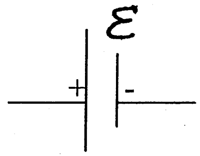
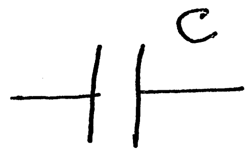
DEVICES – AC CIRCUITS



Battery Source of Coulomb $\vec{E}$ -field
Output is *emf*: ε

Capacitor: Container for $\vec{E}$ -field $C = \frac{Q}{V}$

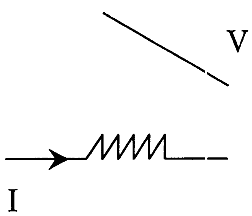


Potential Energy $U_E = Q^2/2C$

$\eta_E =$ Energy stored per $m^3 = \frac{1}{2} \epsilon_0 E^2$

$\epsilon_0 = 9 \times 10^{-12}\ F/m$

Resistor: Represents that it costs energy to transport charge through a conductor

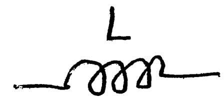


$R = \frac{V}{I}$ $\left[\vec{J} = \sigma \vec{E}\right]$

Power loss $P = I^2 R = \frac{V^2}{R}$

Inductor: A time varying current causes a Non-Coulomb $\vec{E}$ -field, or an induced *emf*, $L = \frac{-\varepsilon}{\left(\Delta i / \Delta t\right)}$

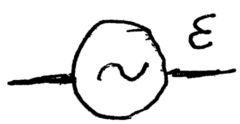
Container for $\vec{B}$ -field, Potential Energy $U_B = \frac{1}{2} L i^2$



$\eta_B =$ Energy stored per $m^3 = \frac{B^2}{2\mu_0}$

A.C. Generator: Wire loops of area A rotated at ω rad/s in a $\vec{B}$ -field. Generates non-coulomb $\vec{E}$ -field in the loops, produces an *emf*: $\varepsilon = \omega NBA\ Sin(\omega t)$

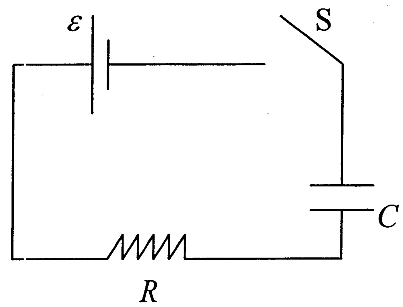
Where N= # of turns in the loop. Hence *ac-generator*

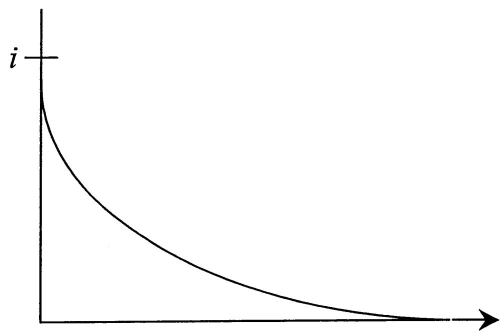


$\Phi_B = NBA\ Cos(\theta)$
where θ is angle
between $\hat{n}$ and $\vec{B}$
Rotn by ω rad/s makes $\theta = \omega t$, $\Phi_B = NBA\ Cos\ \omega t$
so $\frac{\Delta \Phi_B}{\Delta t} = -NBA\omega\ Sin\ \omega t$, $\mathcal{E} = -\frac{\Delta \Phi_B}{\Delta t} = \omega NBA\ Sin\ \omega t$

