

MAXWELL'S EQUATIONS: RADIATION ⇒ LIGHT

To summarize, the field Equations derived from Experiments are:

GAUSS' LAW FOR COULOMB \vec{E}

$$\Sigma_C \vec{E} \cdot \vec{\Delta A} = \frac{1}{\epsilon_0} \Sigma Q_i \quad (1)$$

GAUSS' LAW FOR \vec{B}

$$\Sigma_C \vec{B} \cdot \vec{\Delta A} = 0 \quad (2)$$

LENZ'S LAW

$$\Sigma_C \vec{E}_{NC} \cdot \vec{\Delta l} = - \frac{\Delta \Phi_B}{\Delta t} \quad (3)$$

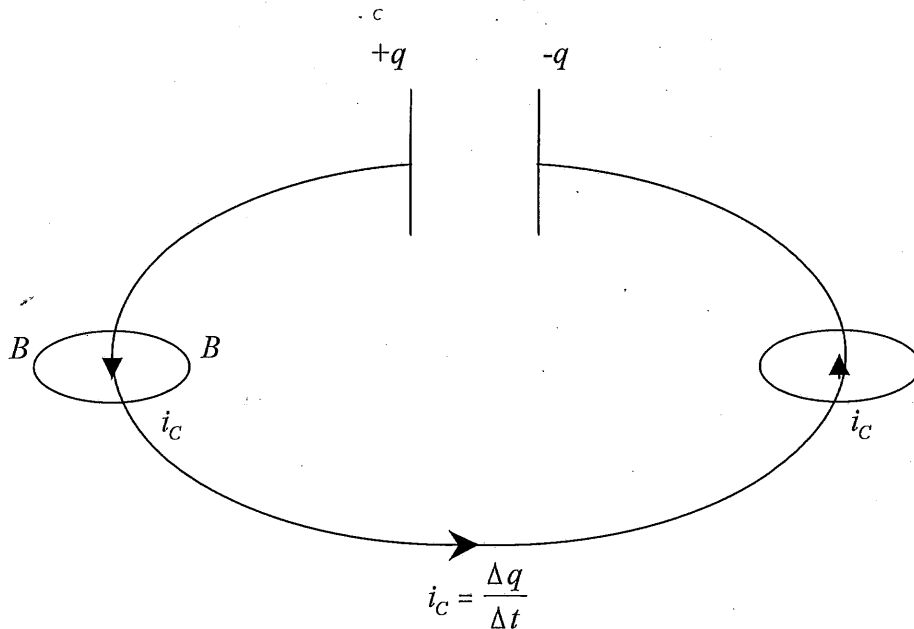
AMPERE'S LAW

$$\Sigma_C \vec{B} \cdot \vec{\Delta l} = \mu_0 \Sigma I_i \quad (4)$$

When Maxwell began to study these equations, he realized that there was a serious problem. Scientists believe that at its most fundamental level nature must be symmetric.

Maxwell noticed that whereas a time varying flux of \vec{B} gave rise to an \vec{E} -field [\vec{E}_{NC} in Eq.(3)]

there was no corresponding term in Eq. (4'). He immediately asserted that the above field equations could not be regarded as being complete. This was a FUNDAMENTAL PROBLEM. Maxwell also noted a "PRACTICAL PROBLEM" in using Eq. (4'). Imagine that we charge a capacitor to $\pm q$ and then connect a wire between the two plates as shown.



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It is clear that a conduction current $\frac{\Delta q}{\Delta t}$ begins to flow through the wire and so [using Eq. (4')] it must create a \vec{B} -field encircling the wire as shown. However, as soon as you cross one of the capacitor plates, both the current and \vec{B} must be zero. Again, Maxwell asserted that such a discontinuity cannot be physically meaningful.

To resolve the fundamental problem Maxwell postulated that if the flux of \vec{E} varies with time it must be equivalent to a current. He called this new type of current a displacement current and introduced the definition $i_D = \epsilon_0 \frac{\Delta \phi_E}{\Delta t}$ (5)

Of course, Eq. (4') implies that every current generates a \vec{B} so Maxwell "completed" Eq. (4') by writing $\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \Sigma I_C + \mu_0 \epsilon_0 \frac{\Delta \phi_E}{\Delta t}$ (4)

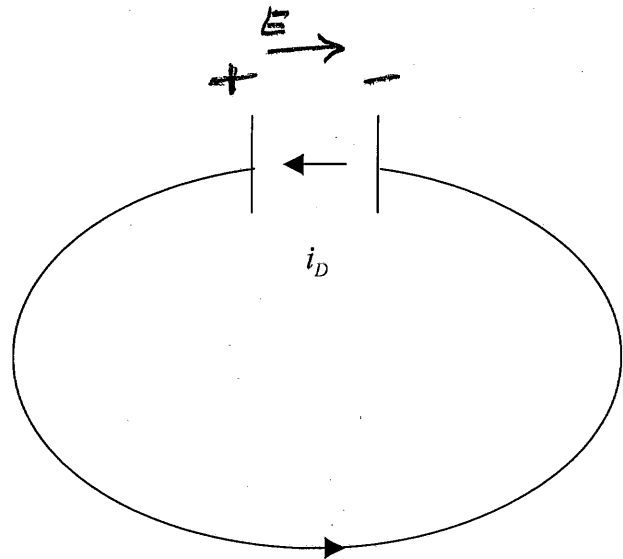
Where I_C explicitly signifies a conduction current = flow of charge in a conductor while the second term on the right comes from i_D [Eq. (5)].

