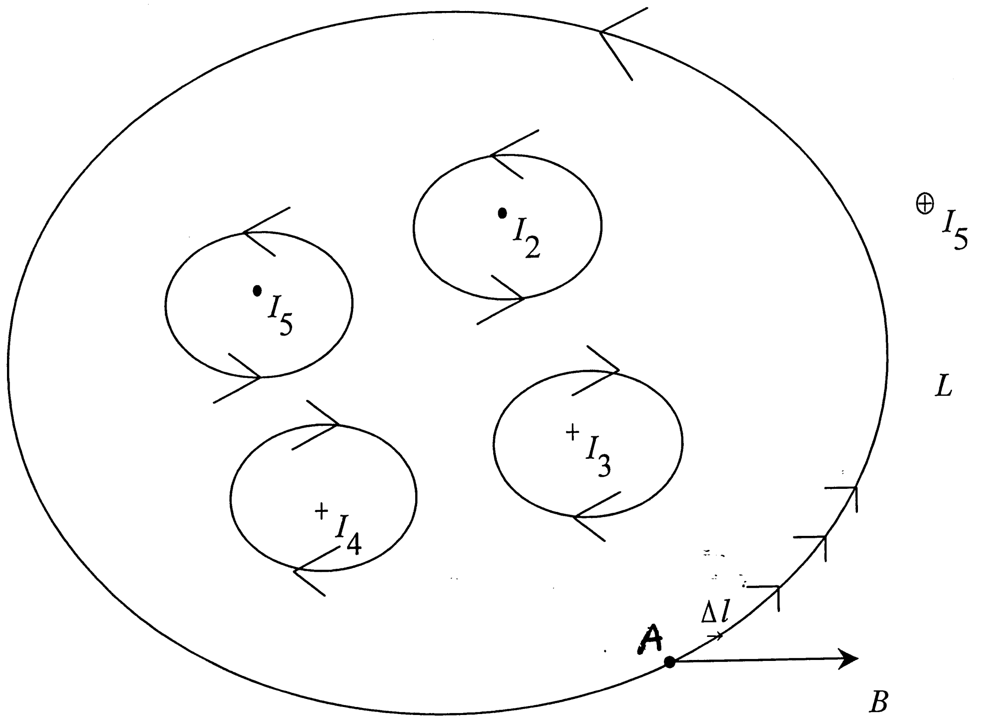
Ampere's Law - Applications

Consider the situation shown schematically in the diagram. Currents I_1, I_2, I_3, I_4, I_5 are flowing out of $(\cdot)$ or $(+)$ into the paper. The corresponding $\vec{B}$-fields swirl around their sources as shown. The main point is that the $\vec{B}$-field lines circulate around the currents. Choose a closed loop (L). Start at A, measure B, choose small stop $\Delta \vec{l}$ along loop. Calculate Dot product (component of $\vec{B}$ along $\Delta \vec{l}$ multiplied by Δl)

$$\vec{B} \cdot \Delta \vec{l} = B \Delta l \cos(\vec{B}, \Delta \vec{l}).$$

If $\vec{B} \perp \Delta \vec{l}$, $\vec{B} \cdot \Delta \vec{l} = 0$.



Rpeat this calculation at every step as shown. $\vec{B} \cdot \Delta \vec{l}$

Write out the sum

$$\Sigma_c \, \vec{B} \cdot \Delta \vec{l};$$

c: closed loop.

This sum is called circulation of $\vec{B}$ around closed loop and Ampere says that it is determined solely by currents threading through the surface on which the loop is drawn & only currents within the loop contribute, i.e. exclude I_5. The mathematical Equation is:

