

FIELDS: GRAVITATIONAL, COULOMB  $\vec{E}, \vec{B}$

$G$ : A mass  $m$  located in a Gravitational field feels a force  $\vec{F}_G = m\vec{G}_F$

Measure  $\vec{F}_G$ , map out  $\vec{G}_F$ .

A mass  $M$  located at the origin creates a  $\vec{G}_F$

$$\vec{G}_F = -\frac{GM}{r^2}\hat{r}$$

consequently,  $\sum_c \vec{G}_F \cdot \Delta\vec{A} = -4\pi G \sum M_i$

FLUX OF  $\vec{G}_F$  Through a closed surface is determined solely by the masses enclosed by the surface.

$E$ : A charge  $q$  located in an  $\vec{E}$ -field feels a force  $\vec{F}_E = q\vec{E}$ .


Measure  $\vec{F}_E$ , map out  $\vec{E}$ .

A stationary charge  $Q$  located at  $r=0$  generates a coulomb  $\vec{E}$ -field

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}$$

[+Q (source), -Q (sink)] consequently,  $\sum_c \vec{E} \cdot \Delta\vec{A} = \frac{1}{\epsilon_0} \sum Qi$

FLUX OF  $\vec{E}$  THROUGH A CLOSED SURFACE IS DETERMINED SOLELY BY THE CHARGES ENCLOSED BY THE SURFACE.

The Devices resulting from this are: (i) Battery 

(ii) Capacitor  $C = \frac{Q}{V}$  which leads to energy density  $\eta_E = \frac{1}{2}\epsilon_0 E^2$

That is the energy contained in  $1m^3$  vol. of  $\vec{E}$ -field

(iii) Resistor  $R = \frac{V}{I}$  which leads to  $\vec{J} = \sigma\vec{E}$ , because  $I = \vec{J} \cdot \vec{A}$

That is, if you apply  $\vec{E}$  to a Conductor it responds by setting up a current density  $\vec{J}$  whose magnitude is determined by the electrical conductivity  $\sigma$ .

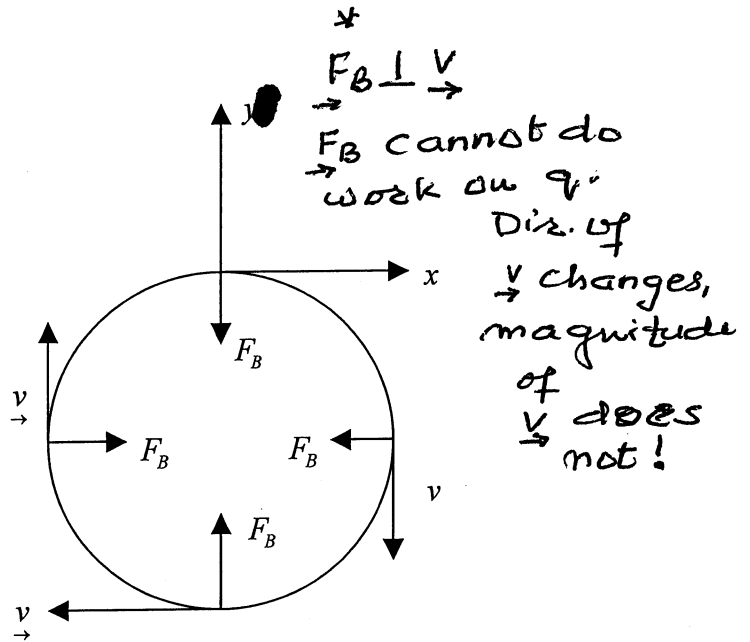
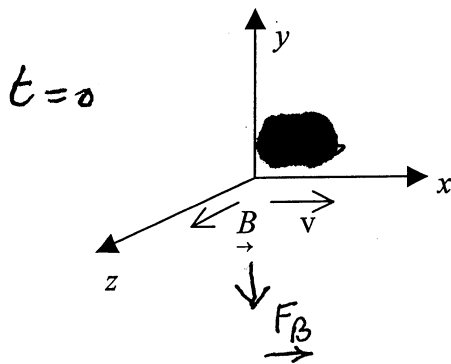
Magnetic ( $\vec{B}$ ): If a  $\vec{B}$ -field is present, a charge  $q$  moving with velocity  $\vec{v}$  will experience a

$$\text{force } \vec{F}_B = q[\vec{v} \times \vec{B}]$$

Magnitude of  $F_B = qvB \sin(\vec{v}, \vec{B})$  [Equivalently, if a moving charge experiences a force which is always perpendicular to its velocity, and there is no visible agency applying the force, then the charge must be moving in a  $\vec{B}$ -field].