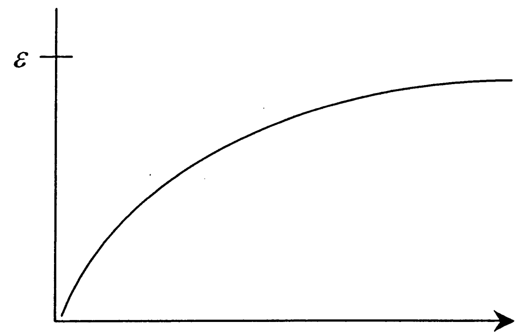
CIRCUITS

I. RC with battery, close switch at t=0, current flows immediately, potential across C appears later $\varepsilon = q/C + iR$





$$i = \frac{\varepsilon}{R} e^{-t/RC}$$



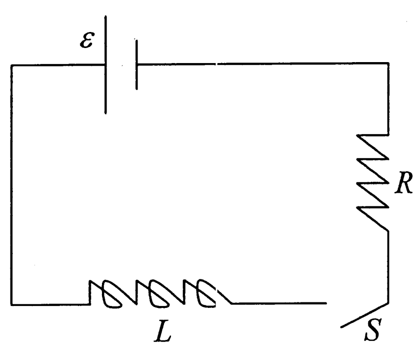
$$v_c = \varepsilon \left[1 - e^{-t/RC}\right]$$

$$\tau = RC$$

<u>N.B.</u> Current first, voltage later.

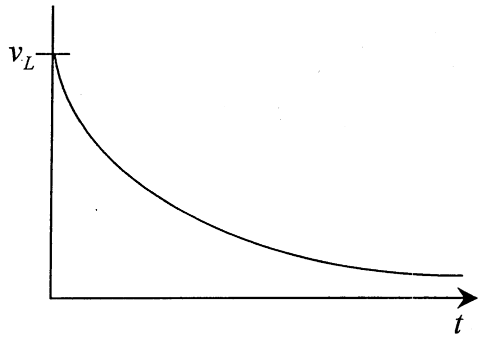
II. <u>RL With Battery</u>

Close switch at t=0, v_L immediately jumps to $|\varepsilon|$

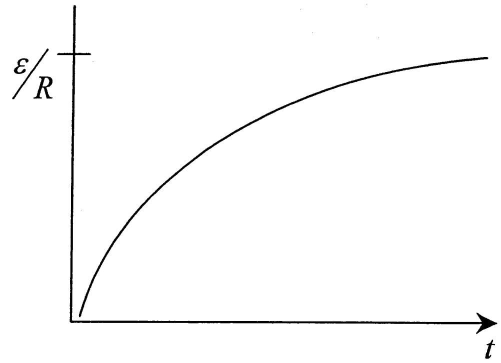


Current builds slowly.

$$\varepsilon = L\frac{\Delta i}{\Delta t} + iR$$



$$v_L = \varepsilon e^{-Rt/L}$$



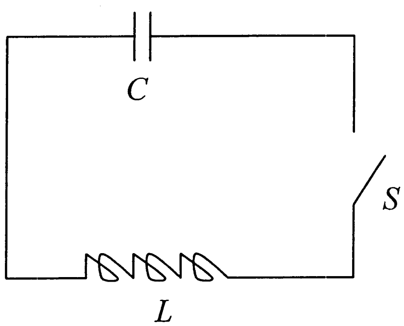
$$i = \frac{\varepsilon}{R}\left[1 - e^{-Rt/L}\right]$$

$$\tau = \frac{L}{R}$$

Note: Voltage First, Current Later.

III. LC-Circuit: Undamped Oscillator

First charge C to Q_0. Close switch at t=0. Energy stored in capacitor $U_E = \frac{Q_0^2}{2C}$

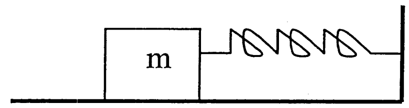


Charge begins to flow. Total Energy = (Energy in $\vec{E}$-field) + (Energy in $\vec{B}$-field)
= (Energy in C) + (Energy in L)

$$\frac{Q_0^2}{2C} = \frac{q^2}{2C} + \frac{1}{2}L\left(\frac{\Delta q}{\Delta t}\right)^2$$

Recognize, similarity to spring-mass oscillator

$$\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}m\left(\frac{\Delta x}{\Delta t}\right)^2$$



