SOLUTIONS – 5

25. Three identical charges ($q = -5.0\ \mu C$) lie along a circle of radius 2.0 m at angles of 30°, 150°, and 270°, as shown in Figure P15.25. What is the resultant electric field at the center of the circle?

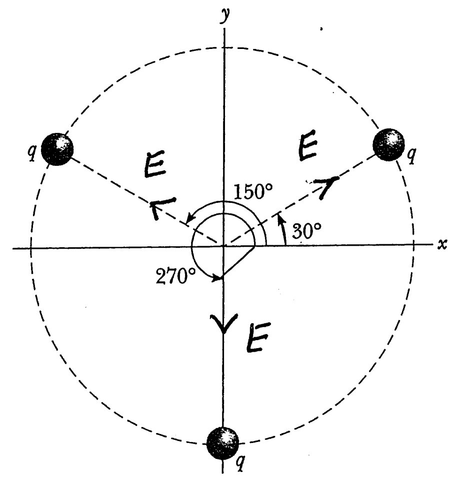


Figure P15.25

All the fields have the same magnitudes

$$E = \frac{9 \times 10^{9} \times 5 \times 10^{-6}}{(2)^2}\ N/C$$

$$= 11.25 \times 10^{3}\ N/C.$$

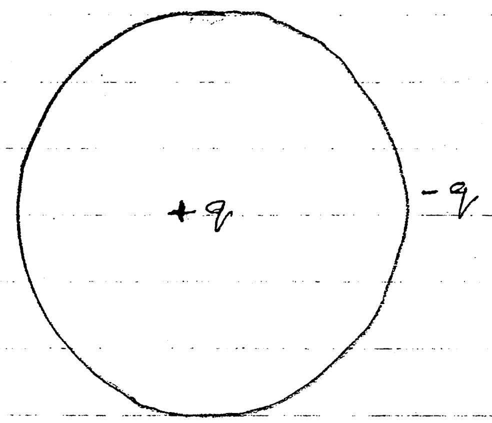
Along X $\quad E_x = E \cos\theta - E\cos\theta$

$$= E \cos 30 - E \cos 30 = 0.$$

Along Y $\quad E_y = E \cos 60 + E \cos 60 - E$

$$= 0.5E + 0.5E - E = 0.$$

Hence $\vec{E}$ at the center is zero!



15-43

There is spherical symmetry about the center of the shell so the E-field can be a function of r only and must be along $\pm\hat{r}$

15-43 (a) For any point outside the ~~surf~~ sphere, any closed surface enclosing the sphere has a net charge $+q-q=0$.

Hence $\Sigma_c \underset{\rightarrow}{E}\cdot\Delta\underset{\rightarrow}{A}=0$ for any spherical surface, so $\underset{\rightarrow}{E}=0$.

(b) Inside the shell, any enclosed surface that are spherical and centered at $+q$ got an enclosed charge $+q$. The electric flux through such a surface is

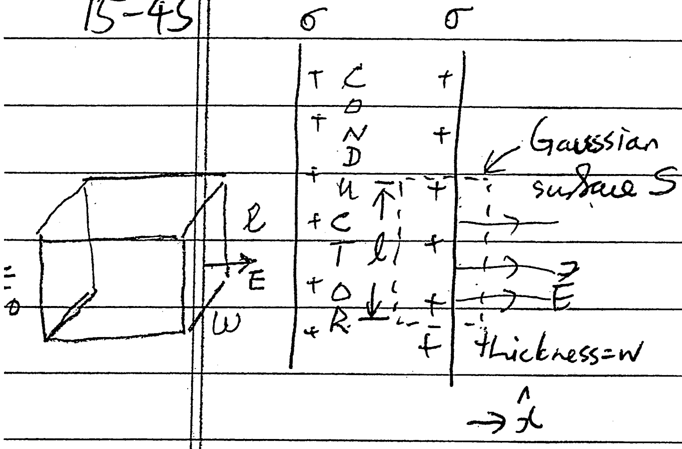
$$4\pi r^2 E=\frac{q}{\varepsilon_0}$$

$$\Rightarrow E=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}=\frac{k_e q}{r^2}$$

Since the enclosed charge is positive, $\vec{E}$ is radially outward and therefore,

$$\vec{E}=\frac{k_e q}{r^2}\hat{r}.$$

15-45



Conducting charges
Inside the sheet, ~~any closed surface got zero charge. Hence any electric~~ are not allowed to move. The $\vec{E}$ field ~~inside the sheet~~ must be zero.

