{p} \qquad (3)$$

as expected from the formula for m [Note: image is inverted].

Next, consider angles $P'FP$ and $I'FI$

$$\frac{II'}{IF} = \frac{PP'}{PF}$$

$$\frac{II'}{PP'} = \frac{II'}{OO'} = \frac{q-f}{f} \qquad (4)$$

From (3) and (4)

$$\frac{q}{p} = \frac{q-f}{f}.$$

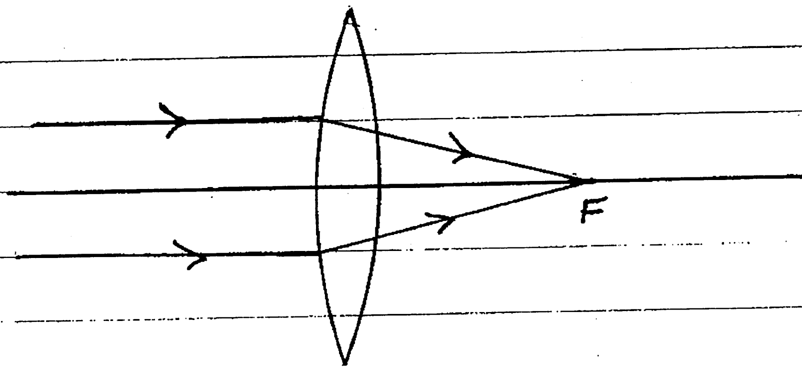
or $$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

and combined with $m = -\frac{q}{p}$

we have a complete description for all possible images formed by a convergent lens.

SPECIAL CASES

a. Object far away, $p \to \infty$, $q \to f$, $m \to 0$. Parallel light falling on lens converges to F.



NOTE: With a Convergent lens, Real Image can never be closer than f

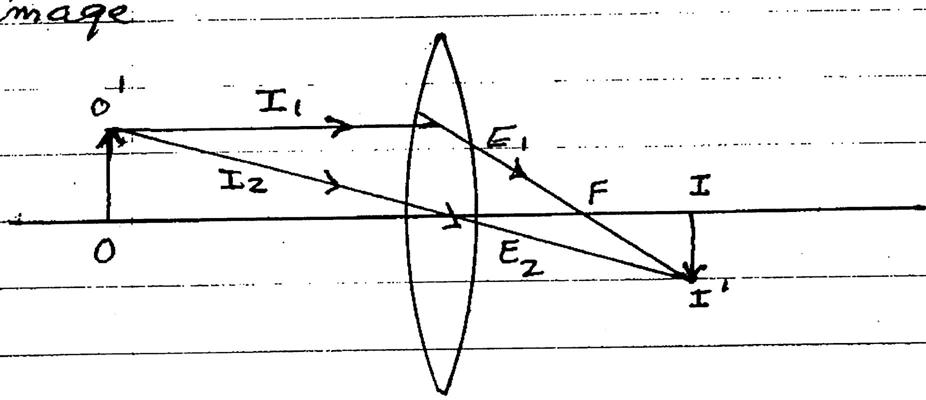
b $p > 2f$

since $\frac{p}{q} = \frac{p}{f} - 1$

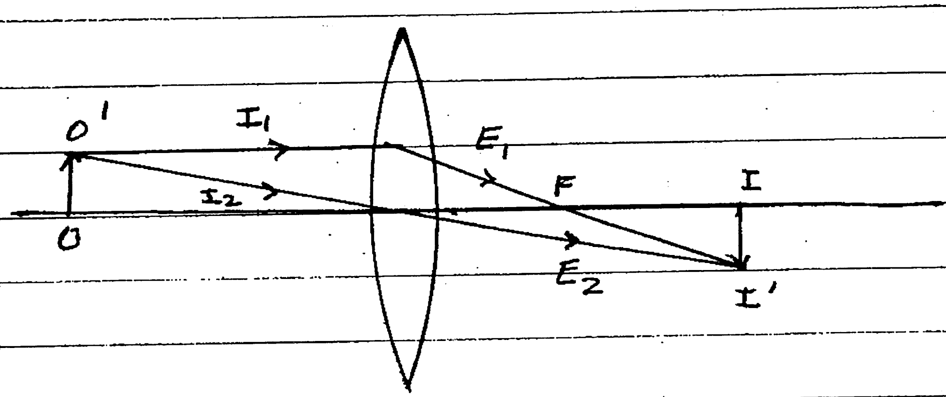
if $p > 2f$, $\frac{p}{q} > 1$ so $q < 2f$.

$|m| < 1$.

and we will form a Real, Inverted, reduced image



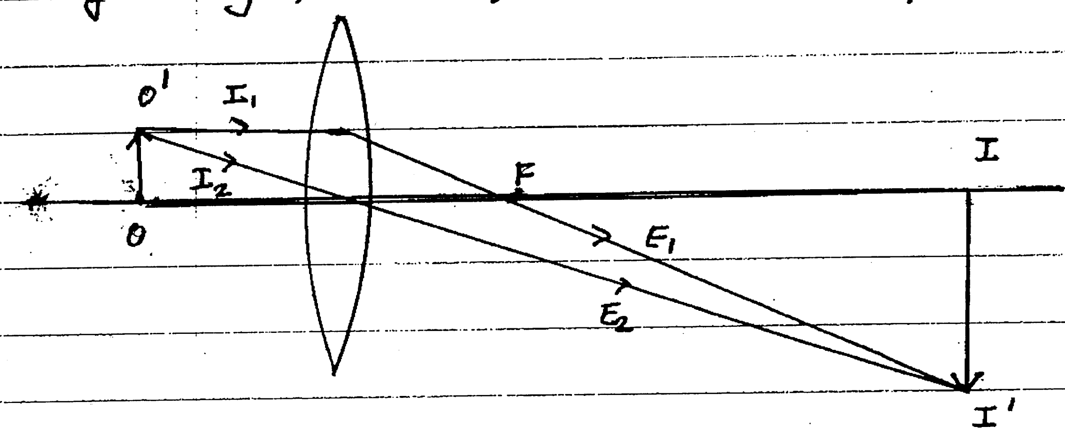
c $p = 2f$, $q = 2f$, $|m| = 1$. Real, Inverted image, Same size as object.