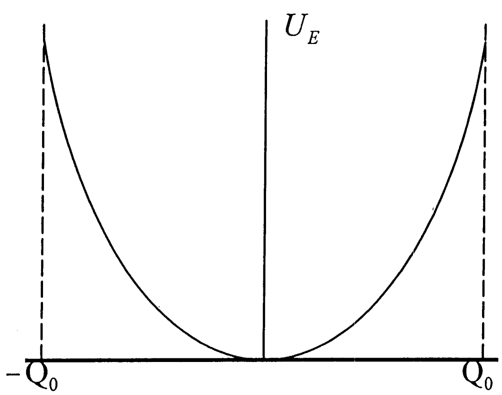
$x \rightarrow q$ $\qquad$ $x = A\ Cos\,\omega t$

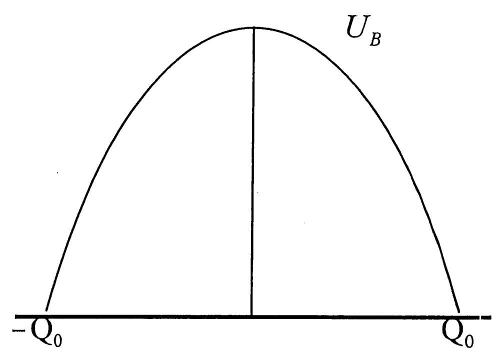
$m \rightarrow L$ $\qquad$ $\omega_0 = \sqrt{\dfrac{k}{m}}$

$k \rightarrow \dfrac{1}{C}$

Now $\qquad$ $q = Q_0\ Cos\,\omega_0 t$

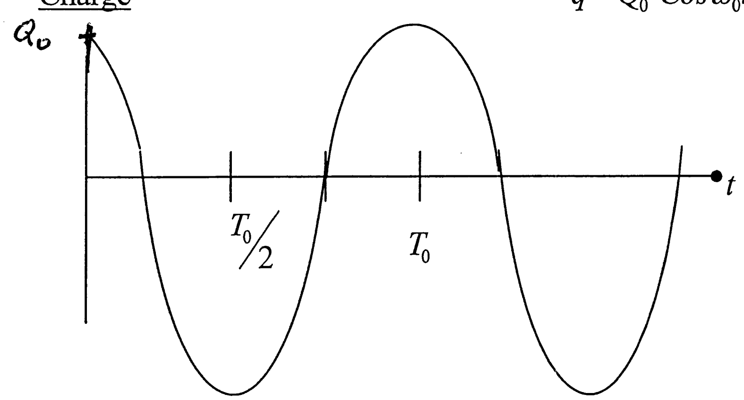
$$\omega_0 = \frac{1}{\sqrt{LC}}$$



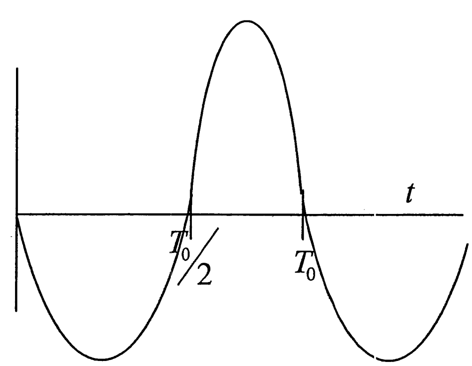


$\vec{E}$ -field collapses giving rise to $\vec{B}$ -field and vice versa.

<u>Charge</u> $\qquad$ $q = Q_0\ Cos\,\omega_0 t$



<u>Current</u>

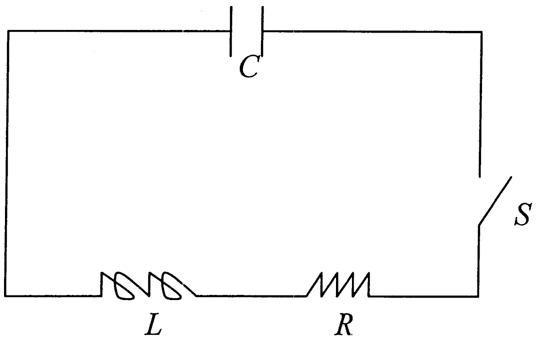


$i = -Q_0\omega_0\ Sin\,\omega_0 t$

Period $T = \dfrac{2\pi}{\omega_0}$

IV. LCR-CIRCUIT: DAMPED OSCILLATOR.

At t=0, charge C to Q_0 close switch. Now driving i through R costs i^2R per second.



