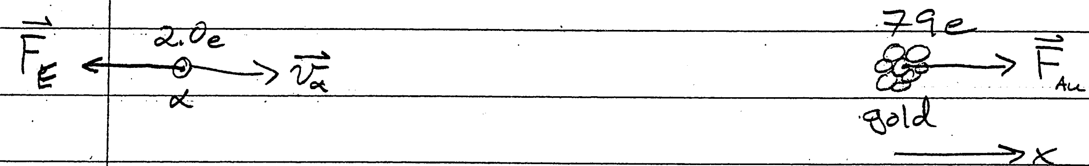
# WEEK-4 (SOLNS)

15-3 An alpha particle (charge $= +2.0\,e$) is sent @ high speed toward a gold nucleus (charge $= +79e$). What is electrical force acting on the alpha particle when it is $2.0 \times 10^{-14}$ m from the gold nucleus?




If the alpha particle is located as shown..., then we know the direction of the force is in the negative x direction because both charges are positive. To find the magnitude of the force, use

$$F = \frac{k_e |q_1||q_2|}{r^2}$$

$$= \frac{8.99 \times 10^9 \frac{Nm^2}{C^2} (2e)(79e)}{(2.0 \times 10^{-14}\,m)^2}$$

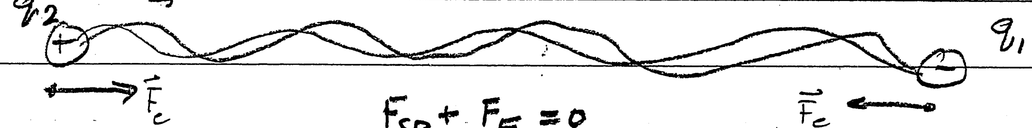
$$= \frac{(8.99)(158)(1.6)^2}{4} \times 10^{9+28-38}\ N$$

$$= 910 \times 10^{-1}\ N = 91\ N \qquad \vec{F}_E = -91\,N\,\hat{x}$$

15-6 A molecule of DNA is $2.17\,\mu m$ long. The ends of the molecule become singly ionized — negative on one end, positive on the other. The helical molecule acts like a spring and compresses $1.00\%$ upon becoming charged. Determine the effective spring constant of the molecule

$$\vec{F}_E = \cancel{+}(k_e |q_1||q_2| / r^2)\,\hat{x}$$

$$\vec{F}_{sp} = -k\,\Delta x\,\hat{x} \quad \text{and } \Delta x \text{ is } -\text{ive!}$$




$$\vec{F}_{sp} + \vec{F}_E = 0$$

When the force of the "spring" plus $\vec{F}_E$ equal ~~equal to zero~~ force, the system will be in equilibrium. The electric force is trying to compress the molecule, and the spring is pushing the ions apart.

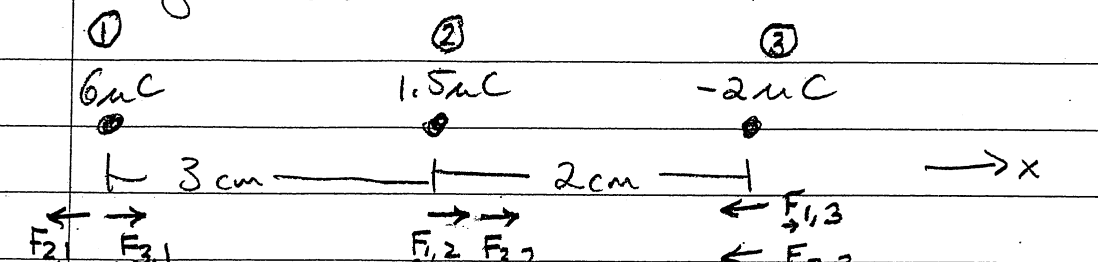
so $\vec{F}_{spring} \cancel{=} \vec{F}_E$

$$k\,\Delta X = K_e \frac{q_1 q_2}{r^2}$$

$$k = K_e \frac{q_1 q_2}{\Delta X\, r^2} = \frac{8.99 \times 10^9 \frac{Nm^2}{C^2} (e)^2}{(0.01)(2.17\,\mu m)(2.17\,\mu m)^2}$$

$$= \frac{(8.99)(1.6)^2}{(2.17)^3} \times 10^{9-38+2+18} \frac{N}{m} = 2.25 \times 10^{-9}\ N/m$$

15-10 Calculate the magnitude and direction of The Coulomb force on each of the three charges shown here




We can shorten things using the third law.

