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PHYS 122 **EXAM 1** **October 4, 2007**
Prof. S.M. Bhagat

Name: SOLUTION

(Sign in ink, print in pencil)

Notes

1. There are six (6) problems in this exam. Please make sure that your copy has all of them.
2. Please show your work, indicating clearly what formula you used and what the symbols mean. Just writing the answer will not get you full credit. In stating vectors, give both magnitude and direction.
3. Write your answers on the sheets provided.
4. Do not forget to write the units.
5. Do not hesitate to ask for clarification at any time during the exam. You may buy a formula at the cost of one point.

Take Care! God Bless You!

$$k_e = 9\times 10^9 \frac{N\cdot m^2}{C^2},\ \mu_0 = 4\pi x 10^{-7} \frac{T-m}{A}$$

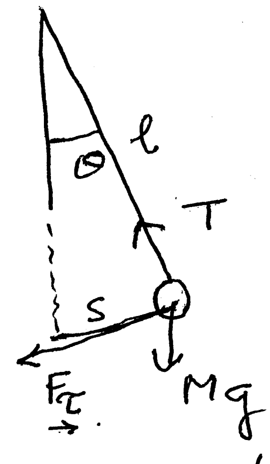
$$\varepsilon_0 = 9\times 10^{-12} F/m$$

Mass of proton	$m_p = 1.6\times 10^{-27} kg$
Mass of electron	$m_e = 9\times 10^{-31} kg$
Elementary Charge	$e = 1.6\times 10^{-19} C$

Problem 1

(a) In the experiment with a simple pendulum, why do you need small amplitudes to obtain linear harmonic oscillations? (5)

To initiate the oscillation you must pull the mass away from vertical; say angle θ. The forces acting on M are $-Mg\hat{y}$ and T along the string. Split ~~Mg~~ into components:



Forces Along string $T - Mg\cos\theta = \frac{Mv^2}{l}$ (centripetal force).

$\perp$string along tangent $\vec{F}_{\tau} = -Mg\sin\theta\,\hat{\tau}$ is the restoring force (minus sign) which causes oscn. However, for linear oscn. it must vary as θ. $\sin\theta \simeq \theta$ for small θ. Then $\vec{F}_{\tau} = -Mg\theta\hat{\tau}$
$= -Mg(S/l)\hat{\tau}$

LINEAR!

(b) A pendulum has a period of 1 second on Earth ($g = 9.8\ m/s^2$). What is its length? If you take it to the moon ($g = g_{Earth}/6$), will the period increase or decrease? By what factor? (5,6)

$$T_E = 2\pi\sqrt{\frac{l}{g_E}} \qquad T^2 = 4\pi^2\frac{l}{g}$$

$T = 1\text{sec}$ $\qquad \frac{g_E}{\pi^2} \approx 1$

$$l = \frac{T^2 \cdot g}{4\pi^2} \simeq \frac{1}{4}\ \text{meter}$$

$$T_M = 2\pi\sqrt{\frac{l}{g_M}} \qquad \text{Since } g_M = \frac{g_E}{6}.$$

$$\frac{T_M}{T_E} = \sqrt{\frac{g_E}{g_M}} = \sqrt{6} \simeq 2.45$$

Problem 2

(a) Fill in the blanks in the equation

$$D = A \; \mathrm{Sin}\, 2\pi\frac{(x - vt)}{\lambda},$$

for a traveling wave and explain the physical meaning of the symbols that you introduce. (5)

(i) D is a physical quantity so it must have units/dimensions. However, Sin function is dimensionless so we insert A to have the same dimensions as D. Sin varies between +1 and −1 so D varies between A and −A and thus A is called <u>AMPLITUDE</u>

(ii) The argument of the sin function cannot have dimensions so we divide (x−vt) by a length, λ.

(iii) The Sin function has a period of 2π and inserting 2π makes this explicit.

(iv) Including 2π makes the definition of λ simple. λ becomes the repeat distance (that is, the wave repeats itself every λ meters) hence it is called <u>WAVELENGTH</u>

(b) We have shown that on a stretched string a small amplitude periodic wave of amplitude A, angular frequency w, and speed v transports

$$\eta = \frac{1}{2} A^2 w^2 \frac{F}{v}$$

Jules of energy per second, where F is the tension in the string. (15)

How would η change if
i) A is doubled,
ii) w is doubled,
iii) F is increased by a factor of 4

i) $\eta \propto A^2$ so double A, η increases by a factor of 4.

ii) $\eta \propto \omega^2$ so double ω, η increases by a factor of 4.

(iii) Increasing F by 4 also changes $v = \sqrt{\frac{F}{\mu}}$ so $\eta \propto \sqrt{F}$ and changes by a factor of 2.

Problem 3

(a) What is sound? (6)

Any mechanical wave whose frequency lies between 20Hz and 20,000 Hz.

(b) The speed of sound in a gas is written as

$$v_s = \sqrt{\frac{\gamma k_B T}{m}}$$

Why is there a $\gamma\left(=\frac{C_p}{C_v}\right)$ in this equation? (10)

Sound can be thougat of as a displacement wave $S = S_m \sin[kx - \omega t]$.