So $\left(\frac{q^2}{2C}+\frac{1}{2}Li^2\right)$ IS NOT CONSTANT

$$\frac{\Delta\left(\frac{q^2}{2C}+\frac{1}{2}Li^2\right)}{\Delta t} = -\,i^2R$$

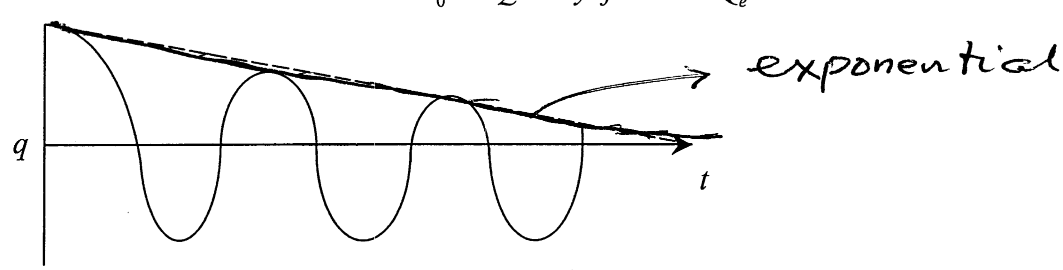
-ive sign on right because energy is being lost (R is getting warmer).

Now $q = Q_0 e^{-Rt/2L}\, Cos\,\omega t$

$$\omega = \omega_0\left[1-\frac{1}{(2\omega_0\tau)^2}\right]^{1/2}$$

$$\tau = \frac{L}{R}$$

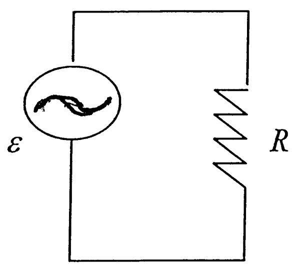
$$\omega_0\tau = Quality\ factor = Q_e$$



Note 1: smaller R, larger the duration for which the oscillations persist.
Note 2: R plays role of friction; as always energy lost goes to raise temperature. Electrical Equivalent of Heat.

CIRCUITS: AC

I. Resistor and Generator



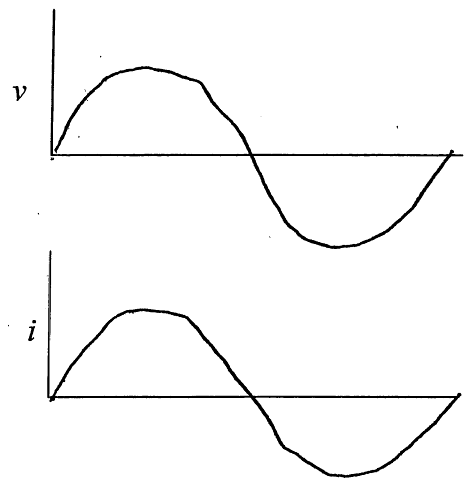
$$V_R = IR$$

so

If $\varepsilon = \varepsilon_0 \, Sin\, \omega t$

$$i = \frac{\varepsilon_0}{R} Sin\, \omega t$$

Current and voltage are in phase.



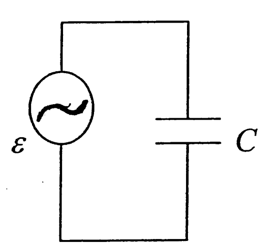
Power

$$P(t) = iv$$

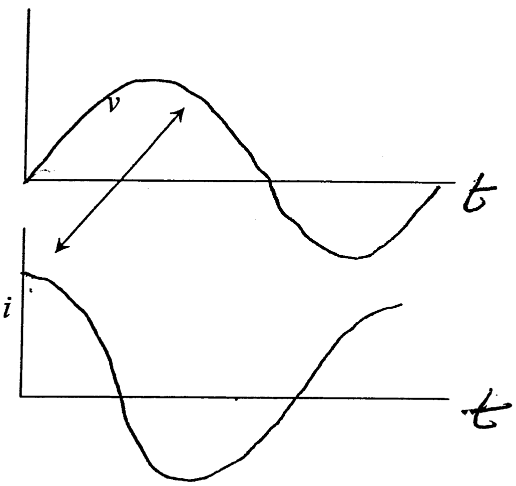
$$= \frac{\varepsilon_0}{R} Sin^2\ \omega t$$