Let us see if introduction of i_D also solves the practical problem. If the capacitor plates have an area A the \vec{E} -field between them is

$$\vec{E} = \frac{q}{\epsilon_0 A} \hat{x}, \quad A = A \hat{x}$$

$$\text{so } \Phi_E = \frac{q}{\epsilon_0}$$

$$\text{and } i_D = \epsilon_0 \frac{\Delta \Phi_E}{\Delta t} = \frac{\Delta q}{\Delta t} = i_C!$$



[i_D is from -ive to +ive because of $\frac{\Delta q}{\Delta t}$ is -ive]

Since $i_D = i_C$ we will have no discontinuity in either the current or the \vec{B} -field on crossing the capacitor plate.

Maxwell has solved both the fundamental and the practical problem by proposing Eq. (5).

MAXWELL'S EQUATIONS

GAUSS' LAW FOR COULOMB \vec{E} :

Since a stationary charge generates a Coulomb \vec{E} field, the TOTAL flux of \vec{E}_{Coul} THROUGH a closed surface is determined solely by the charges located in the volume enclosed by that surface:

$$\sum_c \vec{E}_{\text{Coul}} \cdot \vec{\Delta A} = \frac{1}{\epsilon_0} \sum Q_i \quad (1)$$

GAUSS' LAW FOR \vec{B}

Since the elementary generators of \vec{B} are point magnetic dipoles the TOTAL flux of \vec{B} THROUGH A closed surface is always zero:

$$\sum_c \vec{B} \cdot \vec{\Delta A} \equiv 0 \quad (2)$$

FARADAY-LENZ LAW

IF THE FLUX OF \vec{B} VARIES WITH TIME A NON-COULOMB \vec{E} FIELD WILL APPEAR IN EVERY CLOSED "LOOP" SURROUNDING THE REGION WHERE THE FLUX OF \vec{B} IS VARYING.

THE SENSE OF \vec{E}_{ENC} IS INVARIABLY SUCH AS OPPOSE THE VARIATION IN THE FLUX OF \vec{B} THAT CAUSES IT. HENCE, CIRCULATION OF

\vec{E}_{NC} around a closed loop is determined by the ~~time~~ time rate of change of flux of \vec{B} through the area within the loop; [Note: Crucial -ive sign].

$$\sum \vec{E}_{NC} \cdot \Delta \vec{l} = - \frac{\Delta \Phi_B}{\Delta t} \quad (3)$$

MAXWELL-AMPERE LAW

Every current generates a \vec{B} field that circulates around it. There are two types of current: i) conduction current which involves flow of charge in a conductor and ii) displacement current which arises when flux of \vec{E} field varies with time. Hence, Circulation of \vec{B} around a closed loop is determined by the ~~currents~~ currents encircling the surface on which the loop is drawn:

$$\sum_C \vec{B} \cdot \Delta \vec{l} = \mu_0 \sum I_c + \mu_0 \epsilon_0 \frac{\Delta \Phi_E}{\Delta t} \quad (4)$$

CAUTION: i_D exists in vacuum. It never involves flow of charge. No conduction current can exist inside the capacitor!!!

Maxwell's *Equations (1) through (4)* have profound consequences. Let us recall his work using these in outer space, where there is vacuum, $q=0, i_C = 0$ so the Equations become:

$$\Sigma_C \vec{E} \cdot \Delta \vec{A} = 0 \quad I$$

$$\Sigma_C \vec{B} \cdot \Delta \vec{A} = 0 \quad II$$

$$\Sigma_C \vec{E}_{NC} \cdot \Delta \vec{l} = - \frac{\Delta \phi_B}{\Delta t} \quad III$$

$$\Sigma_C \vec{B} \cdot \Delta \vec{l} = \mu_0 \epsilon_0 \frac{\Delta \phi_E}{\Delta t} \quad IV$$

and now indeed there is total symmetry with respect to \vec{E} and \vec{B} . This is what led Maxwell to propose that rather than think of \vec{E} and \vec{B} fields, one should think of a single entity:

Electromagnetic or EM field

And call *Equations I through IV*, EM field Equations. He next used these Equations to predict that in vacuum there must exist EM-waves! He was able to show that the structure of these Equations is such that both the \vec{E} and \vec{B} have the functional form (propagation along x for example) $f(x \pm ct)$. That is, they propagate as an Electromagnetic wave with the enormous

speed $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s}$. This was a giant step forward: Maxwell had solved the problem

of the nature of Radiation or Radiant energy. \Rightarrow Radiation is an Electromagnetic wave. Our observable universe = Matter + Radiation

Incidentally, Einstein demonstrated that matter and radiation convert into one another there by further simplifying our picture of the universe.

- \rightarrow Heat
- \rightarrow Light
- \rightarrow x-rays
- \rightarrow radiowaves

are all cases of EM waves. They are distinguished only by their frequencies (or wavelengths). We will concentrate on

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FINALLY WE COME TO \rightarrow LIGHT

LIGHT: is a transverse EM wave (\vec{E} and \vec{B} fields perpendicular to direction of propagation and also $\vec{E} \perp \vec{B}$) whose wavelength lies between 400 nm and 800 nm and whose speed in vacuum is $3 \times 10^8 \text{ m/s}$. As always, light waves transport energy. Let us compare transport of Energy by:

Wave on a string: Power

$$P = \frac{1}{2} \mu A^2 \omega^2 v$$

$$v = \sqrt{\frac{F}{\mu}}$$

Av.
[Energy Stored per unit length multiplied by velocity]

Sound: Intensity

$$I = \frac{1}{2} \rho_0 S_m^2 \omega^2 v$$

$$v = \sqrt{\frac{\gamma P_0}{\rho_0}}$$

Av.
[Energy Stored per unit volume multiplied by velocity].

EM-wave Light: Intensity

$$I = \frac{B_m^2}{2\mu_0} c = \frac{1}{2} \epsilon_0 E_m^2 c$$

$$c = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$$

Av.
[Energy Stored per unit length multiplied by velocity].