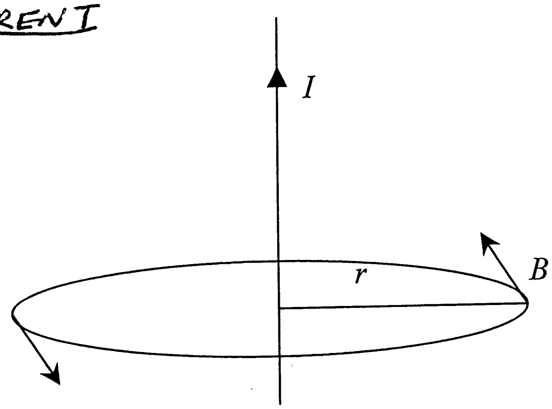
$$\Sigma_c \, \vec{B} \cdot \Delta \vec{l} = \mu_0 \, \Sigma I_i, \mu_0 = 4\pi \times 10^{-7} \, \frac{T-m}{A}$$

In words, circulation of $\underset{\rightarrow}{B}$ around a closed loop is proportional to the algebraic sum of the currents threading the ~~loop~~ surface on which the loop is drawn.

Note: As in case of Gauss' Law, Ampere's law gives circulation but not $\underset{\rightarrow}{B}$. To get $\underset{\rightarrow}{B}$ you need high symmetry!

<u>Applications</u>

1.) <u>SINGLE CURRENT</u>



<u>Single wire with current</u> I, there is cylindrical symmetry so $\underset{\rightarrow}{B}$ can be a function of r only & must encircle I. [$\underset{\rightarrow}{B}$ and $\Delta \ell$ are parallel to one another]
Appropriate loop is circle of radius r centered on the wire

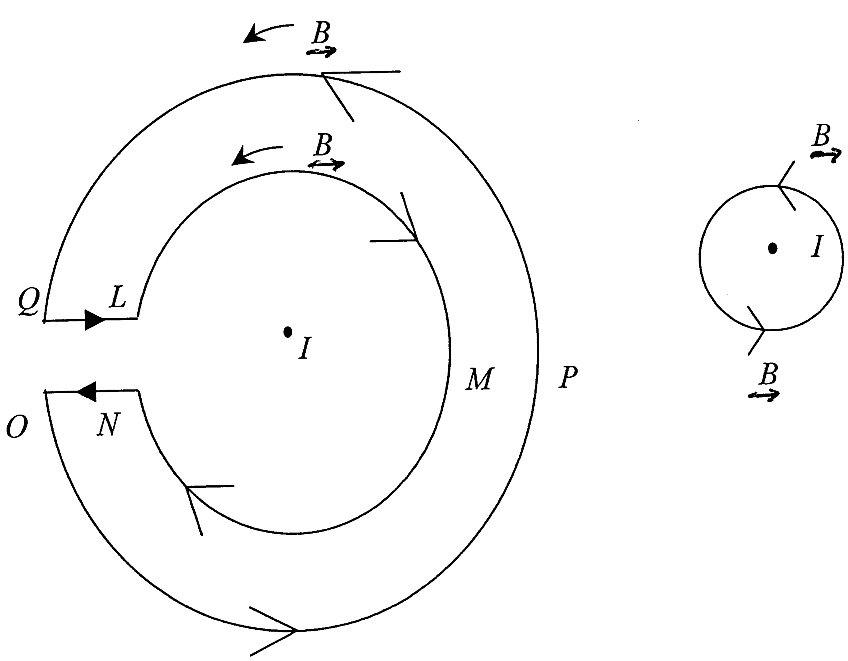
$$B \cdot 2\pi r = \mu_0 I$$

$$\text{so, } \underset{\rightarrow}{B} = \frac{\mu_0 I}{2\pi r}\hat{\varnothing}$$

as claimed previously

2.) Next, we begin by showing that if current is outside the loop it contributes nothing to the circulation. Choose $L\,M\,N\,O\,P\,Q$ with I at the center of the circles of radii r_1, and r_2

I is out of paper.