averaged over a cycle. $< P >= \frac{\varepsilon_0^{\ 2}}{2R} = \frac{i_0 \varepsilon_0}{2} = \frac{i_0^{\ 2} R}{2}$ and the power loss is as if R was connected to a battery who $\varepsilon = \frac{\varepsilon_0}{\sqrt{2}}$. In this sense one talks of $\frac{\varepsilon_0}{\sqrt{2}}$ *and* $\frac{i_0}{\sqrt{2}}$ as root-mean-square or r.m.s. voltage and current.

II. Generator and Capacitor



now $q = Cv$



so charge and voltage in phase.

$v = \varepsilon_0 \, Sin\, \omega t$

$q = \varepsilon_0 \, Sin\, \omega t$

$$i = \frac{\Delta q}{\Delta t} = + \varepsilon_0 C \omega \, Cos\, \omega t$$

i and v are not in phase i leads v by $\pi/2$

$\frac{1}{\omega C}$ has dimensions of R and we define capacitive reactance

$$X_C = \frac{1}{\omega C}$$

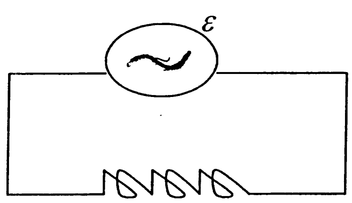
$$i = \frac{\varepsilon_0}{X_C} Cos\, \omega t$$

$$\text{Now} \quad P(t) = \frac{\varepsilon_0^2}{X_C} Sin\, \omega t \; Cos\, \omega t$$

$$\text{Average } \langle P \rangle = 0$$

No power loss on average.

III.



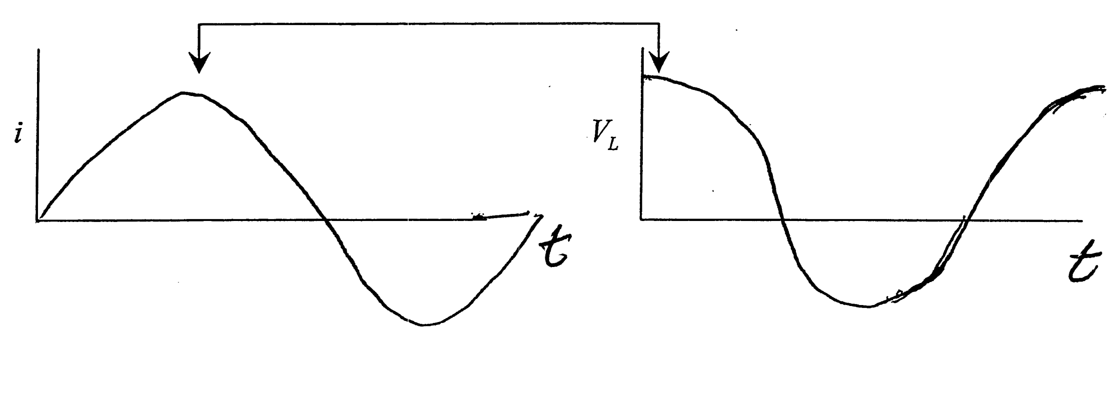
Now voltage in phase with slope of i vs t curve.