Direction of  $\vec{F}_B \rightarrow$  *rt hand rule*  $\left\{ q \vec{v} \parallel \text{Thumb}, \vec{B} \parallel \text{fingers}, \vec{F}_B \perp \text{Palm} \right\}$

Problem I: At  $t=0$ , charge  $q$  is at origin and has velocity  $\vec{v} = v\hat{x}$ . Turn on a field  $\vec{B} = B\hat{z}$ . Immediately, it experiences  $\vec{F}_B$  along  $-\hat{y}$ . Makes  $\vec{v}$  turn, but  $\vec{F}_B$  turns also. Net result is as shown in Figure.  $q$  goes around in circle,  $\vec{F}_B \perp \vec{v}$  always so Kinetic Energy fixed, magnitude of  $\vec{v}$  does not change.



Particle moves under influence of  $\vec{F}_B = -qvB\hat{r}$  [ $\vec{v}$  &  $\vec{B}$  are  $\perp$  to one another]

Note: Plane of orbit  $\perp$  to  $\vec{B}$  field.

Note: Uniform circular motion needs a centripetal force.

$$\vec{F}_C = \frac{-Mv^2}{r} \hat{r}$$

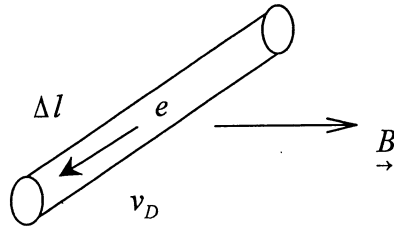
$\vec{F}_B$  provides it.

$$\vec{F}_B = \vec{F}_C \text{ so } r = \frac{Mv}{qB}$$

angular velocity  $\vec{w} = \frac{-qB}{m} \hat{z}$  (see picture above)

Note:  $w$  independent of  $v$ .

Problem II: Force on Current Carrying conductor of length  $l$ ; Cross. Sec  $A$ , charge density  $n_e$  each electron feels  $\vec{F}_B = (-e)[\vec{v}_D \times \vec{B}]$



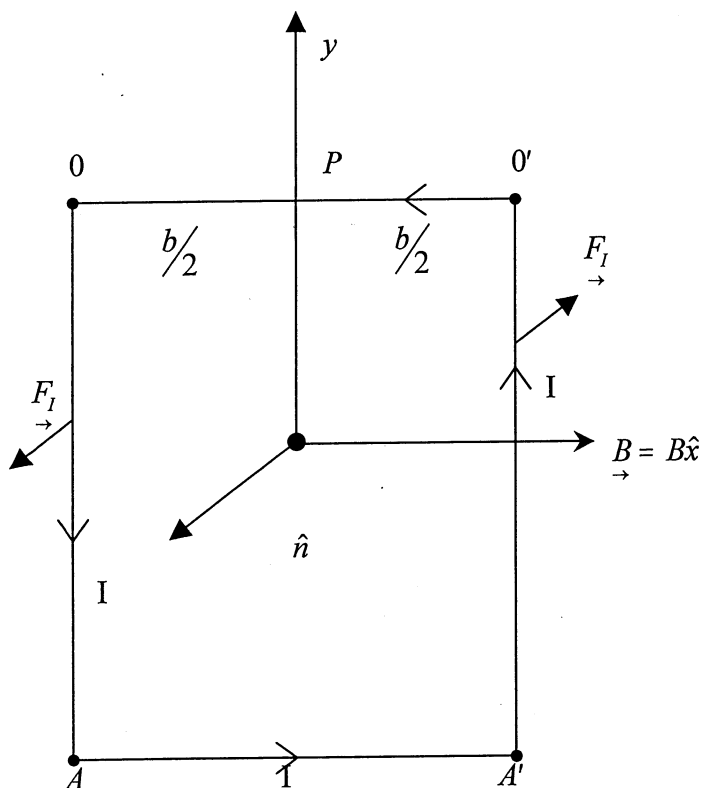
# of electrons =  $n_e A \Delta l$

so total force on conductor  $\vec{F}_I = n_e A_e \Delta l [\vec{v}_D \times \vec{B}]$

electrons constrained to move along  $\Delta l$  so  $\vec{F}_I = n_e A_e v_D [\Delta l \times \vec{B}] = I \Delta l \times \vec{B}$

Problem III

Rectangular loop of wire suspended in a  $\vec{B}$ -field with current in loop as shown, start with loop in  $xy$ -plane, at  $t=0$ .



