THE FIELD EQUATIONS [SO FAR]

I. GRAVITATION:

$$\underset{\rightarrow}{F_G} = m\underset{\rightarrow}{G_F} \qquad (1)$$

$$\underset{\rightarrow}{G_F} = -\frac{GM}{r^2}\hat{r} \qquad (2)$$

i) If a mass m experiences a force (with no visible entity applying the force) there must be a gravitational field present at that point. [Eq. (1)]

ii) A point mass sitting at r=0, produces a gravitational field $\underset{\rightarrow}{G_F}$ at $\underset{\rightarrow}{r}$ [Eq. (2)]

II. COULOMB $\underset{\rightarrow}{E}$-FIELD:

$$\underset{\rightarrow}{F_E} = q\,\underset{\rightarrow}{E} \qquad (3)$$

$$\underset{\rightarrow}{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \qquad (4)$$

i) If a point charge q experiences a force (with no visible entity applying the force) there must be an $\underset{\rightarrow}{E}$-field present [Eq. (3)]

ii) A coulomb $\underset{\rightarrow}{E}$-field is generated by a stationary charge. A point charge Q located at r=0 creates a coulomb $\underset{\rightarrow}{E}$-field at $\underset{\rightarrow}{r}$ according to [Eq. (4)].

iii) A positive charge acts as a "source" for coulomb $\underset{\rightarrow}{E}$, a negative charge acts as a "sink". Hence GAUSS' Law. Total flux of $\underset{\rightarrow}{E}$ through a closed surface is determined solely by the charges in the enclosed volume.

$$\boxed{\Sigma_c\, \underset{\rightarrow}{E}\bullet\Delta\underset{\rightarrow}{A} = \frac{1}{\varepsilon_0}\Sigma \mathrm{Q}i} \qquad \mathrm{I}$$

III. $\underset{\rightarrow}{B}$-FIELD

$$\underset{\rightarrow}{F_B} = q[\underset{\rightarrow}{v}\times \underset{\rightarrow}{B}] \qquad (5)$$

$$\underset{\rightarrow}{F_I} = I[\Delta\underset{\rightarrow}{l}\times \underset{\rightarrow}{B}] \qquad (6)$$

$$\underset{\rightarrow}{B_I} = \frac{\mu_0 I}{2\pi r}\hat{\phi} \qquad (7)$$

i) if a charge q traveling with velocity $\underset{\rightarrow}{v}$ experiences a force perpendicular to $\underset{\rightarrow}{v}$, there must be a $\underset{\rightarrow}{B}$-field present, [Eq. (5)]

ii) if a current carrying conductor of length $\Delta\underset{\rightarrow}{l}$ experiences a force perpendicular to $\Delta\underset{\rightarrow}{l}$ it must be located in a $\underset{\rightarrow}{B}$-field [Eq. (5)]

iii) A current I generates a $\underset{\rightarrow}{B}$-field which circulates around I [Eq. (7)].

Consequently, Ampere's Law: circulation of $\vec{B}$ around a closed loop is determined by currents threading the surface on which the loop is drawn

$$\boxed{\Sigma_c \vec{B} \bullet \Delta \vec{l} = \mu_0 \Sigma I_i} \qquad \text{II}$$

Note: Only currents within the loop count.

iv) Elementary Generators of a $\vec{B}$-field are magnetic dipoles which have no spatial extent. That is, if you wish to think in terms of sources and sinks then sources and sinks are at the same point. There are no Magnetic Monopoles. Gauss' law for ~~the~~ $\vec{B}$-field says that total flux of $\vec{B}$ through any closed surface must always be equal to zero.

$$\boxed{\Sigma_c \vec{B} \bullet \Delta \vec{A} \equiv 0} \qquad \text{III}$$

IV. NON-COULOMB $\vec{E}$ ($\vec{E}_{NC}$)

If flux of $\vec{B}$ varies with time, a non-coulomb $\vec{E}$ will appear in every loop encircling the region where $\varnothing_B$ is changing.

The sense of $\vec{E}_{NC}$ must be such as to oppose the change in $\varnothing_B$ that gave rise to $\vec{E}_{NC}$. Hence, Lenz's Law (Farady's Discovery of $\vec{E}_{NC}$):

Circulation of $\vec{E}_{NC}$ around a closed loop is determined by the time rate of change of the flux of $\vec{B}$ through the area inside the loop.

$$\boxed{\Sigma \vec{E}_{NC} \bullet \Delta \vec{l} = -\frac{\Delta \varnothing_B}{\Delta t}} \qquad \text{IV}$$

The "minus" sign on the right is crucial. It ensures that the sense of $\vec{E}_{NC}$ is to oppose the change in $\varnothing_B$.

Also, $\sum_C \vec{E}_{NC} \cdot \Delta \vec{l} = \mathcal{E}$

the INDUCED emf in the loop.