Since the charge is positive, the electric field is away from the sheet. Consider the Gaussian surface S. $\vec{E}$ is in $+\hat{x}$ direction.

By Gauss law, $E(lw) = (\sigma l w)/\varepsilon_0$

$$\Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

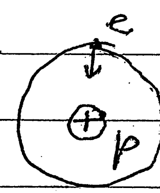
$$\Rightarrow \vec{E} = \frac{\sigma}{\varepsilon_0}\hat{x}$$

15-48 (a) The force acting on each particle is given by the Coulomb's law:

$$F = \frac{k_e |q_1||q_2|}{r^2} = \frac{k_e e^2}{r^2}$$

$$= \frac{(8.99\times10^{9}\ Nm^2C^{-2})(1.60\times10^{-19}\ C)^2}{(0.53\times10^{-10}\ m)^2}$$

$$= 8.2\times10^{-8}\ N$$



ON Electron

$$\vec{F}_E = -8.2\times10^{-8} N\hat{r}$$

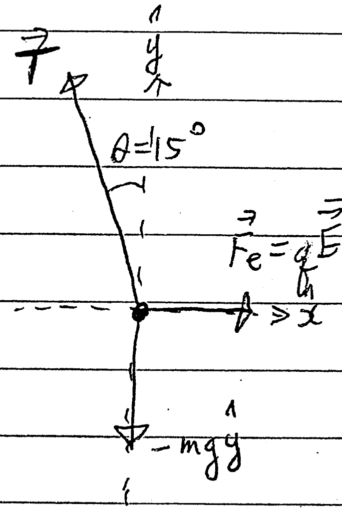
b) You need

$$\vec{F}_e = \frac{-mv^2}{r}\hat{r} \Rightarrow v = \sqrt{\frac{rF}{m}}$$

$$= \sqrt{\frac{(0.53\times10^{-10}\ m)(8.2\times10^{-8}\ N)}{9.11\times10^{-31}\ kg}}$$

$$= 2.2\times10^{6}\ ms^{-1}$$

15-50



$\vec{E} = (1.00 \times 10^3 \, N C^{-1})\hat{x}$, $m = 2g = 2 \times 10^{-3} kg$

For equilibrium

$$\vec{T} + \vec{F}_e - mg\hat{y} = \vec{0}$$

$$\Rightarrow (-T\sin\theta + qE)\hat{x} + (T\cos\theta - mg)\hat{y} = \vec{0}$$

$$\Rightarrow \begin{cases} T\sin\theta = qE \\ T\cos\theta = mg \end{cases}$$

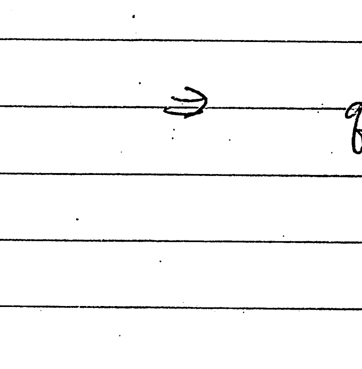
$$\Rightarrow \tan\theta = \frac{qE}{mg}$$

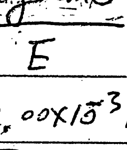
$$\Rightarrow q = \frac{mg\tan\theta}{E}$$

$$= \frac{(2.00 \times 10^{-3} kg)(9.8 m s^{-2}) \tan 15^\circ}{1.00 \times 10^3 N C^{-1}}$$

$$= 5.25 \times 10^{-6} C = 5.25 \mu C$$

15-57 The positive charge experiences an electric force to the right and the negative to the left. Now use the ~~...~~ ~~symmetric~~ symmetry to look at one + charge





$q = 5 \times 10^{-8} C$

$m = 2 \times 10^{-3} kg$

$$\vec{F}_E = -\frac{k_e q^2}{r^2}\hat{r}$$

Let $\vec{E} = E\hat{x}$, and $\vec{F}_E = -F_E\hat{x}$.

