

Name: SOLUTION

(Sign in ink, print in pencil)

Notes

- 1) There are four (4) problems in this exam. Please make sure that your copy has all of them.
- 2) Please show your work indicating clearly what formula you used and what the symbols mean. Just writing the answer will not get you full credit. In stating vectors give both magnitude and direction.
- 3) Write your answers on the sheet provided.
- 4) Do not forget to write the units
- 5) Do not hesitate to ask for clarification at any time during the exam. You may buy a formula at the cost of one point.

Take Care! God Bless You!

$$k_e = 9 \times 10^9 \frac{N \cdot m^2}{C^2}, \mu_0 = 4\pi \times 10^{-7} \frac{T \cdot m}{A}$$

$$\epsilon_0 = 9 \times 10^{-12} \frac{F}{m}$$

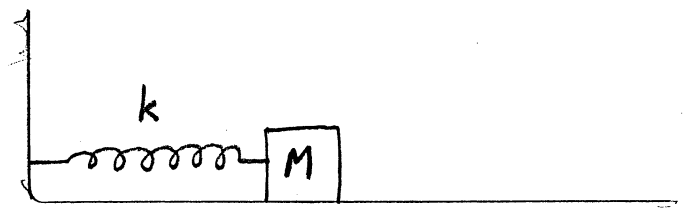
$$\text{Mass of proton} \quad m_p = 1.6 \times 10^{-27} \text{ kg}$$

$$\text{Mass of electron} \quad m_e = 9 \times 10^{-31} \text{ kg}$$

$$\text{Elementary Charge} \quad e = 1.6 \times 10^{-19} \text{ C}$$

Problem 1

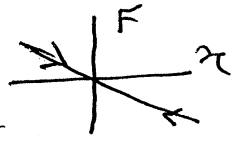
A spring-mass system consists of a mass  $M$ , attached to a spring of spring constant  $k$ , and placed, as shown, on a frictionless, horizontal table. The spring is unstretched at  $x = 0$ .



a) If you pull the mass to  $x = A$ , and then let go, why does it oscillate? (7)

FROM a force pt. of view once the mass is at  $x \neq 0$  it is acted on by a force  $F = -kx \hat{x}$ . So if you let go at  $x = A$ ,  $F$  brings it back to  $x = 0$  but then it is moving toward  $-x$  so it keeps going, when it stops at  $x = -A$ ,  $F$  becomes  $k(-A) \hat{x} = kA \hat{x}$  and makes it return to zero & so it goes.

From an energy pt. of view, it has potl. energy  $P_{sp} = \frac{1}{2}kA^2$ . When you let go  $P_{sp}$  reduces while kinetic energy increases. At  $x = 0$ , it is all kinetic mass keeps going to  $-A$  and builds up  $P_{sp} = \frac{1}{2}kA^2$  when it must return.



b) By what factor would you change  $M$  if you wish to increase the frequency by a factor of 3? Why? (5)

frequency  $f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{M}}$

Since  $f$  is proportional to  $\frac{1}{\sqrt{M}}$  you must ~~reduce~~  $\frac{1}{\sqrt{M}}$  by a factor of 3 what is   
 reduce  $M$  by a factor of 9.

c) At what values of  $x$  will the velocity be maximum? Why? (5)

Energy Conservation:  $\frac{1}{2} k A^2 = \frac{1}{2} k x^2 + \frac{1}{2} M V^2$   
at  $x=0$ ,

$$\frac{1}{2} k A^2 = \frac{1}{2} M V^2$$

So  $V$  is maximum at  $x=0$ !

d) At what values of  $x$  is the kinetic energy equal to the potential energy? Why? (8)