Since S varies with x, Vol. (V) must change & hence Pressure (P) must change

so Sound is also a pressure wave

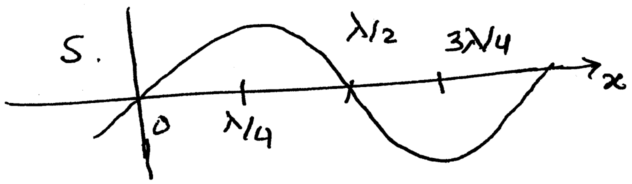
To go further we must know relationship of P and V. Since freq is high there is not enough time to exchange heat with the surroundings ($\Delta Q = 0$). The process becomes adiabatic and the PV relation is

$$PV^\gamma = \text{Const.}$$

Problem 4

(a) Sound can be thought of as a displacement wave or as a pressure wave. Why is there a phase difference of $\pi/2$ between the displacement and the pressure? (8)

Draw the displacement wave at $t=0$



i) Near $x=0$ displacements vary rapidly: ←←•0•→→ The gas is Expanding so pressure is reduced below average, (ii) near $x=\lambda/4$ displacements are large but all nearly equal, no change in vol; no change in pressure. (iii) near $x=\lambda/2$ displacements vary rapidly →→•λ/2•←←, Gas is ~~Contract~~ Contracting, pressure rises to maximum. So where displacement is large pressure variation is zero & vice versa.

(b) Draw a periodic (sine) sound wave as

i) a displacement, and

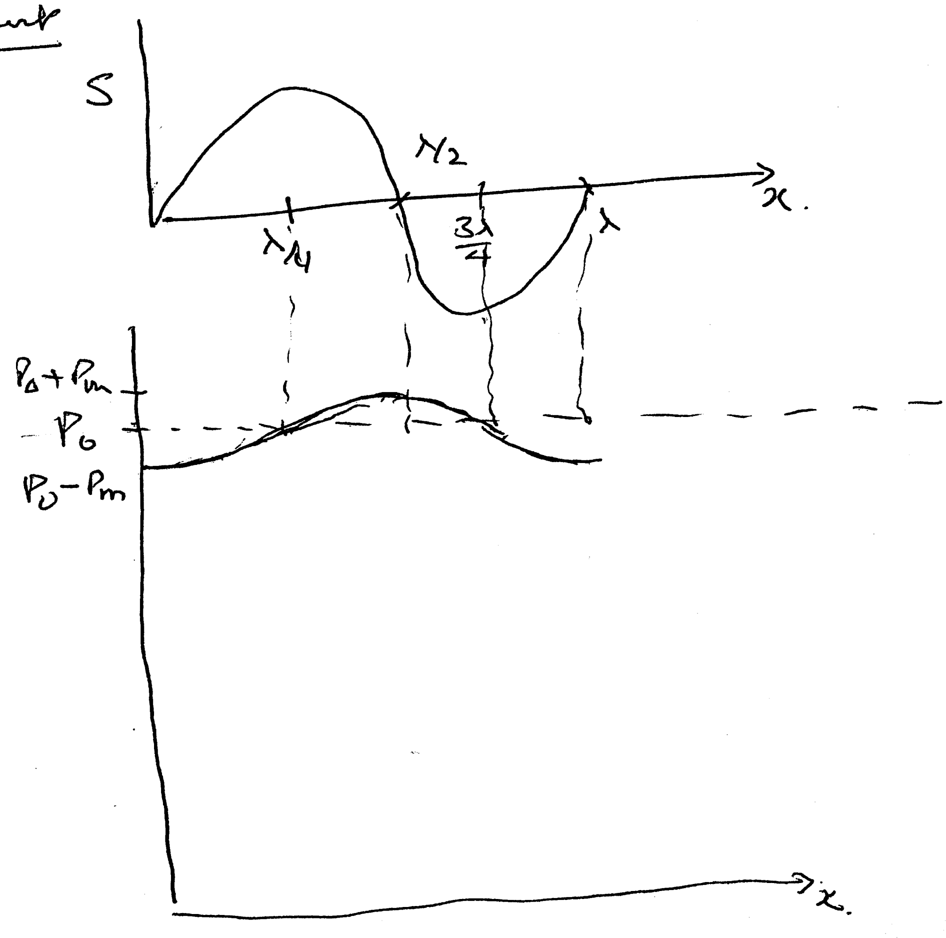
ii) as the corresponding pressure wave

at t = 0. (4, 4)

$t=0$

Displacement

Pressure



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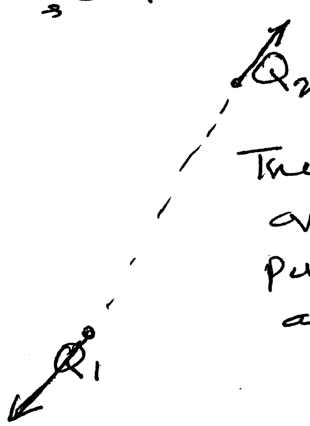
Problem 5

(a) Show that the equation

$$\underrightarrow{F_E} = k_e \frac{Q_1 Q_2}{r^2}\hat{r}$$

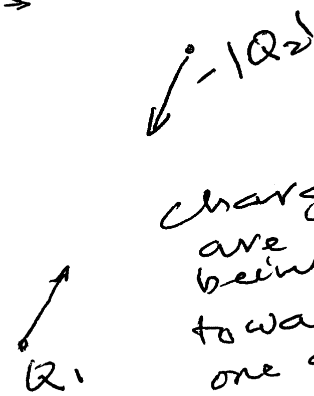
Implies attraction between unlike charges and repulsion between like charges. (6)

If Q_1, Q_2 have the same sign $Q_1 Q_2$ is +ive $\vec{F}_E \parallel +\hat{r}$



The charges are being pushed apart.