d. p slightly larger than f

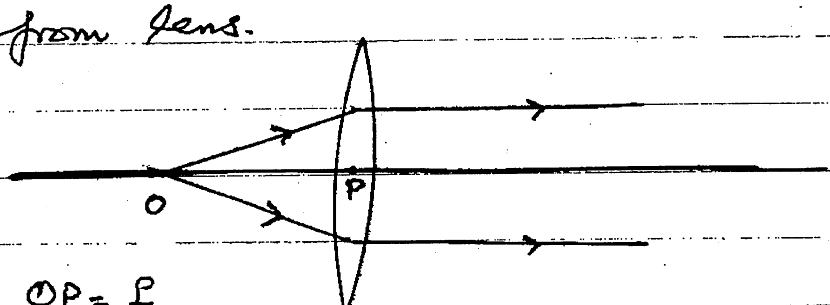
$$\frac{p}{q} = \frac{p}{f} - 1 \qquad \frac{p}{q} \ll 1 \text{ so } q \text{ very large}$$
$$|m| \gg 1.$$

Very large, Real, inverted image



SLIDE PROJECTOR WORKS ON THIS PRINCIPLE

e. $p = f$, $\frac{1}{q} = 0$, $q \to \infty$. Light becomes parallel on emerging from lens.

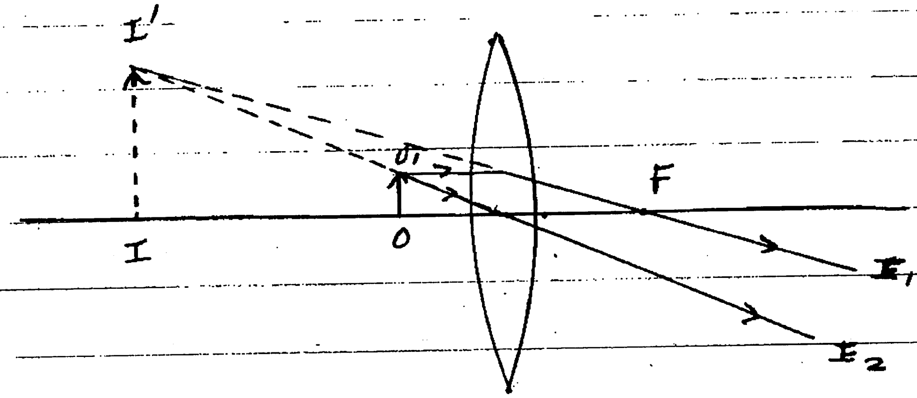


$\underline{f}$ $p < f$ $\qquad \frac{p}{q} = \frac{p}{f} - 1$.

so q must be <u>negative</u>, $m = q/p$

The image switches to "upstream" side of lens.

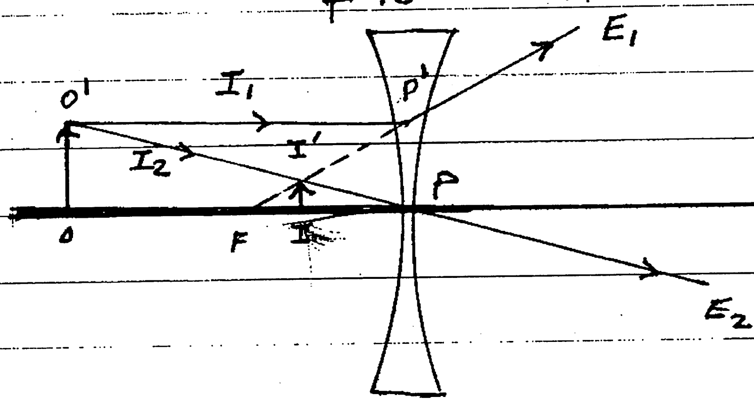
The image is virtual, enlarged and upright.



<u>Note</u>. This is how a Magnifying Glass Works.

<u>IMAGE FORMATION - DIVERGENT LENS</u>

f is $-$ive.



$OP = p$

$IP = q$ [$-$ive]

$FP = f$ [$-$ive]

as before $\frac{II'}{OO'} = \frac{q}{p}$ $(= m)$

$$\frac{II'}{PP'} = \frac{q}{p} = \frac{f-q}{f}$$

$$\frac{1}{p} - \frac{1}{q} = -\frac{1}{f}$$

However, both q and f are negative so we can write

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \left[\begin{matrix} q & -ive \\ f & -ive \end{matrix}\right].$$

$$m = -\frac{q}{p}.$$

for a divergent lens as well.
In this case [as in divergent mirror].
the image is always virtual, always reduced, and always upright.