$$\underset{\rightarrow}{B}(r_1) = \frac{\mu_0 I}{2\pi r_1}\hat{\varnothing}$$

$$\underset{\rightarrow}{B} \perp \Delta \underset{\rightarrow}{l}$$

$$\underset{\rightarrow}{B}(r_2) = \frac{\mu_0 I}{2\pi r_2}\hat{\varnothing}$$

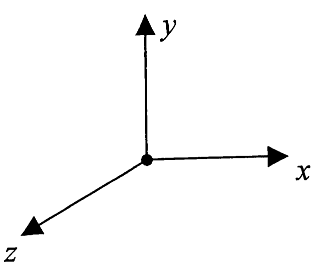
$$\sum_c \underset{\rightarrow}{B}\cdot \Delta \underset{\rightarrow}{l} = \frac{\mu_0 I}{2\pi r_1}\cdot 2\pi r_1 + O + \frac{\mu_0 I}{2\pi r_2}\cdot 2\pi r_2 + O$$

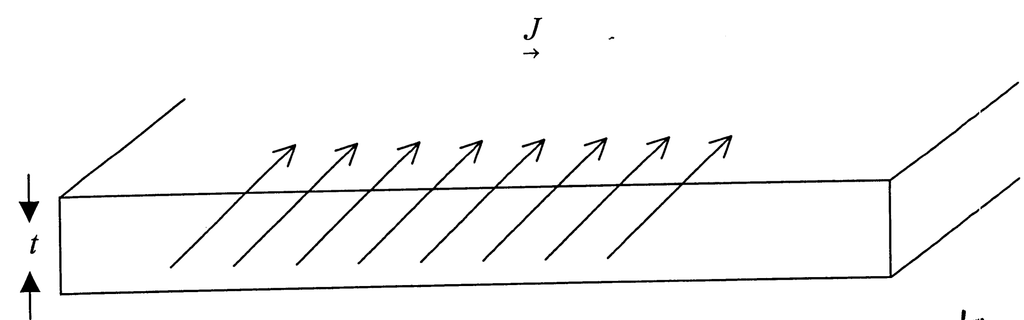
$$= L\rightarrow M\rightarrow N + N\rightarrow 0 + O\rightarrow P\rightarrow Q + Q\rightarrow L$$

$$= O$$

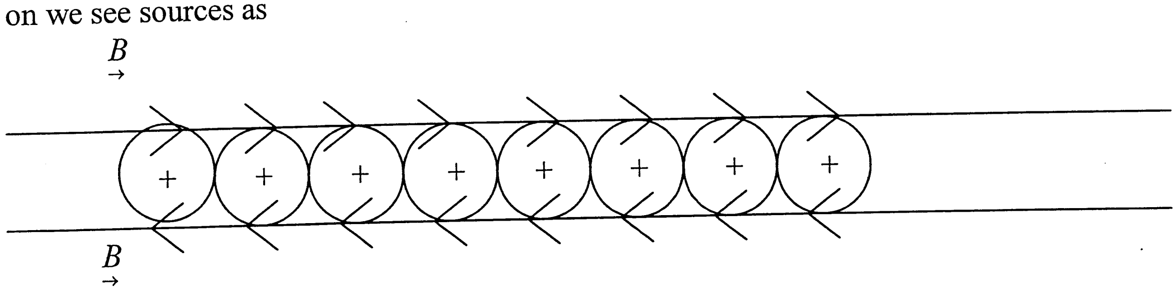
[The first term is -ive.]
$\underset{\rightarrow}{B}$ and $\Delta \underset{\rightarrow}{l}$ are opposite to one another

3.)