$$\vec{F}_I = I l B \hat{z} \text{ on } 0A$$

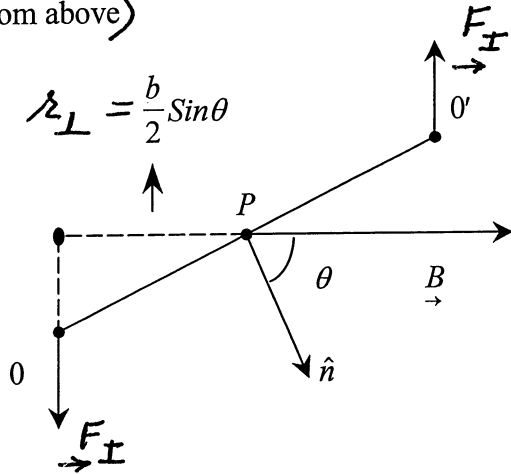
$\vec{F}_I = -I l B \hat{z} \text{ on } 0'A'$

$$\vec{F}_I = -I l B \hat{z} \text{ on } 0'A'$$

Net force is zero. However, torque is given by

Torque  $\rightarrow \tau = IlB\frac{b}{2}\hat{y} + IlB\frac{b}{2}\hat{y}$  (1)  
 $= IlBb\hat{y}$

Rotate (look from above)

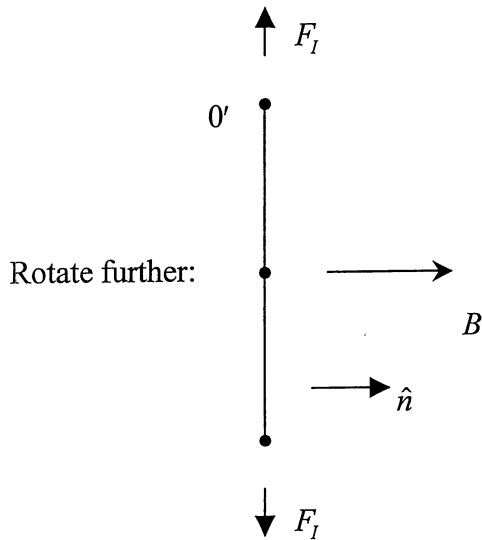


$$\vec{\tau} = [\vec{r} \times \vec{F}]$$

$$\tau = r_{\perp} F$$

Note I and B still at right angles to one another,  $F_I$  does not change but now  $r_{\perp} = \frac{b}{2} \sin \theta$ .

[Direction of  $\hat{n}$  also fixed by right hand rule]  $\tau = IlBb \sin \theta \hat{y}$  (2)



$$\tau = 0$$
 (3)

$$\begin{aligned} \hat{x} \times \hat{y} &= \hat{z} \\ \hat{y} \times \hat{z} &= \hat{x} \\ \hat{z} \times \hat{x} &= \hat{y} \end{aligned}$$

Equations (1), (2), (3) combine to give  $\tau = Ilb\hat{n} \times \vec{B}$

Define Magnetic (Dipole) moment  $\vec{\mu} = I \vec{b} \hat{n} = IA \hat{n}$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

If the coil has  $N$  turns  $\vec{\mu} = IAN \hat{n}$

Note: The top ( $OO'$ ) and bottom ( $AA'$ ) wires have equal and opposite Forces. They will make the coil out of shape but have no other effect.

### Generation of B-field

We have seen that a stationary charge experiences a force in an  $\vec{E}$ -field and a stationary charge creates a (coulomb)  $\vec{E}$ -field. Now we know that a moving charge experiences a force in a  $\vec{B}$ -field so it is natural to expect that a moving charge will generate a  $\vec{B}$ -field. This is indeed the content of the so-called Biot Savart Law.

$$\vec{B}(r) = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \vec{r}}{r^3}$$

where  $\mu_0 = 4\pi \times 10^{-7} \frac{T \cdot m}{A}$  is universal constant. This equation has the immediate consequence that for a current  $I$  in a conductor of length  $\Delta l$ .