If $i = i_0 \, Sin\, \omega t$

$$\varepsilon - \frac{L \Delta i}{\Delta t} = 0$$

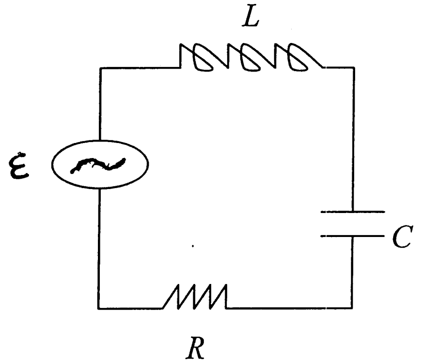
$$v_L = \frac{L \Delta i}{\Delta t}$$

$v_L = i_0 \, \omega L \, Cos\, \omega t$ and now voltage leads current by $\frac{\pi}{2}$



Again $X_L = \omega L$ and $\langle P \rangle = 0$

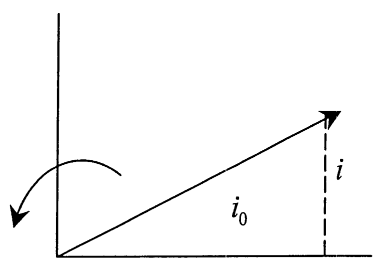
IV. Generator with all three.



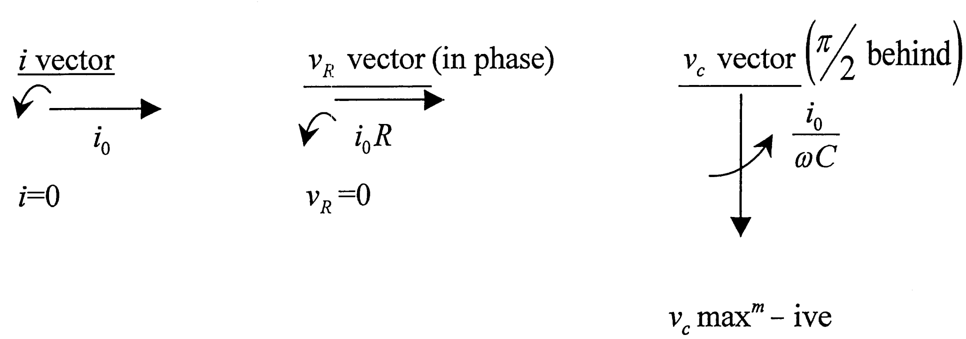
To work out behavior of L-C-R circuit we have to keep track of the relative phases in *R, C,* and *L.*

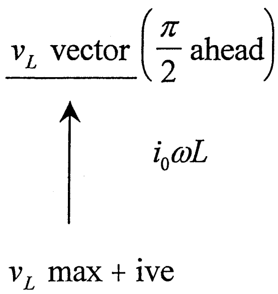
We introduce the concept of a Phasor. We have a series loop so current has to be same at all points. We begin with the current and build the voltage vector.

Take $i = i_0 \, Sin \, \omega t$ and represent it by current Phasor: vector of magnitude i_0 rotating at angular velocity ω, i.e.

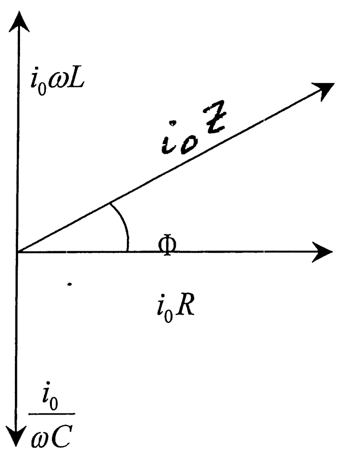


Concentrate on *t*=0





Total voltage vector



$$\varepsilon_0 = v_0 = i_0\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}$$

$$\tan\Phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$i = i_0 \, Sin\, \omega t$$
$$v = \varepsilon_0 \, Sin(\omega t + \varnothing)$$

<u>Power in Circuit</u>

$$P = iv = i_0 \varepsilon_0 \, Sin\,\omega t \; Sin(\omega t + \varnothing)$$
$$= i_0 \varepsilon_0 \left[Sin^2 \omega t \, Cos \varnothing + Sin\, \omega t \, Cos\, \omega t \, Sin \varnothing\right]$$
$$\langle P \rangle = \frac{i_0 \varepsilon_0}{2} Cos \varnothing$$