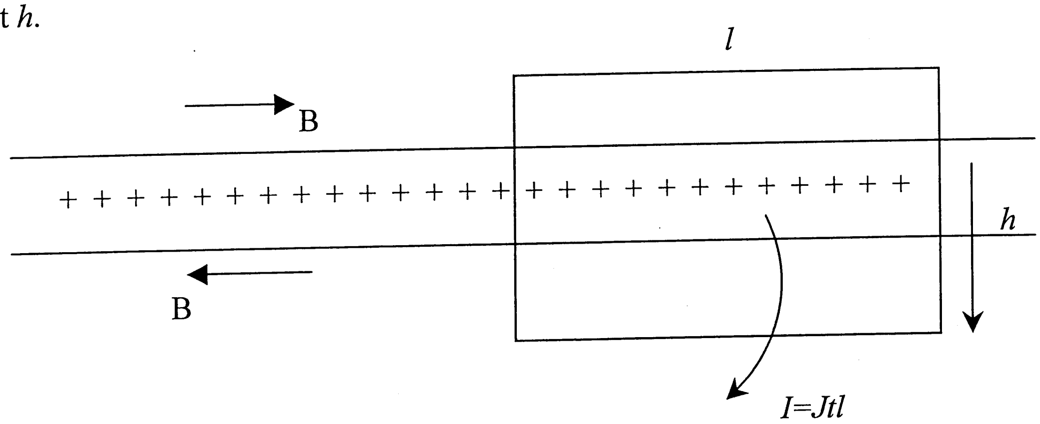




At y=0 there is a Current sheet of thickness t carrying current density $\underset{\rightarrow}{J} = -J\hat{z}$. Looking at it end-on we see sources as



and we see that y-components of $\underset{\to}{B}$ cancel out. $\underset{\to}{B} \| \hat{x}$ survives. Let us take loop of width l and height h.

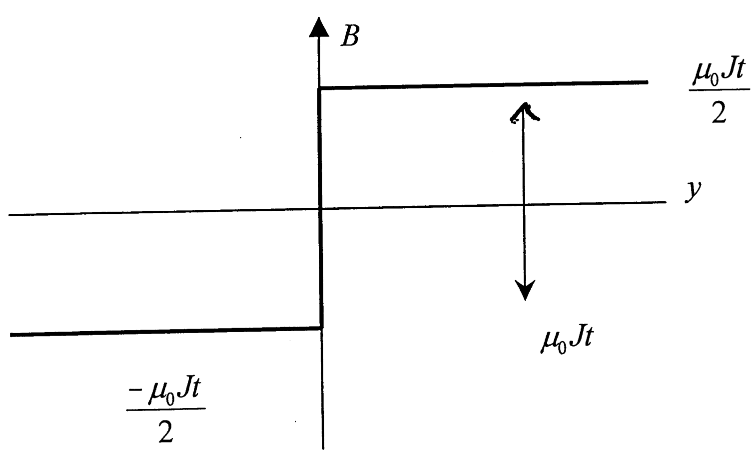


$$\Sigma_c \underset{\to}{B} \bullet \Delta \underset{\to}{l} = Bl + 0 + Bl + 0$$
$$= 2Bl$$
$$= \mu_0 Jtl$$

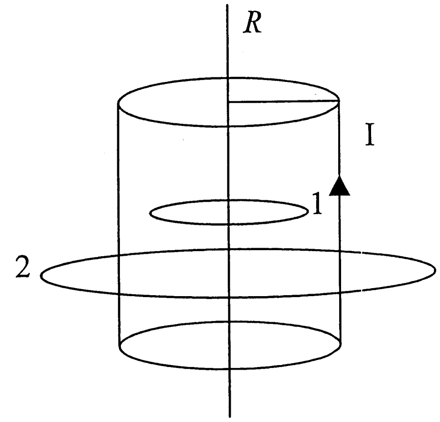
so,

$$B = \frac{\mu_0 Jt}{2} \;\&\; \underset{\to}{B} = \frac{\mu_0 Jt}{2}\hat{x} \qquad y > 0$$
$$= \frac{-\mu_0 Jt}{2}\hat{x} \qquad y > 0$$

That is, $\underset{\to}{B}$-field will jump by $\mu_0 Jt$ on crossing the current sheet from y<0 to y>0.