$$\vec{F}_{1,2} = k_e \frac{(6\mu C)(1.5\mu C)}{(3cm)^2}\hat{x} = \frac{(8.99)(6)(1.5)}{(3)^2} \times 10^{9+4-12}\, N\,\hat{x}$$

$$\vec{F}_{1,3} = k_e \frac{(-2\mu C)(6\mu C)}{(5cm)^2}\hat{x} = \frac{-8.99(2)(6)}{3^2} \times 10^{9+4-12}\, N\,\hat{x}$$

$$\vec{F}_{2,3} = k_e \frac{(1.5\mu C)(-2\mu C)}{(2cm)^2} = \frac{-8.99(1.5)(2)}{(4)^{2}} \times 10^{9+4-12}\, N$$

$\vec{F}_{1,2} = 8.99 \times 10^1\, N\hat{x} = 89.9\, N\hat{x}$

$\vec{F}_{1,3} = -4.32 \times 10^1\, N\hat{x} = -43.2\, N\hat{x}$

$\vec{F}_{2,3} = -6.74 \times 10^1\, N\hat{x} = -67.4\, N\hat{x}$

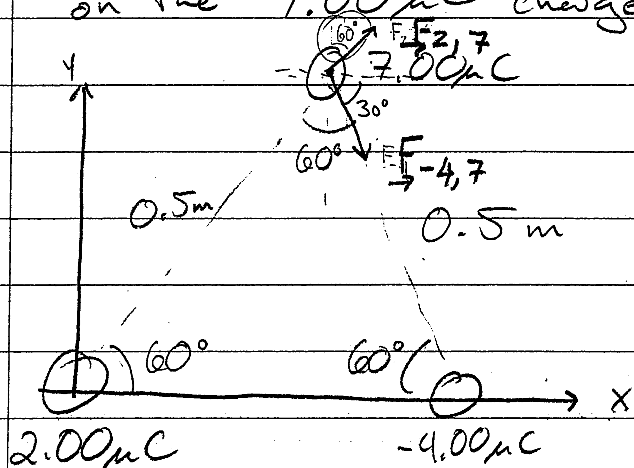
Now when finding the total force on each ion, just be careful with direction.

$\vec{F}_1 = (-8.99 + 4.32) \times 10^1\, N\hat{x} = -46.7\, N\hat{x}$

$\vec{F}_2 = (+89.9 + 67.4)\, N\hat{x} = 157\, N\hat{x}$

$\vec{F}_3 = (-43.2 - 67.4)\, N = -111\, N\hat{x}$

15-13 Calculate the net electric force on the $7.00\mu C$ charge.




Magnitudes of

Force from $2\mu C$ $\rightarrow$ $F_{2,7} = \dfrac{k_e(2\mu C)(7.00\mu C)}{(0.5m)^2}$

Force from $-4\mu C$ $\quad F_{-4,7} = \dfrac{k_e(4\mu C)(7\mu C)}{(0.5m)^2}$

We need to consider the vector nature of the forces.

| $F_x$ | $F_y$ |
|---|---|
| $F_{2,7}\cos 30°$ | $F_{2,7}\sin 30°$ |
| $F_{-4,7}\cos 30°$ | $-F_{-4,7}\sin 30°$ |
| $(F_{2,7}+F_{-4,7})\cos 30°$ | $(F_{2,7}-F_{-4,7})\sin 30°$ |
| $0.755N$ | $-0.436N$ |

$F_2 = \dfrac{8.99(2)(7)}{(0.5)^2} \times 10^{9+(-12)} N$

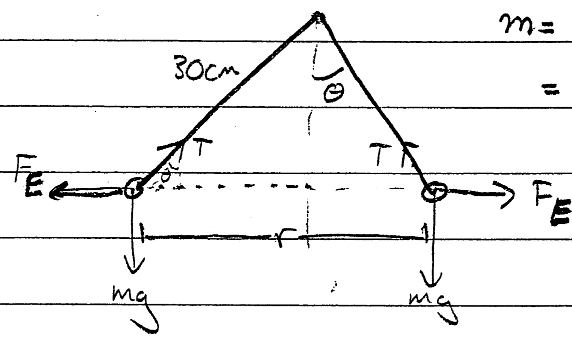
$F_{2,7} = 503 \times 10^{-3} N$

$F_{4,7} = 101 \times 10^{-3} N$

$\vec{F}_7 = 0.755N\hat{x} - 0.436N\hat{y} \qquad |\vec{F}| = 0.872N$

15-15

Two small spheres of mass 0.2g are suspended by pendulums and have equal charges. Given the angle, find the charge




