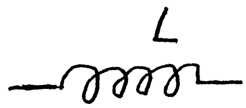
Devices: L-R CIRCUIT

We already know that a
Resistor: represents that it costs energy to transport charge through a conductor. Establish a current i, a voltage drop v appears across R such that

$$R = \frac{v}{i}$$

Faraday's discoveries allow us to define a new device:
Inductor: (L) i causes $\vec{B}$

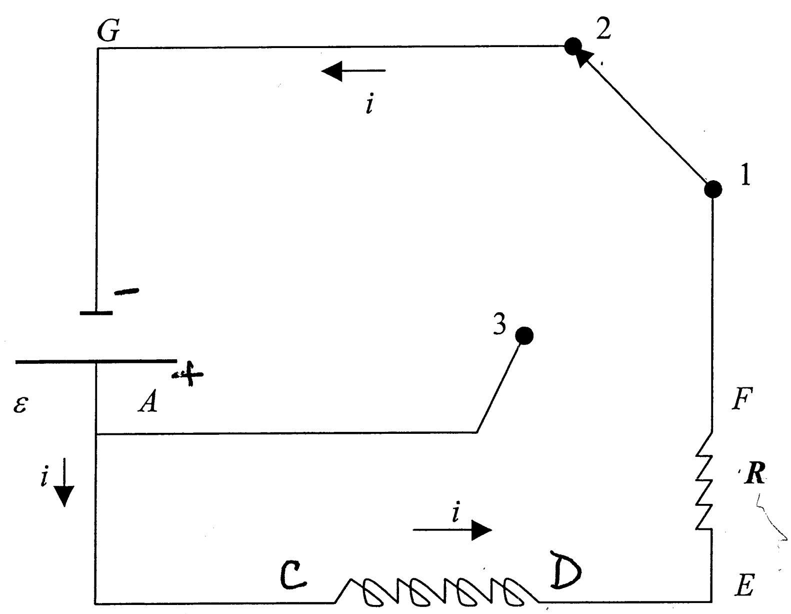
$$\frac{\Delta i}{\Delta t} \text{ causes } \frac{\Delta \vec{B}}{\Delta t} \text{ causes } \frac{\Delta \Phi_B}{\Delta t} \text{ causes } (-\varepsilon)$$



L is defined by $L = \dfrac{-\varepsilon}{\left(\Delta i / \Delta t\right)}$

And we showed in class that the self Inductance of a solenoid is $L = \mu_0 n^2 V$, where
n= # of turns per meter
V= volume of solenoid

Now we want to study what happens when we connect our 3-devices, Battery, R and L in the circuit shown.
At t=0, put the switch as shown (1→ 2). The battery immediately wants to set up a current.
However, L immediately objects* by setting up a "back" *emf*, which will oppose ε, $\varepsilon_B = -L\left(\dfrac{\Delta i}{\Delta t}\right)_0$

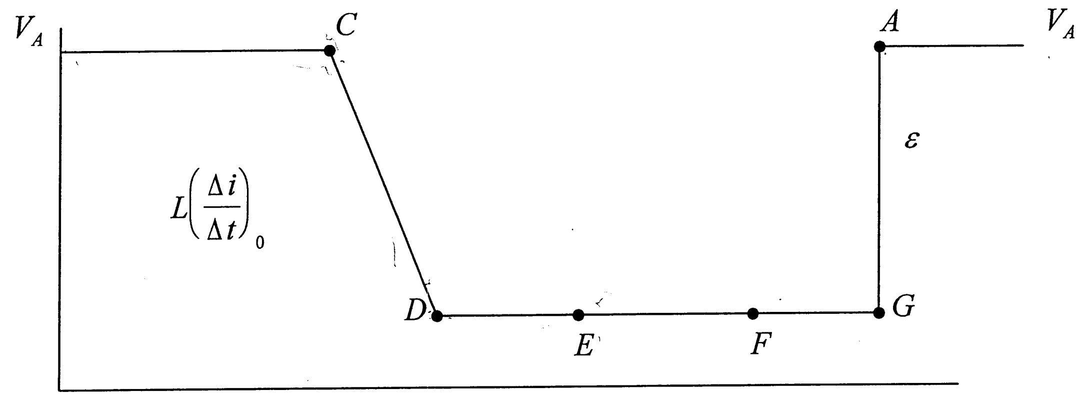


* The "minus" sign in the equation ensures that an Inductor always opposes any change in current through it.

Thus it will take time for the current to grow.