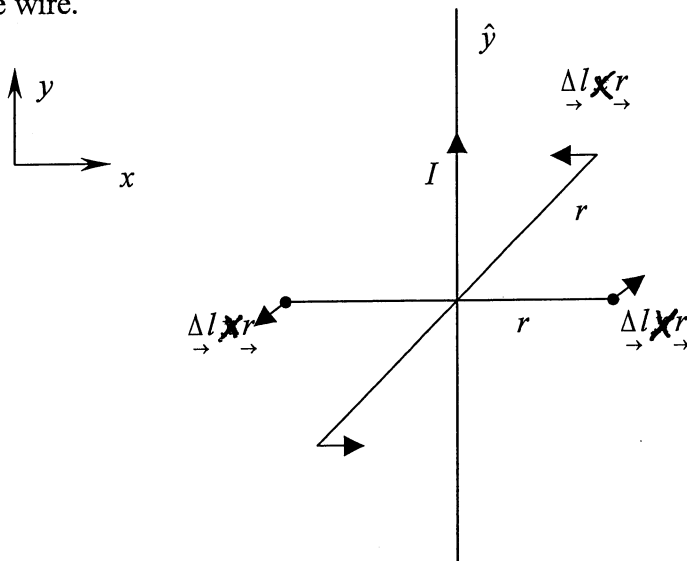
$$\vec{B}(r) = \frac{\mu_0}{4\pi} I \frac{\Delta \vec{l} \times \vec{r}}{r^3}$$

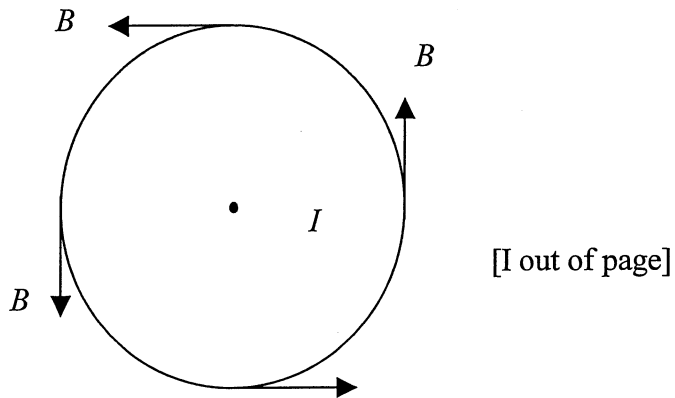
We will not use these equations in detail.

### CASES OF SPECIAL INTEREST.

Single current –  $I$  in a long wire: What can we say about  $\vec{B}$ -field at a distance  $r$  from the wire?

Notice that  $\Delta l \parallel \hat{y}$ . And the vector  $\Delta \vec{l} \times \vec{r}$  is perpendicular to  $\vec{r}$  so we can say that  $\vec{B}$  must curl around the wire.





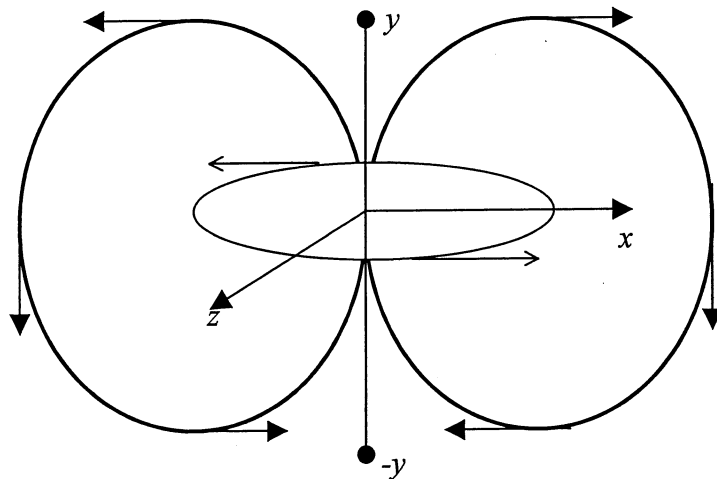
Looking end on (see picture) we have cylindrical symmetry so  $\vec{B}$  can be a function of  $r$  only. It

turns out that  $B = \frac{\mu_0 I}{2\pi r}$

so,  $B = \frac{\mu_0 I}{2\pi r} \hat{\phi}$

Thus,  $\vec{B}$  is said to be Azimuthal,  $\hat{\phi}$  is the direction which curls around I. [check with the sheet on right hand rules].

Next, take the wire and make a circular loop out of it, put it in  $xz$ -pl. with center at the origin.  
What is the  $\vec{B}$ -field at  $y$  or  $-y$ ?



