MECHANICAL WAVES (TRAVELLING)

We begin our discussion of the wave phenomenon by considering waves in matter. The simplest definition of a wave is to call it a traveling disturbance (or equivalently, deviation from equilibrium). For instance, if you drop a stone on the surface of an undisturbed body of water you can watch the "disturbance" traveling radially out of the "point" of contact.

Formally, we can "construct" a wave in several steps. For simplicity, we take a wave traveling along x-axis.

Step 1. We need a disturbance $D$.
Step 2. $D$ must be a function of $x$.
Step 3. $D$ must also be a function of $t$.
Step 4. If $x$ and $t$ appear in the function in the combinations $(x \mp vt)$ the disturbance $D$ cannot be stationary. It must travel along $x$ with speed $v$.

Further,

$(x-vt)$ implies $\underset{\rightarrow}{v} = v\hat{x}[travel\ in + ive\, x - direction]$

$(x+vt)$ implies $\underset{\rightarrow}{v} = -v\hat{x}[travel\ in - ive\, x - direction]$

EXERCIZE: Put $D = A(x-t)^2$ and show that "parabola" travels.

Periodic Waves

The simplest wave is when $(x-vt)$ appears in a *sin* or *cos* function. $D = sin\ (x-vt)$ But this equation is not justified. First, since $D$ is a disturbance it must have dimensions so we need

$$D = A\ Sin(x - vt)$$

Where $A$ has the dimensions of $D$. Next, argument of *Sin* cannot have dimensions, so we need

$$D = A\ Sin\frac{(x-vt)}{\lambda}$$

*Where* $\lambda$ is a length. Since $\frac{v}{\lambda}$ has dimension of (1/Time), put $\frac{v}{\lambda} = \frac{1}{T}$

Next, introduce a phase angle $\varnothing$ and we get $D = A\ Sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T} + \varnothing\right)$ as the most general periodic wave. Note that $2\pi$ has been put in, as we know repeat angle for *Sin*. If you put $\varnothing = \pi$ you recover the Equation in some books.

$$D = A\ Sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$$

As shown in class

$\lambda$ = Repeat Distance= wavelength

T = period, $\frac{1}{T} = f$ (frequency)

And $v = \lambda f$

Next, define $k = \frac{2\pi}{\lambda}$ (wave vector)

$\omega = 2\pi f$ (angular frequency)

$\omega = vk$

And we can write $D = A\ Sin(kx - wt + \varnothing)$ for any periodic wave traveling a long $+ive$ $x$-axis with velocity $\underset{\rightarrow}{v} = \frac{\omega}{k}\hat{x}$

Similarly, $D = A\ Sin(kx + wt + \varnothing)$ is any periodic wave along –ive $x$-axis with

$$\underset{\rightarrow}{v} = -\frac{w}{k}\hat{x}$$