Step 1 at t=0, i=0, No potential drop across R. As a function of position on the circuit the potential values are

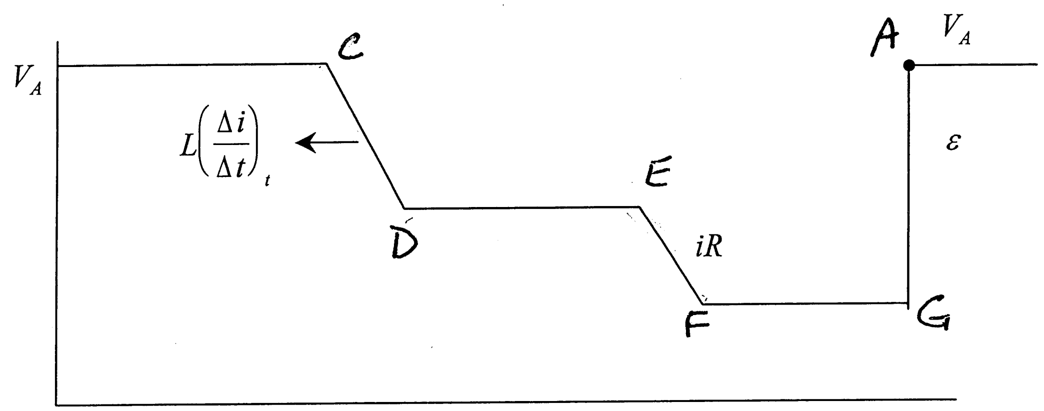
t=0
i=0



$$\varepsilon - L\left(\frac{\Delta i}{\Delta t}\right)_0 = 0$$

Step 2 A little time later i has grown some so now $v_R = iR$ and the potentials become

a little later

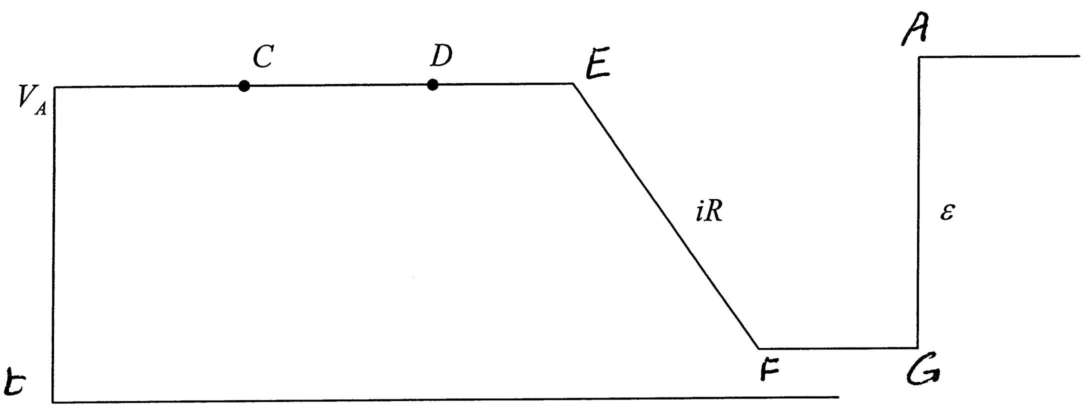


$$\varepsilon - L\left(\frac{\Delta i}{\Delta t}\right)_t = iR$$

Clearly, $\left(\frac{\Delta i}{\Delta t}\right)_t < \left(\frac{\Delta i}{\Delta t}\right)_0$ Rate of growth has slowed down.

<u>Step 3</u> A long time later i has grown to its full value, i stops changing with time, $\frac{\Delta i}{\Delta t} \to 0$ so no more back *emf* from L. Potential variation looks like.

Long time later i is constant



$$i(t \to \infty) = \frac{\varepsilon}{R}$$

<u>Note:</u>

$V_L = \varepsilon_B$ is ε at t=0, i=0

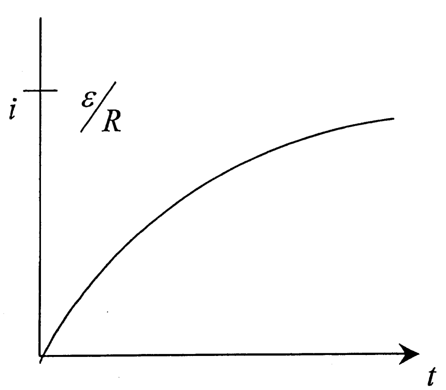
$V_L = \varepsilon_B$ is 0 at $t \to \infty$, $i = \frac{\varepsilon}{R}$

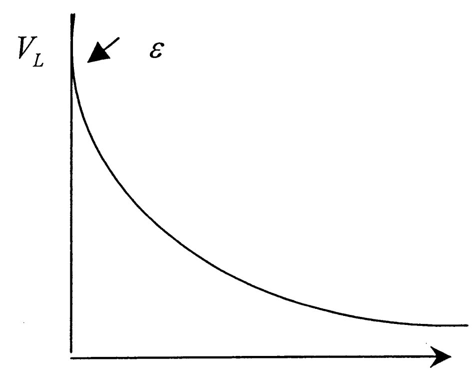
The time variations are controlled by the characteristic time

