ENERGY CONSERVATION PRINCIPLE REVISITED : ELECTRIC POTENTIAL

Now that we have a new force

$$\underset{\rightarrow}{F_E} = \frac{k_e Q_1 Q_2}{r^2} \hat{r} \longrightarrow (1)$$

and a new field:

Call
$$\underset{\rightarrow}{E} = \frac{\underset{\rightarrow}{F_E}}{q} \longrightarrow (2)$$

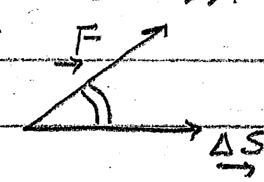
we need to take another look at the principle of CONSERVATION OF ENERGY

First, let us recall some of our discussion from 121 where we talked only of mechanical Energy:

<u>MECHANICAL WORK</u>

$$\Delta W = \underset{\rightarrow}{F} \cdot \underset{\rightarrow}{\Delta S} = F \Delta S \cos(\underset{\rightarrow}{F}, \underset{\rightarrow}{\Delta S})$$

where



$\underset{\rightarrow}{F}$ = Force Vector

$\underset{\rightarrow}{\Delta S}$ = Displacement Vector

<u>NOTE</u> $\Delta W = 0$ if $\underset{\rightarrow}{F} \perp \underset{\rightarrow}{\Delta S}$, that is only component of $\underset{\rightarrow}{F} \parallel \underset{\rightarrow}{\Delta S}$ does work.

<u>KINETIC ENERGY</u>

WORK STORED IN MOTION: if an object of Mass M is sitting at rest, the work required to give it a speed v is stored as Kinetic Energy

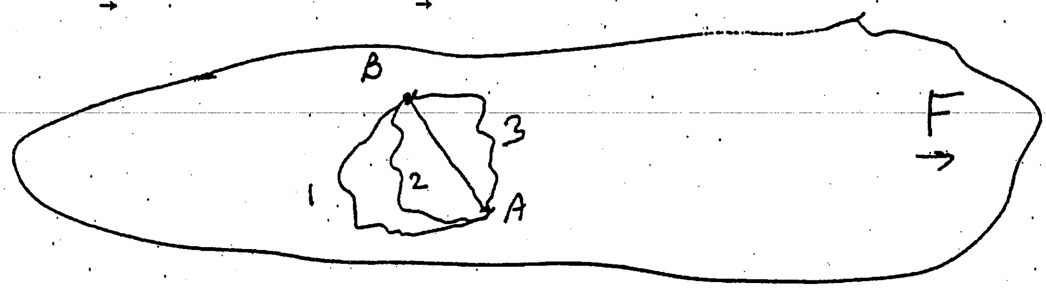
$$K = \frac{1}{2} M v^2$$

or since linear momentum $\underset{\rightarrow}{p} = M\underset{\rightarrow}{v}$ we can write $K = \frac{p^2}{2M}$

Potential Energy (U) presents a greater conceptual challenge.

P is the mechanical work stored in a system when it is prepared (or put together) in the presence of a prevailing conservative force.

Suppose we have a region of space in which there is a prevailing force (weight near Earth's surface comes to mind). That is, at every point in this region an object will experience a force. Let the object be at point B [First, notice that you cant let the object go as $\vec{F}$ will immediately cause $\vec{a}$ and object will move].



To define U at B we have to calculate how much work was needed to put the object at B in the presence of $\vec{F}$. Let us pick some point A, where we can claim that U is known, and calculate the work needed to go from A to B. As soon as we try to do that we realize that the only way we can get a meaningful answer is if the work required to go from A to B is independent of the path taken. So our prevailing force has to be special. Such a force is called a CONSERVATIVE FORCE – WORK DONE DEPENDS ONLY ON END-POINTS AND NOT ON THE PATH TAKEN.

If that is true we have a unique answer

$$\Delta w_1 = \Delta w_2 = \Delta w_3 = \Delta w_{AB}$$

and we can use this fact to calculate the change in U in going from A to B

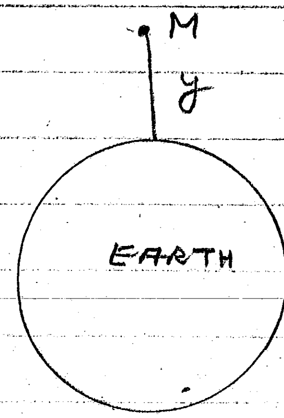
$$\Delta U_{AB} = -\vec{F} \bullet \Delta \vec{S}_{AB}$$

NOTE THE –SIGN: It comes about because as stated above we cannot let the object go. In fact, the displacement from A to B must be carried out in such a way that the object cannot change its speed (if any). That is, we need to apply a force $-\vec{F}$ to balance the ambient $\vec{F}$ at every point. The net force will become close to zero at all points. ΔU_{AB} is work being done by $-\vec{F}$.

So when $\vec{F}$ is conservative ΔU_{AB} is unique. In the final step we can choose A such that $U_A = 0$ $U_A = 0$. Then $U_B = -\vec{F} \bullet \Delta \vec{S}_{AB}$

using the above definition for $\mathcal{U}$
we discussed two cases

(i) $\mathcal{U}$ for Earth - Mass system: Near Earth
the Conservative force
is $\underset{\rightarrow}{F}_g = -Mg\,\hat{y}$



hence $\mathcal{U}_g(y) = Mgy$ (3)
taking $P = 0$, at $y = 0$.