$$\vec{F} + \vec{T} + q\vec{E} - mg\hat{y} = 0$$

$$-F_E\hat{x} + (-T\sin\theta\hat{x} + T\cos\theta\hat{y}) + eE\hat{x} - mg\hat{y} = 0$$

$$(-F_E - T\sin\theta + qE)\hat{x} + (T\cos\theta - mg)\hat{y} = 0$$

15-57

$$\Rightarrow \begin{cases} T\sin\theta = eE - F \\ T\cos\theta = mg \end{cases}$$

$$\Rightarrow \tan\theta = \frac{eE - F}{mg} \Rightarrow E = \frac{mg\tan\theta + F}{q}$$

By Coulomb's law,

$$F = \frac{k_e |q_1||q_2|}{(2l\sin\theta)^2} = \frac{(8.99\times10^9)(5\times10^{-8})^2}{(2\times0.1\times\sin10^\circ)^2} = 5.71\times10^{-4}\,N$$

Then $$E = \frac{(2\times10^{-3})(9.8)\tan10^\circ + 5.71\times10^{-4}}{5\times10^{-8}}$$

$$= 4.4\times10^{5}\,NC^{-1}$$

16-3 The work done to move the charge (Na^+) out of the cell is

$=(1.60\times10^{-19}C)(90\times10^{-3}V)$

$=1.4\times10^{-20}J$

[Singly ionized Na^+ carries $+1.6\times10^{-19}C$]

16-7 (a) $E = \frac{\Delta V}{d} = \frac{600\ V}{5.33\times10^{-3}\ m} = 1.13\times10^{5}\ V m^{-1}$ (or NC^{-1})

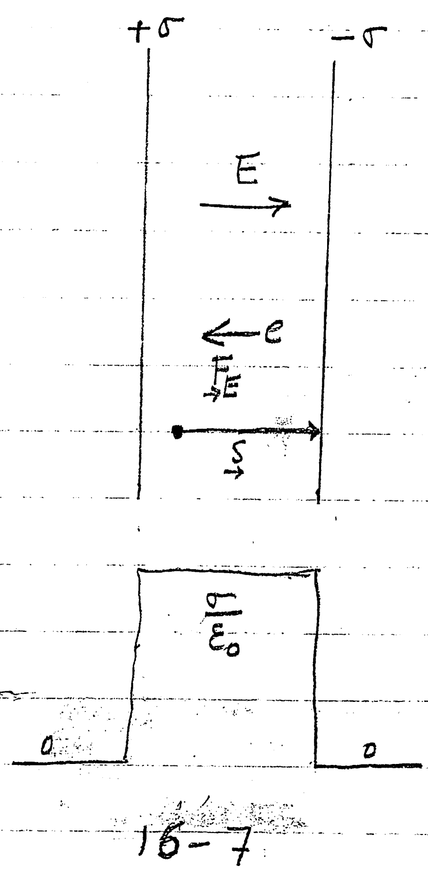
see figure $\vec{E} = +1.13\times10^{5}\ V/m\ \hat{x}$

(b) $F = qE = (1.60\times10^{-19} C)(1.13\times10^{5}\ V m^{-1}) = 1.80\times10^{-14} N$

see figure $\vec{F}_E(\text{electron}) = -1.80\times10^{-14} N\,\hat{x}$

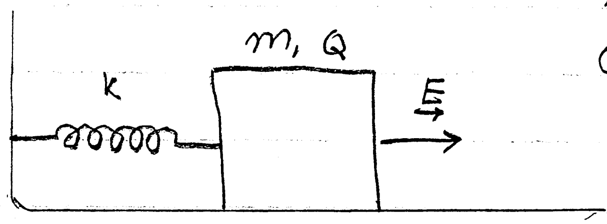
(c) $W = \vec{F}\cdot\vec{s} = Fs\cos\theta = Fs\cos 180^{\circ}$

$= -(1.80\times10^{-14}N)\left[(5.33-2.90)\times10^{-3}\ m\right] = -4.38\times10^{-17}\ J$