$m = 0.2\,g$
$= 0.2 \times 10^{-3}\,kg$

$\sin\theta = \frac{r}{2 \times 0.3}$

The forces need to be balanced

x: $F_E = T\sin\theta$

y: $mg = T\cos\theta$

divide the two equations to eliminate T.

$$\frac{F_E}{mg} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

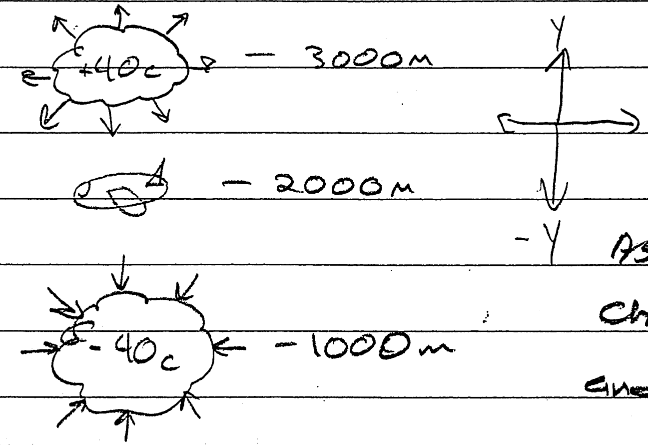
$$F_E = k_e\frac{q^2}{r^2} = mg\tan\theta \;,\quad q = \sqrt{\frac{r^2 mg\tan\theta}{k_e}}$$

but what is r? $30_{cm}\sin\theta = r/2$

$$q = \left(\frac{(60cm\sin\theta)^2(0.2g)(9.8\,m/s^2)\tan\theta}{8.99\,Nm^2/C^2}\right)^{1/2}$$

$q = 7.2\,NC$

15-19 An airplane is flying through a thunderstorm @ 2000m. If there are charge concentration of 40c @ 3000m + -40.0 c @ 1000m, what is $\vec{E}$ @ the aircraft?




For a pt. charge $\underset{\rightarrow}{E} = \frac{k_e Q}{r^2}\hat{r}$

Assume point charges at 3000m and 1000m

What is the contribution to $\vec{E}$ @ the aircraft due to charge on top?

$$\vec{E}_+ = -k_e \frac{40.0\,c}{(1000m)^2}(\hat{y}) = -\frac{(8.99\times10^9)\,40}{1000^2}\frac{N}{C} = -3.60\times10^5 \frac{N}{C}$$

The cloud of negative charge will contribute exactly the same field at that point.

$$\vec{E} = \vec{E}_+ + \vec{E}_- = 2\vec{E}_+ = -7.20\times10^5 \tfrac{N}{C}(\hat{y})$$

15-22

What magnitude and direction does an electric field need to stop a proton w/ $3.25 \times 10^{-15}$ J in a distance of 1.25 m?

Conservation of Energy $P_f + K_f = P_i + K_i$

$$P_f + 0 = P_i + K_i$$

$$P_f - P_i = K_i$$

The change in potential energy is equal to the initial kinetic energy, the particle will have stopped.

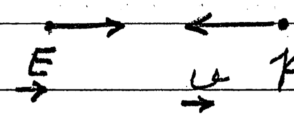
Change in P is:

$$qE\Delta x = 3.25 \times 10^{-15} \text{ J}, \quad q = 1.6 \times 10^{-19} \text{ C}$$

$$\Delta x = 1.25 \text{ m}$$

$$E = \frac{3.25}{(1.25)(1.6)} \times 10^{-15+19} \frac{N}{C}$$

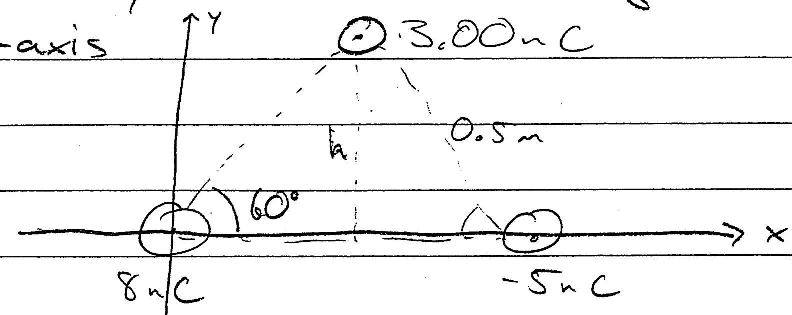
$$= 1.63 \times 10^4 \text{ N/C}$$




For the direction, we know the force will have to be opposite of the initial velocity to slow it down. Since charge is positive, we can say the field must point in the direction opposite of the initial velocity.