$$\tau = \frac{L}{R} \text{ (verify that this has dimensions of time)}$$

and look like

GROWTH OF i





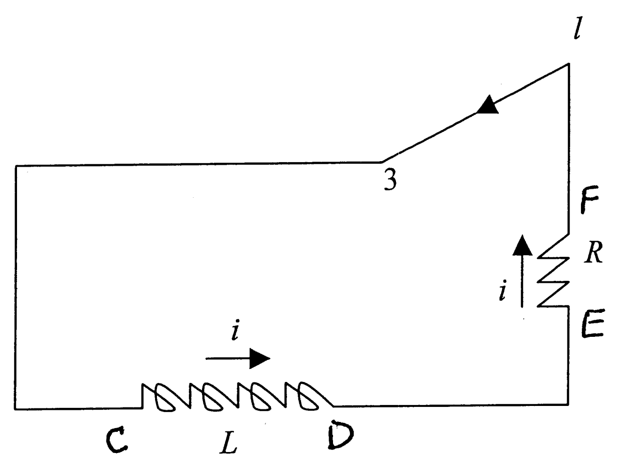
$$i = \frac{\varepsilon}{R}\left[1 - e^{-Rt/L}\right]$$

$$V_L = \varepsilon e^{-Rt/L}$$

$V_R = iR$

$= \varepsilon\left[1 - e^{-Rt/L}\right]$.

After this long time $i = \frac{\varepsilon}{R}$ and a magnetic field has been established in L and stores $\frac{B^2}{2\mu_0}$ of energy per m^3. So now if we move the switch so 1→3 the circuit becomes



No Battery in Circuit. Magnetic Energy Stored in L is source of Current!

Let us start the clock again:

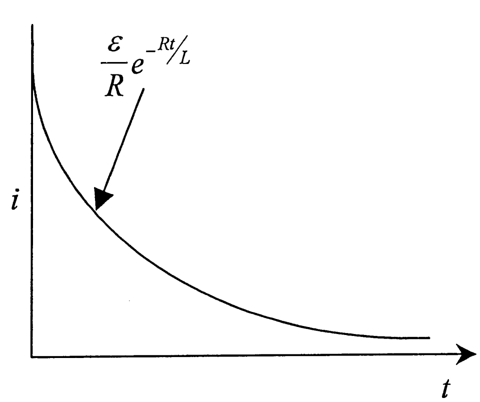
Step 1 t=0, $i = \frac{\varepsilon}{R}$ and the inductor wants to keep i going. Since $\frac{\Delta i}{\Delta t}$ is now –ive, V_L will be +ive and immediately jump to ε.

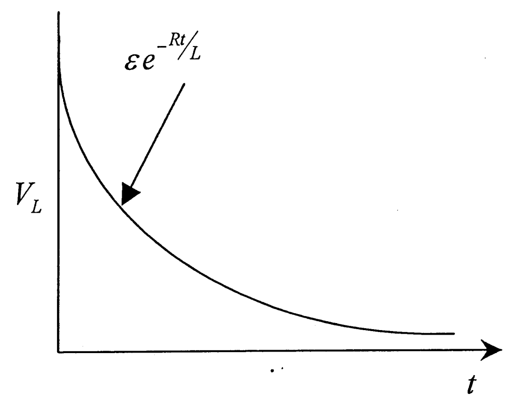
Step 2 As stored energy in L becomes smaller $\left|\frac{\Delta i}{\Delta t}\right|$ reduces with time.

Step 3 Long time later, all of the stored energy has been used up: $\frac{\Delta i}{\Delta t} \to 0$, $i \to 0$.

The time variations are

Decay of i





To Summarize:

i) L keeps i=0 when switch first closed

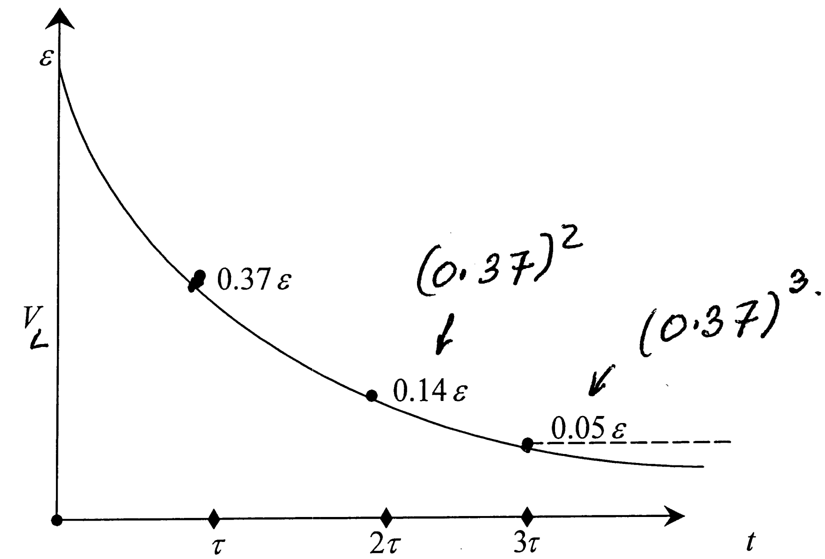
ii) $V_L \rightarrow 0 \; as \; t \rightarrow \infty$

iii) in an L-R circuit potential precedes current.

QUESTION: Why does τ depend on both L and R?

Exponential functions.

$V_L = \varepsilon e^{-t/\tau}$



$V_R = \varepsilon \left[1 - e^{-t/\tau}\right]$