16-7

16-9



$m = 4\,kg$
$Q = 50\,\mu C$
$k = 100\,N/m$
$\vec{E} = 5\times10^5\,V/m$

16-9

Assume that the Electric Potl. energy at $x=0$ is zero.
Notice that the initial kinetic energy is zero.
The Potl. Energy due to spring is zero.
Total Energy is zero initially
If spring extends by x.

$$(\Delta P_E)_{spring} = \frac{1}{2}kx^2$$

$$\Delta P_E)_{Electric} = -Q\vec{E}\cdot\Delta\vec{x} = -QEx.$$

Total Energy is zero at the end

$$\frac{1}{2}kx^2 - Ex$$

$$x = \frac{2EQ}{k}$$

For $\equiv$m of course the total force must be zero $\quad -kx + QE = 0$

$$x = \frac{QE}{k}$$

compare this problem to vertical spring mass oscillator where again if you let go y changes by $-\frac{2Mg}{k}$ while the $\equiv$m occurs for $y = -\frac{Mg}{k}$.

Now use #8

$(\Delta PE)_{spring} = \frac{1}{2}kx_{max}^2$

$(\Delta PE)_e = -QEx_{max}$

$$\frac{1}{2}kx_{max}^2 - QEx_{max} = 0$$

$$\Rightarrow \quad x_{max} = \frac{2QE}{k}$$

$$= \frac{2(50.0\times10^{-6}C)(5.00\times10^{5}\,Vm^{-1})}{100\,Nm^{-1}}$$

$$= 0.5\,m$$

(b) At the equilibrium position,

$$-k\vec{x} + Q\vec{E} = \vec{0}$$

$$\Rightarrow \quad -kx\hat{x} + QE\hat{x} = 0$$

$$\Rightarrow \quad x = \frac{QE}{k} = \frac{(50.0\times10^{-6}C)(5.00\times10^{5}\,Vm^{-1})}{100\,Nm^{-1}}$$

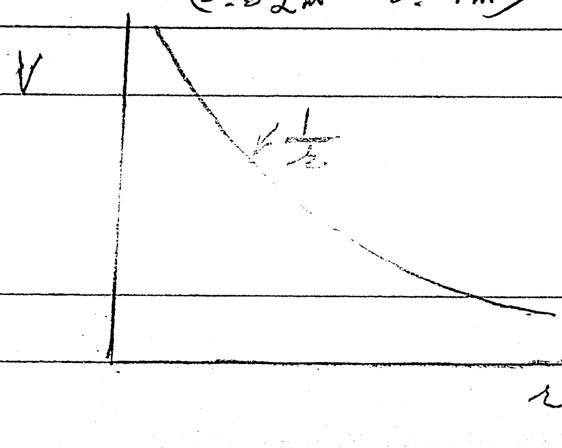
$$= 0.25\,m$$

16-1 (a) $V = \frac{keq}{r} = \frac{(8.99\times10^{9}\,Nm^2C^{-2})(1.60\times10^{-19}C)}{1.00\times10^{-2}m} = 1.44\times10^{-7}\,V$

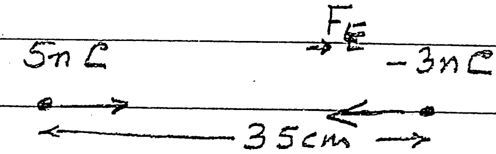
b) $\Delta V = V_2 - V_1 = \frac{keq}{r_2} - \frac{keq}{r_1} = keq\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

$$= (8.99\times10^{9}\,Nm^2C^{-2})(1.60\times10^{-19}C)\left(\frac{1}{0.02m} - \frac{1}{0.01m}\right)$$

$$= -7.19\times10^{-8}\,V$$



16-15(a) $V = \frac{k_e q_1}{r_1} + \frac{k_e q_2}{r_2} = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$



$$= \left(8.99\times10^{9}\,\frac{Nm^2}{C^2}\right)\left(\frac{5.00\times10^{-9}C}{0.175m} - \frac{3.00\times10^{-9}C}{0.175m}\right)$$

$$= 103V$$

(b) $PE = \frac{k_e q_1 q_2}{r_{12}} = \frac{(8.99\times10^{9}\,Nm^2C^{-2})(5.00\times10^{-9}C)(-3.00\times10^{-9}C)}{0.35\,m}$

$$= -3.85\times10^{-7}\,J$$

Negative sign means positive work has to be done to separate the charges. The force between them is an attractive force. To store work you must apply $F \parallel \hat{x}$ if you move -3nC charge.