If Q_1, Q_2 have opp. signs $Q_1 Q_2$ is -ive $\vec{F}_E \parallel -\hat{r}$.



charges are being pulled toward one another

(b) Charges of $1\mu C$ and $25\mu C$ are located at $x = 0$ and $x = 6$m, respectively. Where would you locate a charge of $-5\mu C$, so that it experiences no force? Why? (10)

i) $\vec{F}_E$ acts along line joining the charges so to cancel the two forces $-|Q_3|$ will have to be located on the x-axis.

ii) The two forces must be opposite to one another to add to zero, hence Q_3 must be between 0 and 6m.

$$Q_1 = 1\mu C \quad \overset{x}{\longleftrightarrow} \; -|Q_3| \; \overset{(6-x)}{\longleftrightarrow} \quad Q_2 = 25\mu C$$

$$\vec{F}_{1,3} = -k_e \frac{1 \times 5 \times 10^{-12}}{x^2}\hat{x}$$

and we need $\vec{F}_{1,3} + \vec{F}_{2,3} = 0$.

$$\vec{F}_{2,3} = +k_e \frac{25 \times 5 \times 10^{-12}}{(6-x)^2}\hat{x}$$

$$\frac{-\cancel{k_e} \times 1 \times \cancel{5 \times 10^{-12}}}{x^2} + \frac{\cancel{k_e} \times 25 \times \cancel{5 \times 10^{-12}}}{(6-x)^2} = 0.$$

$$\frac{1}{x^2} = \frac{25}{(6-x)^2}$$

$$\frac{1}{x} = \frac{5}{6-x}$$

$$\underline{x = 1m}$$

Problem 6

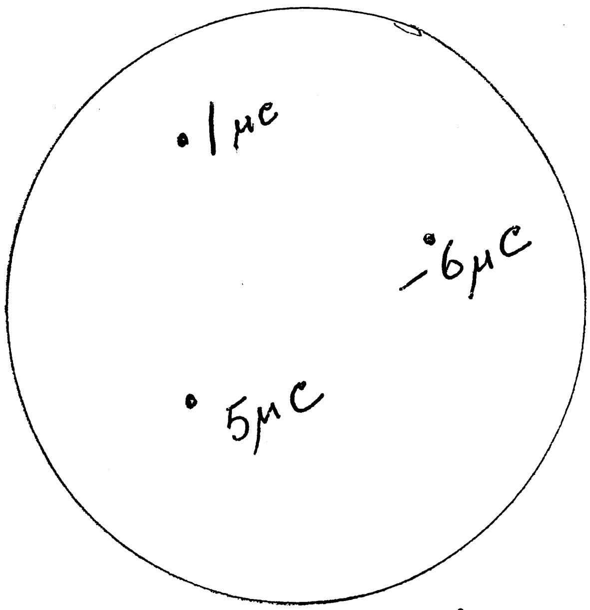
(a) What is Doppler Effect? (4)

If either the SOURCE or the Detector moves, the observed frequency is NOT EQUAL to the frequency emitted by the SOURCE.

(b) If I give you a charge q, how would you discover the presence of an $\underset{\rightarrow}{E}$-field? (4)

An $\underset{\rightarrow}{E}$-field exerts a force on a charge so given a charge q, we should attach it to a force measuring device (such as a spring balance). If we get a reading q is located in an $\underline{E}$-field. Indeed, if we measure the force at every point we can map out the $\underline{E}$ field using $\underset{\rightarrow}{E} = \frac{\underset{\rightarrow}{F_E}}{q}$.

(c) Charges of $1\mu C$, $5\mu C$, and $-6\mu C$ are located inside a spherical shell. What is the total flux of the $\underset{\rightarrow}{E}$- field through the surface of the shell? Why? (8)



A stationary charge generates a Coulomb $\underline{E}$ field. $\underset{\rightarrow}{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$.

Hence, Gauss' law says that total flux of $\underset{\rightarrow}{E}$ through a closed surface is determined solely by the enclosed charges, that is,

$$\sum_c \underset{\rightarrow}{E} \cdot \underset{\rightarrow}{\Delta A} = \frac{1}{\varepsilon_0} \sum Q_i$$

Here $\sum Q_i = 1\times10^{-6} + 5\times10^{-6} - 6\times10^{-6} = 0.$

Hence $\sum_c \underset{\rightarrow}{E} \cdot \underset{\rightarrow}{\Delta A} = 0.$