ELECTRIC POTENTIALS – special cases.

1 Single plate having charge density σ C/m^2

$+\sigma$

$$\Delta V = -\vec{E}\cdot\vec{\Delta S}$$

Since $\vec{E} \parallel \hat{x}$

$\vec{E} = -\frac{\sigma}{2\epsilon_0}\hat{x}$ $\quad$ $\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{x}$

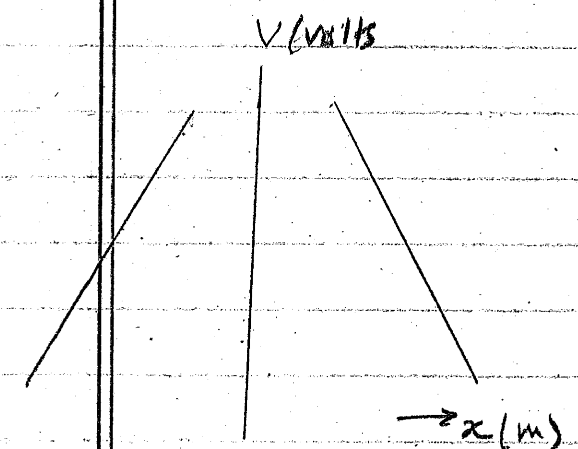
Nonzero ΔV only if $\vec{\Delta S} \parallel \hat{x}$

$x > 0$ $\quad$ $\vec{\Delta S} = x\,\hat{x}$

$$\Delta V = -\frac{\sigma}{2\epsilon_0}x$$

$x < 0$ $\quad$ $\vec{\Delta S} = -x\,\hat{x}$

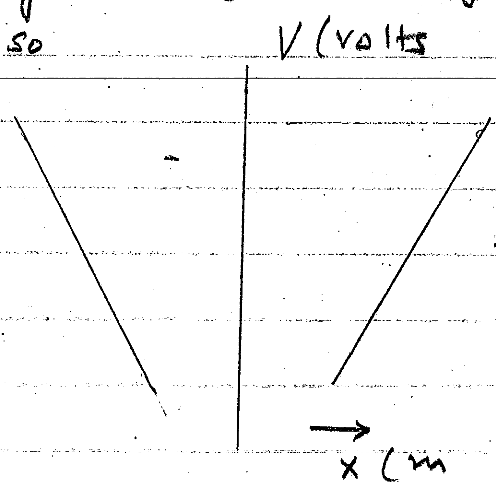
$$\Delta V = \left(+\frac{\sigma}{2\epsilon_0}\right)(-x)$$



2. Single plate having charge density $-\sigma$ C/m^2

Now $\vec{E}$ $\quad$ so

$+\frac{\sigma}{2\epsilon_0}x$ $\quad$ $-\frac{\sigma}{2\epsilon_0}\hat{x}$



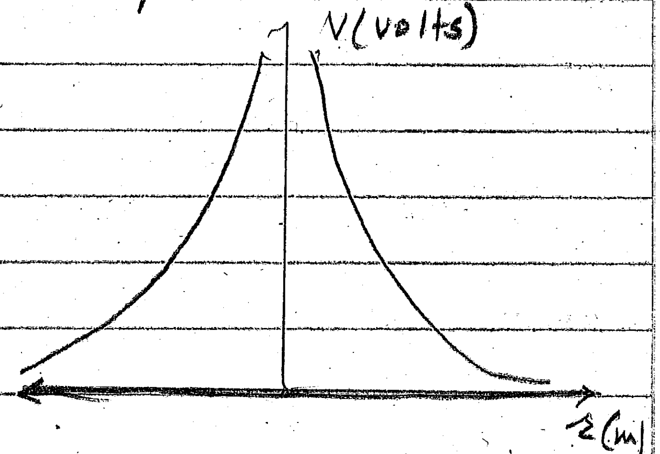
$$\Delta V = \frac{\sigma}{2\epsilon_0}x$$

