

MECHANICAL WAVES (Traveling)

We begin our discussion of the wave phenomenon by considering waves in matter. The simplest definition of a wave is to call it a traveling disturbance (or equivalently, deviation from equilibrium). For instance, if you drop a stone on the surface of an undisturbed body of water you can watch the “disturbance” traveling radially out of the “point” of contact.

Formally, we can “construct” a wave in several steps. For simplicity, we take a wave traveling along x-axis.

Step 1. We need a disturbance D .

Step 2. D must be a function of x .

Step 3. D must also be a function of t .

Step 4. If x and t appear in the function in the combinations $(x \mp vt)$ the disturbance D cannot be stationary. It must travel along x with speed v .

Further,

$$(x-vt) \text{ implies } \underset{\rightarrow}{v} = v \hat{x} [\text{travel in +ive } x \text{ - direction}]$$

$$(x+vt) \text{ implies } \underset{\rightarrow}{v} = -v \hat{x} [\text{travel in -ive } x \text{ - direction}]$$

EXERCISE: Put $D = A(x - t)^2$ and show that “parabola” travels.

Periodic Waves

The simplest wave is when $(x-vt)$ appears in a \sin or \cos function. $D = \sin(x-vt)$ But this equation is not justified. First, since D is a disturbance it must have dimensions so we need

$$D = A \sin(x - vt)$$

Where A has the dimensions of D . Next, argument of \sin cannot have dimensions, so we need

$$D = A \sin \frac{(x - vt)}{\lambda}$$

Where λ is a length. Since $\frac{v}{\lambda}$ has dimension of (1/Time), put $\frac{v}{\lambda} = \frac{1}{T}$

Next, introduce a phase angle ϕ and we get $D = A \sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T} + \phi\right)$ as the most general periodic wave. Note that 2π has been put in, as we know repeat angle for \sin . If you put $\phi = \pi$ you recover the Equation in some books.

$$D = A \sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$$

As shown in class

λ = Repeat Distance= wavelength

T = period, $\frac{1}{T} = f$ (frequency)

And $v = \lambda f$

Next, define $k = \frac{2\pi}{\lambda}$ (wave vector)

$\omega = 2\pi f$ (angular frequency)

$\omega = vk$

And we can write $D = A \sin(kx - \omega t + \phi)$ for any periodic wave traveling a long +ive x -axis with velocity $v = \frac{\omega}{k} \hat{x}$

Similarly, $D = A \sin(kx + \omega t + \phi)$ is any periodic wave along -ive x -axis with

$$v = -\frac{\omega}{k} \hat{x}$$