

DIPOLE IN AN $\vec{E}$ -FIELD

Before solving this problem let us recall the relevant part of Phys 121.

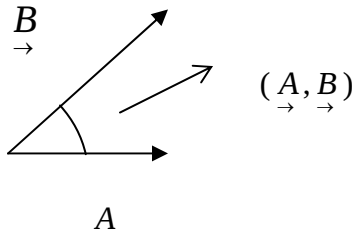
1st

CROSS-PRODUCT OF TWO VECTORS: Given two vectors $\vec{A}$ and $\vec{B}$, their cross-product or vector Product $\vec{C}$ is given by:

$$\vec{C} = \vec{A} \times \vec{B}$$

Magnitude of $\vec{C}$

$$C = AB \sin(\angle \vec{A}, \vec{B})$$



Direction of $\vec{C}$ is normal to $(\vec{A}, \vec{B})$ plane that is,

$$\vec{C} \perp \vec{A}$$

$$\vec{C} \perp \vec{B}$$

Supplemented by the right hand rule

Take

$\vec{A} \parallel$ Thumb

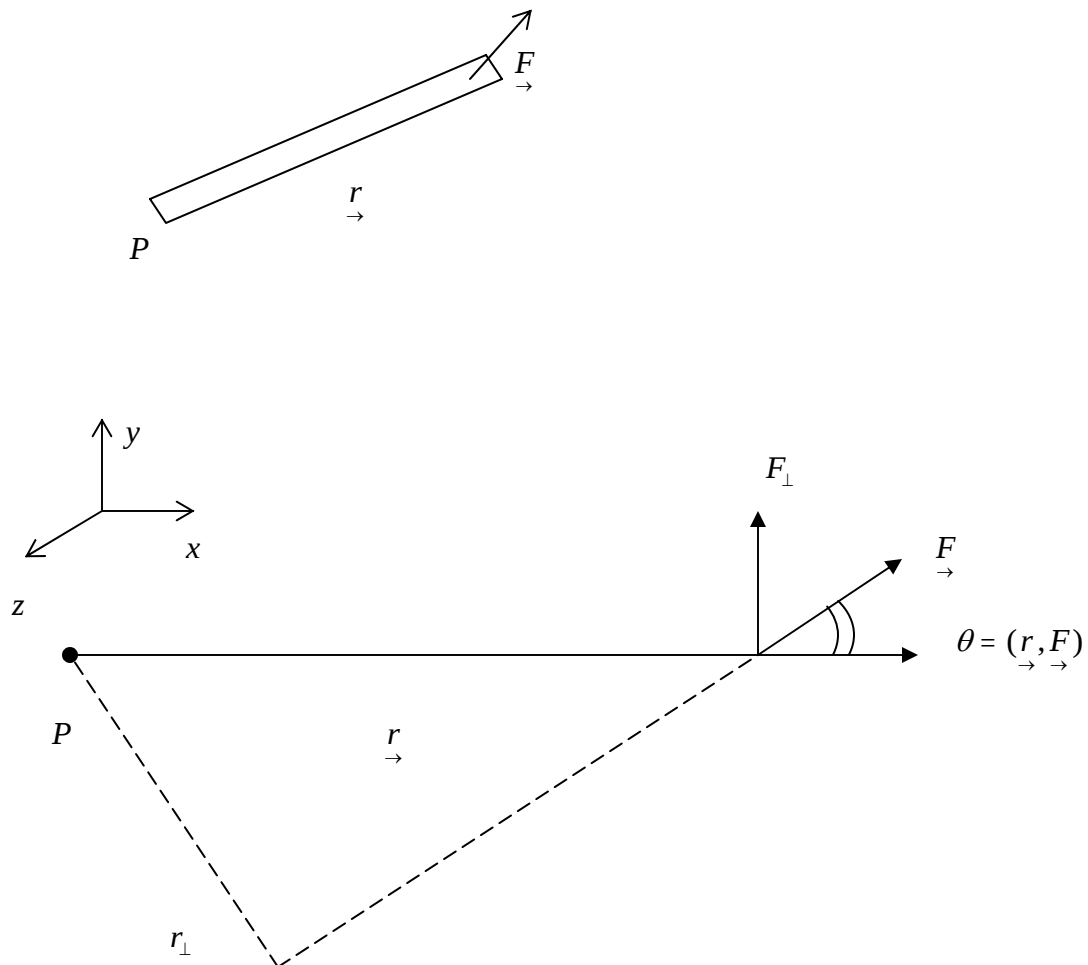
$\vec{B} \parallel$ Fingers

Then

$\vec{C}$ is $\perp$ palm of right hand

2nd

TORQUE: When a force $\vec{F}$ is applied at some distance $\vec{r}$ from a pivot pt. P on an extended object such as a bar, the bar turns about the pivot point on an axis