16-19 By conservation of energy,

$$(KE + \underbrace{PE}_{0})_i = (\underbrace{KE}_{0} + PE)_f$$

$$\frac{1}{2} m_\alpha v_i^2 = \frac{k_e Q(2e)}{r_f}$$

$$\Rightarrow r_f = \frac{4 k_e (79) e^2}{m_\alpha v_i^2}$$

$$= \frac{4\,(8.99\times10^{9}\,Nm^2C^{-2})(79)(1.60\times10^{-19}C)^2}{(6.64\times10^{-27}kg)(2.00\times10^{7}\,ms^{-1})^2}$$

$$= 2.74\times10^{-14}\,m$$

16-23(a) $C = \frac{\epsilon_0 A}{d} = \frac{(8.85\times10^{-12}\,C^2N^{-1}m^{-2})(1.0\times10^{6}\,m^2)}{800\,m} = 1.1\times10^{-8}F$

(b) $Q_{max} = C(\Delta V)_{max} = C E_{max} d$

$$= (1.11\times10^{-8}\,F)(3.0\times10^{6}\,NC^{-1})(800\,m)$$

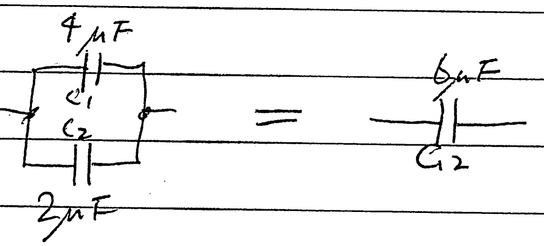
$$= 27\,C$$

$Q = 4\times10^{-12}\,C$

16-27(a) $\Delta V = \frac{Q}{C} = \frac{Qd}{\varepsilon_0 A} = \frac{(400\times10^{-12}\,C)(1.00\times10^{-3}\,m)}{(8.85\times10^{-12}\,C^2 N^{-1} m^{-2})(5.00\times10^{-4}\,m^2)} = 904\,V$

(b) $E = \frac{|\Delta V|}{d} = \frac{90.4\,V}{1.00\times10^{-3}\,m} = 9.04\times10^{4}\,V\,m^{-1}$

16-31(a)

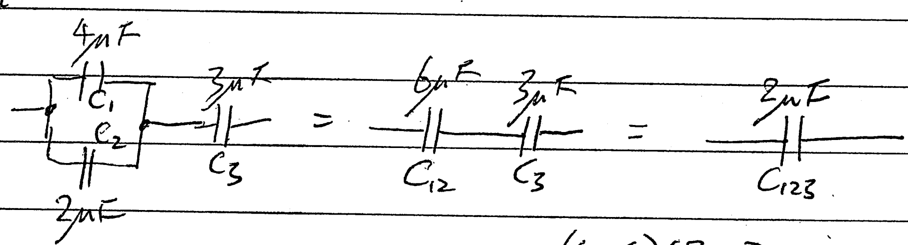


$4\mu F + 2\mu F = 6\mu F$

Rule

$C_p = \sum C_i$

$\frac{1}{C_s} = \sum \frac{1}{C_i}$



$\frac{(6\mu F)(3\mu F)}{6\mu F + 3\mu F} = 2\mu F$

$Q_{123} = C_{123}\,\Delta V = (2\mu F)(12V) = 24\,\mu C$

Then $Q_{12} = Q_3 = Q_{123} = 24\,\mu C$

$\Delta V_3 = \frac{Q_3}{C_3} = \frac{24\,\mu C}{3\,\mu F} = 8V$

$\Delta V_{12} = 12V - 8V = 4V$

$\Delta V_1 = \Delta V_2 = \Delta V_{12} = 4V$

$Q_1 = C_1 \Delta V_1 = (4\mu F)(4V) = 16\,\mu C$

$Q_2 = C_2 \Delta V_2 = (2\mu F)(4V) = 8\,\mu C$