This is the concept of Power factor $Cos\varnothing$

<u>Next,</u> Define Impedance

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{wC}\right)^2}$$

then $i_0 = \frac{\varepsilon_0}{Z}$ & $Cos\varnothing = \frac{R}{Z}$

so $\langle P \rangle = \frac{\varepsilon_0^{\,2}}{2Z} Cos\varnothing = \frac{\varepsilon_0^{\,2}}{2R} Cos^2\varnothing$

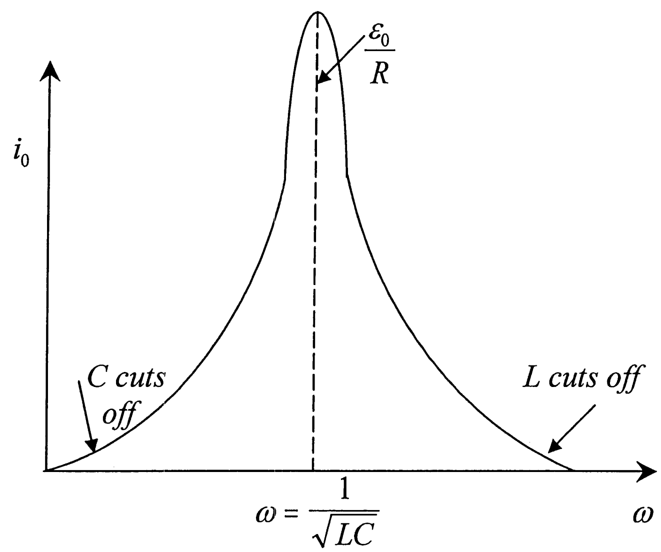
<u>RESONANCE</u> If generator frequency can be varied $i_0 = \dfrac{\varepsilon_0}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$

Gives $\quad i_0 \to 0$ when $\omega \to 0$ b/c $\dfrac{1}{\omega C} \to \infty$ [capacitor cuts off i_0]

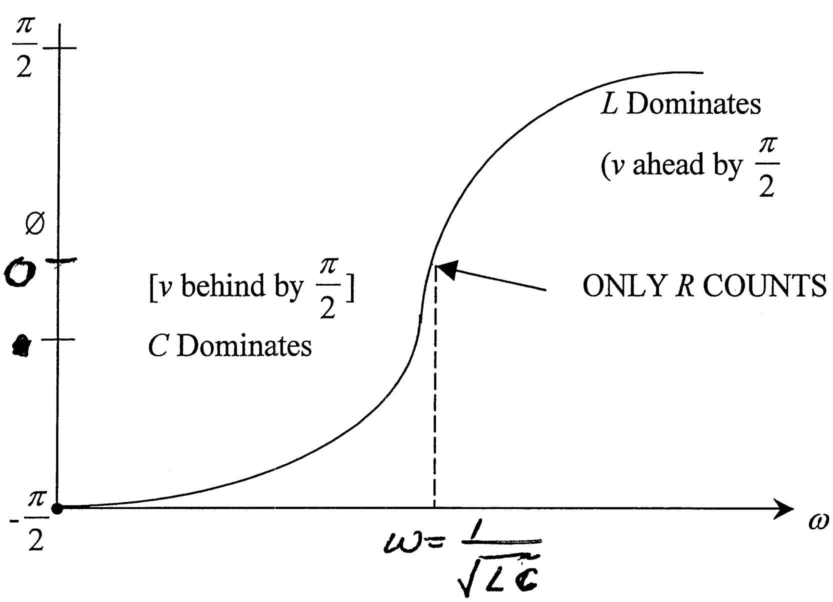
$\quad i_0 \to 0$ when $\omega \to \infty$ b/c $\omega L \to \infty$ [Inductor cuts off i_0]

$\quad i_0$ is max^m when $\omega L = \dfrac{1}{\omega C}$

This is the phenomenon of Resonance



The phase difference ∅ is also a function of frequency



[v and i in phase R gets max^m power hence "RESONANCE"]

$$\omega = \frac{1}{\sqrt{LC}}$$

Note: Resonance occurs when Generator frequency ω is equal to natural frequency of LC Circuit $\omega_0 = \frac{1}{\sqrt{LC}}$