which is perpendicular both to $\vec{r}$ and $\vec{F}$, this turning is controlled by the torque (vector) $\vec{\tau}$. Just as a force is required to cause an object to change its linear momentum (center of mass velocity) Torque is needed to change angular velocity (angular momentum). **FORCE CAUSES LINEAR ACCELERATION, (TRANSLATION) TORQUE CAUSES ANGULAR ACCELERATION (ROTATION).**

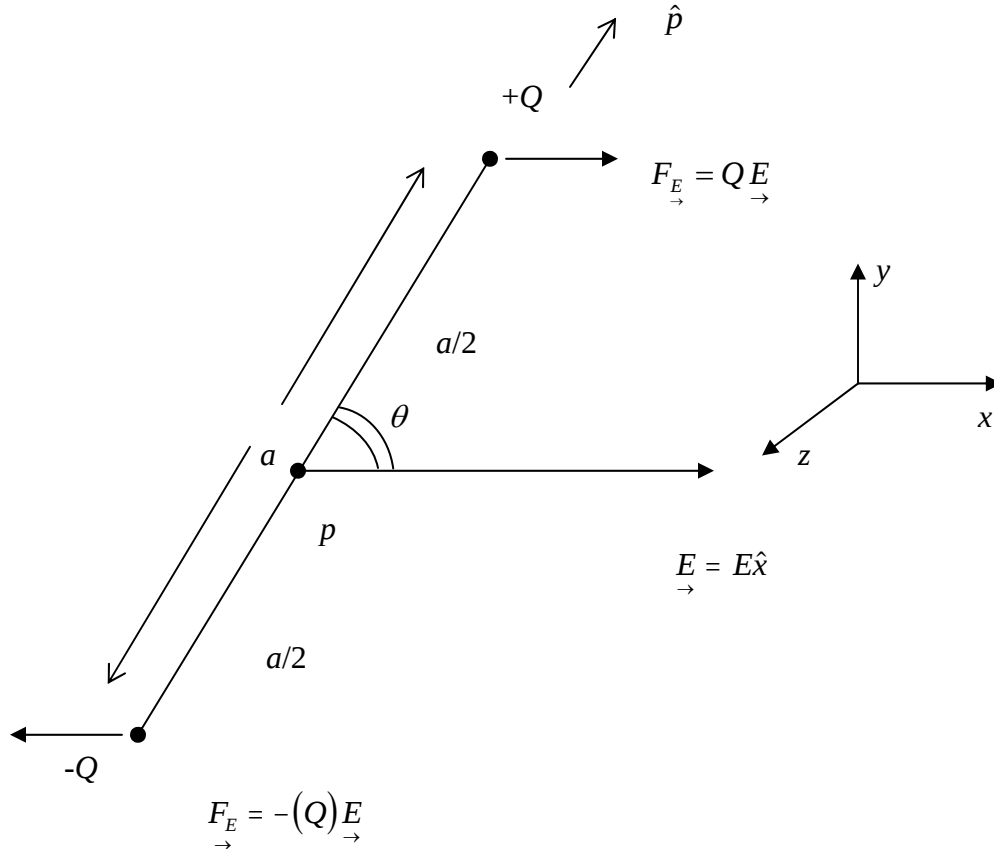


Notice that in turning the bar about an axis through P , only component of $\vec{F} \perp \vec{r}$ matters or alternatively, the moment arm $r_{\perp}$ controls τ .

$$\begin{aligned}\vec{\tau} &= \vec{r} \times \vec{F} \\ &= rF \sin\theta \hat{z}\end{aligned}$$

Thus magnitude of Torque is $\tau = rF_{\perp} = r_{\perp}F = rF \sin(r, F)$ with the right hand rule for the picture above

Now let us put a dipole $\vec{p} = aQ\hat{p}$ in an $\vec{E}$ field.