(ii) $\mathcal{U}$ for Mass attached to a spring.
Here the Conservative force is
$$\underset{\rightarrow}{F} = -kx\,\hat{x}$$
hence
$$\mathcal{U}_{sp}(x) = \frac{1}{2}kx^2 \qquad (4)$$
taking $\mathcal{U}=0$, when $x=0$ [spring unstretched].

We were then able to write the principle of Energy conservation
$$K_f + \mathcal{U}_{gf} + \mathcal{U}_{spf} = K_i + \mathcal{U}_{gi} + \mathcal{U}_{spi} + W_{NCF} \qquad (5)$$

where i and f refer to the initial and final states and W_{NCF} takes account work done by non-conservative forces (friction for instance) in going from i to f.

When we got to thermodynamic systems we learnt that the system can change its energy in 3 ways.

Exchange Heat DQ with its surroundings because of a temperature difference across a conducting wall

Have mechanical work DW done on or by it

Change the internal energy stored with-in it (du)

The Conservation law (FIRST LAW) OF Thermodynamics becomes

$$\pm DQ \pm DW \pm du = 0 \qquad (6)$$

That is, in any thermodynamic process the total change in Energy must be ZERO

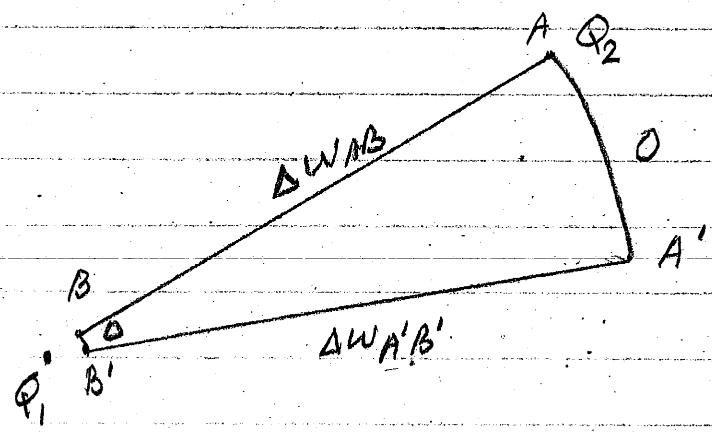
Now, let us look at $\vec{F}_E$ from Eq.(1). The first step is to examine if it is a conservative force. If so, we can define a potential energy for $\vec{F}_E$.

Note: In the following proof it is CRUCIAL to recognize that $\vec{F}_E$ acts only along the line joining the two charges and so work done by $\vec{F}_E$ being

$$\Delta W = \vec{F}_E \cdot \Delta\vec{S}$$

will be Zero for any displacement along the circumference (ie $\perp \hat{r}$).

Let us fix Q_1 and move Q_2 starting at pt. A



First path: $\underset{\rightarrow}{\Delta S}$ is AB along $\hat{\varepsilon}$ work done will be

$$\Delta W_{AB} = -\underset{\rightarrow}{F_E} \cdot \underset{\rightarrow}{\Delta S_{AB}}$$

Note, the "minus" sign. As always, the force which does the work must be equal but opposite to the prevailing force

Second Path

First, go from $A \rightarrow A'$ along circumference

$$\Delta W_{AA'} = 0$$

Next go along $\hat{\varepsilon}$ from A' to B' where $A'B' = AB$

$$\Delta W_{A'B'} = -\underset{\rightarrow}{F_E} \cdot \underset{\rightarrow}{\Delta S_{A'B'}} = \Delta W_{AB}$$

Next go from B' to B along circumference

$$\Delta W_{B'B} = 0$$

Hence

$$\Delta W_{AA'B'B} = \Delta W_{AB}$$

Work is independent of path $\underset{\rightarrow}{F_E}$ is indeed CONSERVATIVE, potential energy is definable.

$$\Delta U_E = -\vec{F}_E \cdot \vec{\Delta S}$$

. Change in Electrostatic potential energy consequent upon a displacement $\vec{\Delta S}$

~~The~~ energy conservation Equation will now read

$$K_f + U_{gf} + U_{spf} + U_{Ef} = K_i + U_{gi} + U_{spi} + U_{Ei} + W_{NCF} \rightarrow (7)$$

where again f and i, respectively, refer to the final and initial states of the system W_{NCF} is work done by non-conservative forces while system goes from i to f.

In the present case it is useful to define a new quantity called Electrostatic Potential which is related to ΔU_E by the equation

$$\Delta V = \frac{\Delta U_E}{q} = -\frac{\vec{F}_E \cdot \vec{\Delta S}}{q}$$

$$= -\vec{E} \cdot \vec{\Delta S}$$

~~Like~~ ΔU_E, ΔV is also a scalar, the dimensions are $ML^2T^{-2}Q^{-1}$ and the unit is Joule/Coulomb which is called a Volt.