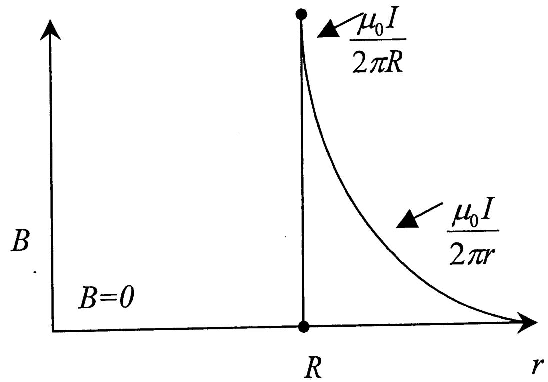


4.) <u>Hollow Cylindrical Conductor</u>- Radius R, carries uniform current. We want $\underset{\rightarrow}{B}$ at a distance r from its axis. Since there is a cylindrical symmetry we should use circles centered on the axis for our closed loop.
For $r<R$. use loop 1.



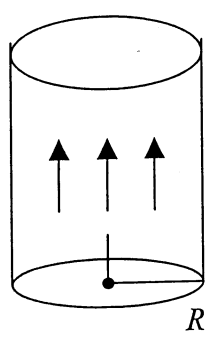
$B \cdot 2\pi r = 0$. No Current threads through loop 1.
for $r>R$
~~so~~, $B \cdot 2\pi r = \mu_0 I$, the entire current threads loop 2.

so, $\underset{\rightarrow}{B} = \frac{\mu_0 I}{2\pi r}\hat{\varnothing}$



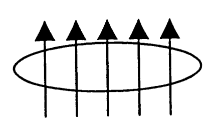
Note: if cylinder has wall thickness t: $I = J \cdot 2\pi Rt$ and field at surface would be $\mu_0 Jt$.
Again field would jump by $\mu_0 Jt$ on crossing a current sheet.

5.) <u>SOLID CYLINDRICAL CONDUCTOR</u> – with uniform current



Define $J = \frac{I}{\pi R^2}$

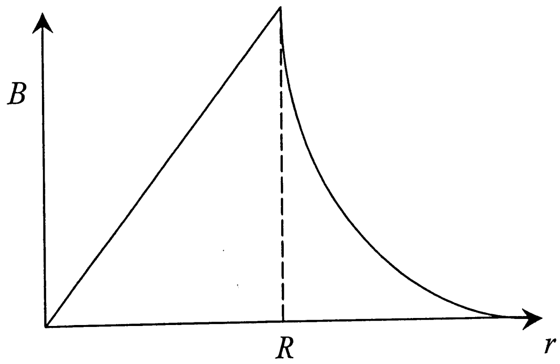
Now for $r<R$ $I = J\pi r^2$



$B \cdot 2\pi r = \mu_0 J\pi r^2$

$\underset{\rightarrow}{B} = \frac{\mu_0 Jr}{2}\hat{\varnothing}$ $\quad r < R$

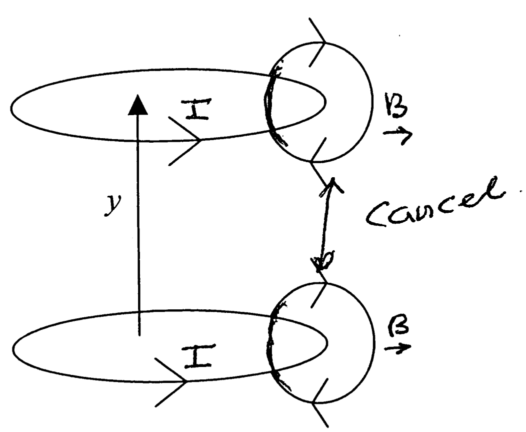
For $>R$, entire I contributes $\underset{\rightarrow}{B} = \frac{\mu_0 I}{2\pi r}\hat{\varnothing}$



6.) Solenoid: Tightly wound, small radius, length much larger than radius:

N turns, L long, $n = \frac{N}{L}$ =# of turns per meter.

Look at two neighboring turns



r-component cancels
B_y inside survives.

Long-narrow solenoid. [B field lines must come out of top, loop around and enter at bottom with no breaks or bends allowed].

$\underset{\rightarrow}{B} \approx 0$ just outside.

Take loop as shown

$$Bl = \mu_0 nIl$$
$$B = \mu_0 nI$$

For case shown $\underset{\rightarrow}{B} = \mu_0 nI\hat{y}$