3. SINGLE POINT CHARGE Q at $r=0$.

$$\vec{E} = \frac{k_e Q}{r^2}\,\hat{r}$$

We will put $V=0$ at large r ($r \to \infty$) b/c E goes to zero at large r. Then calculate the change in V as we come from ∞ to r.

$$\Delta V = -\vec{E}\cdot\Delta\vec{r}$$



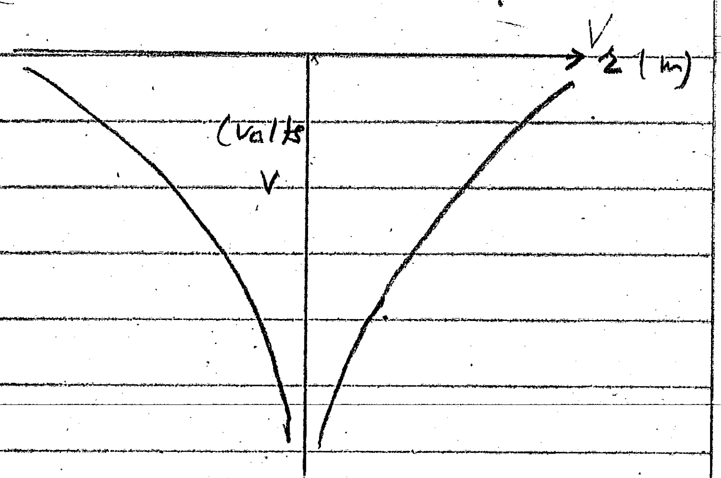
This requires an Integral

$$V(r) = \frac{k_e Q}{r}$$

4. SINGLE $-|Q|$ at $r=0$

$$\vec{E} = \frac{-k_e Q}{r^2}\,\hat{r}$$

so $$V(r) = \frac{-k_e Q}{r}$$



5. spherical shell or spherical conductor of radius R. In this case charge resides only on the surface. Hence

for $r < R$ $\vec{E}=0$ [illegible]

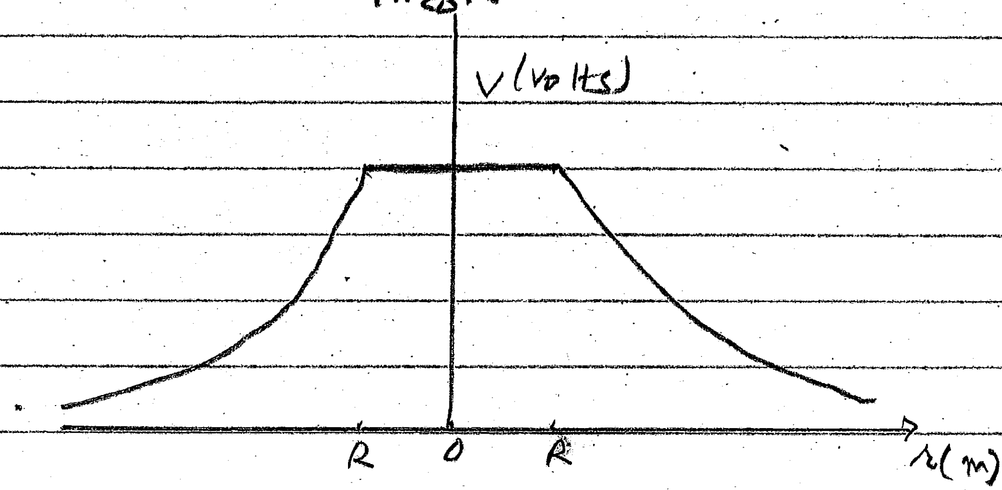
[illegible]

for $r > R$ $\quad \underset{\rightarrow}{E} = \dfrac{Q}{4\pi \epsilon_0 r^2} \hat{r}$