Notice there are two forces on the Dipole. $+QE$ & $-|Q|E$, so total force is zero. Take torque about pivot point P (mid pt. Of Dipole). It is the sum of two torques.

Magnitude

$$\begin{aligned}\tau &= \frac{a}{2}QE \sin\theta + \frac{a}{2}QE \sin\theta \\ &= aQE \sin\theta \\ &= pE \sin\theta\end{aligned}$$

And for the geometry shown and using the right hand rule one can write $\vec{\tau} = \vec{p} \times \vec{E}$

Because mag. is $pE \sin\theta$ & direction is $\perp$ paper pointing down! $\vec{\tau} = -pE \sin\theta \hat{z}$

Potential Energy of DIPOLE IN $\vec{E}$ - field .

$$\text{Work done } \Delta W = \vec{\tau} \cdot \Delta \vec{\theta} \quad [\Delta W = \vec{F} \cdot \Delta \vec{x}]$$

$$\vec{\tau} = -pE \sin \theta \hat{z}$$

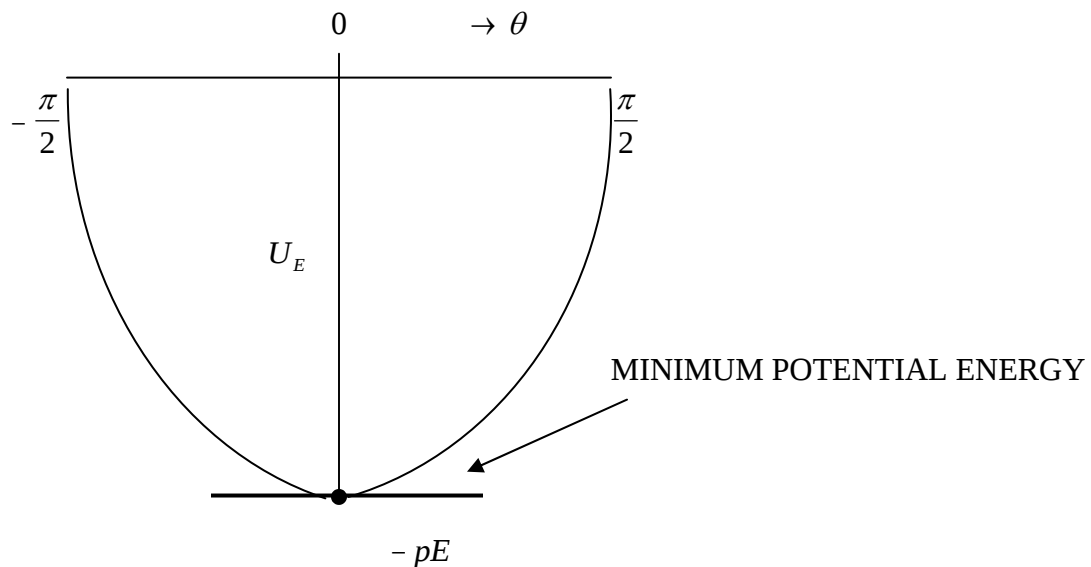
$$\Delta \vec{\theta} = \Delta \theta \hat{z}$$

$$\Delta W = -pE \sin \theta \Delta \theta$$

Change of Potential Energy $\Delta U_E = -\Delta W = +pE \sin \theta \Delta \theta$

Hence $U_E = -(\vec{P} \cdot \vec{E}) = -pE \cos \theta$

SO DIPOLE WILL TURN AND BECOME PARALLEL To $\vec{E}$ field at $\equiv m$.



FINALLY, NOTE THAT FOR $\theta \ll 1$, $U_E \propto \theta^2$ or $\vec{\tau} \propto -\theta \hat{z}$, so you get Linear Harmonic

Oscillations, that is, for small θ dipole will oscillate about $\vec{E}$. In other words, dipole is in $\equiv m$

when it is parallel to $\vec{E}$, if you pull it aside by a small angle θ and let go it will oscillate about $\vec{E}$.