Again energy conservation

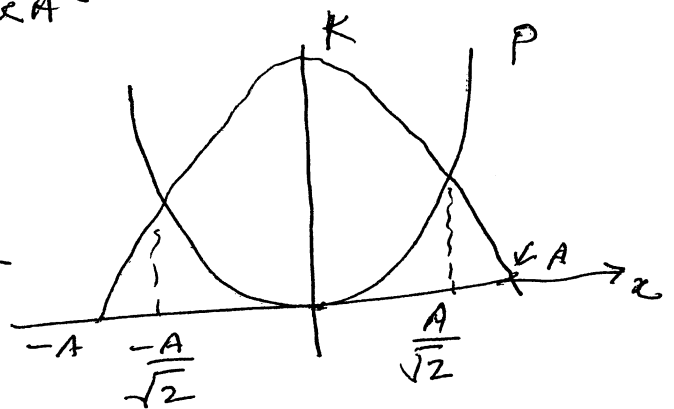
$$\frac{1}{2} k A^2 = \frac{1}{2} M V^2 + \frac{1}{2} k x^2$$

For  $\frac{1}{2} M V^2 = \frac{1}{2} k x^2$  each must be  $\frac{1}{4} k A^2$

$$\frac{1}{2} k x^2 = \frac{1}{4} k A^2$$

$$x^2 = \frac{A^2}{2}$$

$$x = \pm \frac{A}{\sqrt{2}}$$



Problem 2

a) What is a traveling wave? A travelling wave is: (5)

A deviation from equilibrium which is a function of both  $x$  and  $t$  in such a way that  $x$  and  $t$  appear in the combination  
 $(x \mp vt)$

If so, the Deviation travels with a velocity  $\vec{v} = \pm v \hat{x}$ .

b) Fill in the blanks in the equation,

$$D = \sin(x - vt)$$

and explain your reason for introducing a symbol, and what is its physical significance?

(10)

The Completed Equation should read

$$D = A \sin 2\pi \frac{(x - vt)}{\lambda}$$

- i)  $A$  is necessary because  $D$  has units/dimensions while  $\sin$  is dimensionless. Indeed dimensions of  $A$  tell us the value of the wave,  $\sin$  varies between  $\pm 1$  so  $D$  varies between  $\pm A$  hence  $A$  is called amplitude.
- ii) The argument of  $\sin$  cannot have dimensions so we must divide  $(x - vt)$  by a length  $\lambda$ .
- iii) The  $2\pi$  factor is included to explicitly state that  $\sin$  is periodic in  $2\pi$ .
- iv) Advantage of (iii) is that it makes meaning of  $\lambda$  simple. It is the repeat distance and is therefore called wavelength. 4

c) A wave is represented by the equation

$$\underline{y} = 0.05 \sin(6.28x) \cos(12.56t) \hat{y}$$

i) Is this wave longitudinal or transverse? Why?

(5)

The wave is transverse because the amplitude  $0.05 \text{ m } \hat{y}$  is perpendicular to the ~~segment~~  $x$  axis along which the two waves that generated this wave were travelling.

ii) What is the velocity of this wave? Why?

(5)

This is a standing wave with nodes at  $x = 0, \frac{\lambda}{2}, \dots$  so it has no velocity.

Problem 3

a) What is sound?

(5)

Any mechanical wave  
whose frequency lies between  
20 Hz and 20 kHz.

b) What is the speed of sound on the moon? (The moon has no atmosphere.)

(5)

Sound does not exist in a  
vacuum.

c) The intensity of a periodic sound wave of amplitude  $S_m$  and frequency  $w$  is given by the equation

$$I = \frac{1}{2} S_m^2 w^2 \frac{\gamma P_0}{v_s}$$

Where  $P_0$  is the equilibrium pressure and  $v_s$  is the speed of sound.

i) Why is there a  $\gamma = \left(\frac{C_p}{C_v}\right)$  in this equation? (5)

Sound can be thought of as a displacement wave  $S = S_m \sin(kx - wt)$ . Since  $S$  varies with  $x$ , the gas must change its volume  $V$  & hence its pressure. To describe the pressure wave we need to know  $P-V$  relationship. Since freq. is high there is not enough time for heat exchange with surroundings. When  $DQ = 0$ , we have an ADIABATIC process and the  $P-V$  relation is  $PV^\gamma = \text{const.}$

ii) By what factor must you change  $w$  in order to double  $I$ ? Why? (5)

$$I \propto w^2$$

To double  $I$  increase  $w^2$  by 2  
i.e.  $w$  by  $\sqrt{2}$ .