The corresponding potential is

$r > R \quad V(r) = \dfrac{Q}{4\pi\epsilon_0 r}$

$r < R \quad V(r) = \dfrac{Q}{4\pi\epsilon_0 R}$



6 Insulating sphere of radius R which carries a charge Q distributed uniformly over the sphere so one can define a charge density

$$\rho = \frac{Q}{\frac{4\pi}{3} R^3}$$

Now

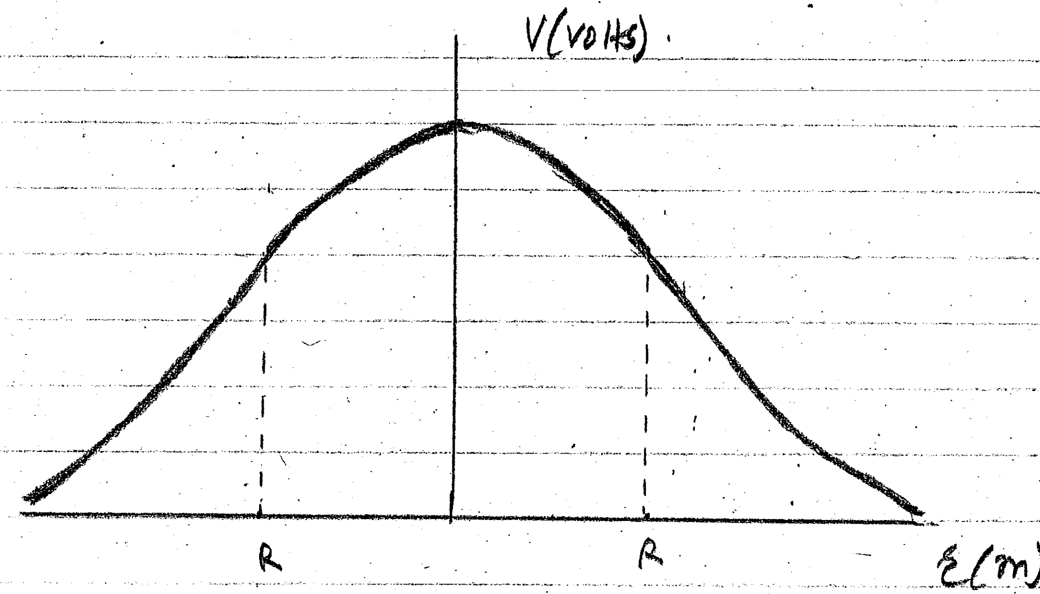
for $r < R$ $\quad \underset{\rightarrow}{E} = \dfrac{\rho\, r}{3\epsilon_0} \hat{r} = \dfrac{Q\, r}{4\pi\epsilon_0 R^3} \hat{r}$

$r > R \quad \underset{\rightarrow}{E} = \dfrac{Q}{4\pi\epsilon_0 R^2}$

so for

$r > R \quad V(r) = \dfrac{Q}{4\pi\varepsilon_0 r}$

$r < R \quad V(r) = \dfrac{Q}{4\pi\varepsilon_0 R}\left[\dfrac{3}{2} - \dfrac{r^2}{2R^2}\right]$



EQUIPOTENTIALS

Curves (in Two Dimensions) and Surfaces (in Three Dimensions) where the Electric potential is constant [V = constant].

(you will do an Experiment to trace equipotential curves)

There are two important properties of an equipotential

(i) If a charge moves on an equipotential it will not cost any energy (reminder: it costs no work to move on a closed loop in a conservative force)

(ii) the $\underline{E}$ field must be perpendicular to an equipotential.

Examples

(i) Plate carrying $+\sigma$ C/m^2; $\Delta V = -\frac{\sigma}{2\epsilon_0} x$

equipotentials are planes parallel to plate.

(ii) Pt. charge Q at $r=0$, $V(r) = \frac{k_e Q}{r}$ equipotentials are spheres of radius r whose center is at $r=0$.

(iii) surface of a conductor in stationary conditions – charge on surface only, $\underline{E} \perp$ surface everywhere so surface is equipotential.