The  $\vec{B}$ -field lines are shown schematically, it turns out that

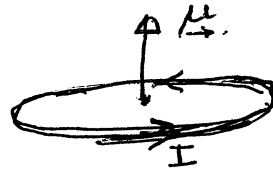
$$\vec{B}(y) = \vec{B}(-y) = \frac{\mu_0}{4\pi} \frac{2I\pi a^2}{(a^2 + y^2)^{3/2}} \hat{y}$$

Once again, we encounter  $I\hat{n}$  so we can write using magnetic (dipole) moment

$$\vec{B}(y) = \frac{\mu_0}{4\pi} \frac{2\vec{\mu}}{(a^2 + y^2)^{3/2}}$$

Far away from  $\mu, y \gg a$

$$\vec{B}(y) = \frac{\mu_0}{4\pi} \frac{2\vec{\mu}}{y^3} \leftarrow \text{Magnetic Dipole}$$

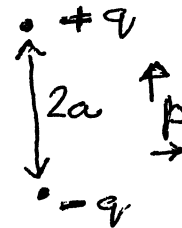


Recall that for an Electric Dipole

$$\vec{p} = 2qa\hat{y}$$

and the  $\vec{E}$ -field at  $y$  is

$$\vec{E}(y) = \frac{1}{4\pi\epsilon_0} \frac{4qa\hat{y}}{(y^2 - a^2)^2}$$



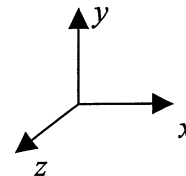
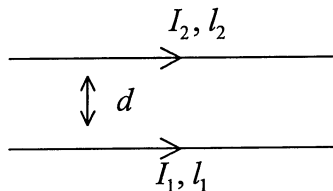
so that at  $y \gg a$

$$\vec{E}(y) = \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{y^3} \leftarrow \text{Electric Dipole}$$

*very imp.*

However, there is a major difference here: along  $y$  the magnetic dipole has no "size" while electric dipole has length ( $2a$ ). You can split the latter but not the former. This has the extremely important consequence that whereas electric-field lines start at  $+q$  and end at  $-q$ , magnetic field lines close on themselves there is no beginning and no end.

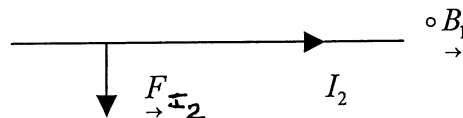
Current-Current force



Two wires of lengths  $l_1, l_2$  carry currents  $I_1, I_2$ . Separation  $d$  along  $y$ , wires parallel to  $x$ . Force on  $I_2$  due to  $I_1$ . To calculate this first. Write  $\vec{B}_1$  at location of  $I_2$

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi d} \hat{z}$$

So  $I_2$  is located in  $\vec{B}_1$  :  $\vec{F}_{I_2}$  looks like

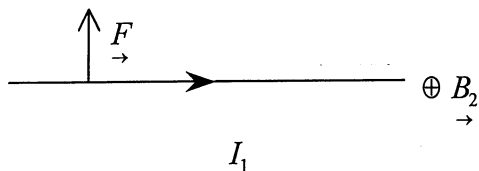


and is given by

$$\begin{aligned} \vec{F}_{I_2 I_1} &= I_2 \Delta l_2 \times \vec{B}_1 \\ &= -\frac{\mu_0 I_1 I_2 l_2}{2\pi d} \hat{y} \end{aligned}$$

Force is attractive  
Force on  $I_1$  due to  $I_2$

$$\begin{aligned} \vec{B}_2 &= \frac{-\mu_0 I_2}{2\pi d} \hat{z} \\ \vec{F}_{I_1 I_2} &= I_1 \Delta l_1 \times \vec{B}_2 \\ &= \frac{\mu_0 I_1 I_2 l_1}{2\pi d} \hat{y} \end{aligned}$$



Force is attractive. If  $l_1 = l_2 = 1 \text{ meter}$  the forces/meter  $\vec{F} = \frac{-\mu_0 I_1 I_2}{2\pi d} \hat{d}$  are an action-reaction pair.

The  $-$  sign with  $\hat{d}$  ensures force is attractive if  $I_1$   $I_2$  parallel  $\rightarrow$  and repulsive when they are anti-parallel  $\rightarrow$   $\leftarrow$ . [You will do an expt. to check this equation]

Incidentally, this is a very fundamental equation as it is used to define the unit of current- The Ampere. That is,

df  
 $I_1 = I_2 = 1 \text{ amp}$

and  $d = 1 \text{ meter}$

Force per meter is  $2 \times 10^{-7} \text{ N}$   $\left( \frac{\mu_0}{2\pi} \right)$

And the claim is that I should be regarded as a "DIMENSION" in place of  $Q$ .

So we can write  $L, T, M, \theta, [IT]$

rather than

$L, T, M, \theta, Q$