d) Do you think it is possible to have a wave given by

$$P = 10 \frac{N}{m^2} - 11 \frac{N}{m^2} \sin(kx - wt) ?$$

Justify your answer. (5)

No!  
The  $\sin$  fn. varies between +1 and -1. When it becomes -1, the  $P$  in this eqn. will become NEGATIVE. That is unphysical. For a physical pressure wave amplitude must be less than the average pressure.

Problem 4

a) What is an  $\underline{E}$ -field? (5)

If a <sup>stationary</sup> charge  $q$  experiences a force and there is no physical agency\* visible, it must be lying in an  $\underline{E}$ -field.

\* No spring, no object touching  $q$ .

b) Which  $\underline{E}$ -field is larger, one at a distance of 4m from a charge of  $16 \mu\text{C}$  or one at a distance of 3m from a charge of  $-|9 \mu\text{C}|$ ? Why? (8)

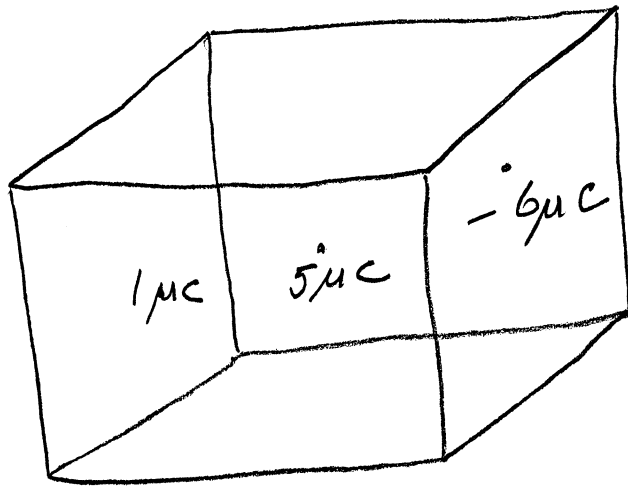
$$\underline{E} = \frac{k_e q}{r^2} \hat{r}$$

$$\underline{E}_{16} = \frac{k_e \times 16 \times 10^{-6}}{16} \text{ N/C } \hat{r} = 10^{-6} k_e \text{ N/C } \hat{r}$$

$$\underline{E}_{-9} = -\frac{k_e \times |9| \times 10^{-6}}{9} \text{ N/C } \hat{r} = -10^{-6} k_e \text{ N/C } \hat{r}$$

Both fields have SAME magnitude only difference is that one is along  $+\hat{r}$  and the other along  $-\hat{r}$ .

c) Charges of  $1 \mu\text{C}$ ,  $5 \mu\text{C}$ , and  $-6 \mu\text{C}$  are located inside a hollow cube. What is the total flux of the  $\underline{E}$ -field through the surfaces of the cube? Why? What can you say about the magnitude of  $\underline{E}$  on the surfaces of the cube? Why? (12)



Gauss' Law says that total flux of  $\underline{E}$  through a closed surface is determined solely by the enclosed charges.

$$\sum_{\vec{c}} \underline{E} \cdot \underline{\Delta A} = \frac{1}{\epsilon_0} \sum Q_i$$

Here  $\sum Q_i = (10^{-6} + 5 \times 10^{-6} - 6 \times 10^{-6}) \text{ C} = 0.$

Hence  $\sum_{\vec{c}} \underline{E} \cdot \underline{\Delta A} = 0.$

However, it is not possible to say anything about the magnitude of  $\underline{E}$  because we don't know where exactly the charges are located and we cannot invoke any symmetry to allow us to calculate  $\sum_{\vec{c}} \underline{E} \cdot \underline{\Delta A}.$