15-24

Calculate the electric field @ a point midway between the charges on the x-axis




First, we'll calculate the contribution from the charges on the x-axis

$$\vec{E}_8 = k_e \frac{(8nC)}{(0.25m)^2}\hat{x} \quad , \quad \vec{E}_{-5} = k_e \frac{(5nC)}{(0.25m)^2}\hat{x}$$

$$\vec{E}_8 = 1150 \times 10^0 \, N/C \, \hat{x} \qquad \vec{E}_{-5} = 719 \times 10^0 \, N/C \, \hat{x}$$

For the third contribution, we need $h$.

$$h = 0.5m \sin 60°$$

$$\vec{E}_3 = -k_e \frac{3nC}{h^2}\hat{y} = \frac{(8.99)(3)}{(0.5 \sin 60°)^2} \times 10^{9-9} \, N/C \, (-\hat{y})$$

$$= -144 \, N/C \, \hat{y}$$

$$\vec{E} = (1150 + 719) \times 10^0 \, N/C \, \hat{x} - 144 \, N/C \, \hat{y}$$

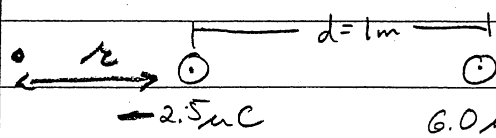
$$= 1880 \, N/C \, \hat{x} - 144 \, N/C \, \hat{y}$$

Magnitude of $\vec{E}$ is

$$|\vec{E}| = 1.88 \times 10^3 \, N/C$$

15-27 Determine the point (other than infinity) @ which $\vec{E} = 0$

$$\vec{E} = \frac{k_e Q}{r^2}\hat{r}$$




To simplify the problem, we can rule out two areas. Realizing that the solution must be on the line going through the charges, we first note that the region between the charges won't work because the fields point in the same direction. The area to the right of the 6μC charge won't work either because the other charge is too small, (does this make sense to you?). So we expect the solution to be to the left of the −2.5μC charge.

$$\vec{E}_{-2.5} = +k_e\frac{(+2.5\mu C)}{r^2}\hat{x} \qquad \vec{E}_6 = -k_e\frac{6\mu C}{(r+1m)^2}\hat{x}$$

$$\vec{E}_{-2.5} + \vec{E}_6 = 0 = k_e\left(\frac{2.5\mu C}{r^2} - \frac{6\mu C}{(r+1m)^2}\right)$$

$$\frac{2.5}{r^2} = \frac{6}{(r+1m)^2} \Rightarrow 2.5(r^2+2r+1m) = 6r^2$$

$$3.5r^2 - 5r - 2.5 = 0, \quad r = \left(\frac{5}{7} + \frac{1}{7}\sqrt{25+35}\right)m = \boxed{1.8m = r}$$

alternately, take sq. root $\frac{\sqrt{2.5}}{r} = \frac{\sqrt{6}}{1+r}$

$\sqrt{2.5}(r+1) = \sqrt{6}\,r$ gives $r = 1.8m$

15-28 a) Determine the ratio $q_1/q_2$

b) What are the signs of $q_1$ + $q_2$?

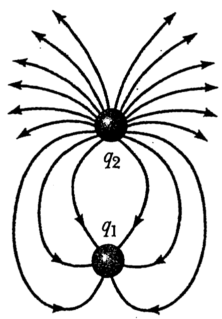
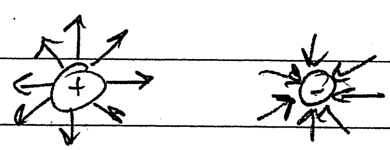


**Figure P15.28**

Use the fact that the number of field lines per unit area through a surface perpendicular to the lines is proportional to the strength of the field @ that surface. This means the number of field lines coming out of a charge is proportional to the magnitude of that charge, (because the field is proportional to the charge). So we simply need to count the lines.

$$\frac{q_1}{q_2} = \frac{\#\text{ of lines coming out of } q_1}{\#\text{ of lines coming out of } q_2} = \frac{6}{18} = \frac{1}{3}$$

The signs on the charges are determined by the directions of the lines.



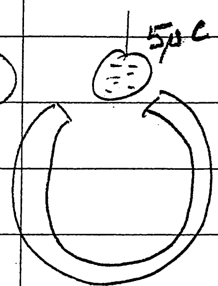

so $q_1$ is negative and $q_2$ is positive

so actually $\frac{q_1}{q_2} = -\frac{1}{3}$ (Notice the minus sign)

15-33 Find the charge on the inside and outside of the hollow conductor for the following configurations.

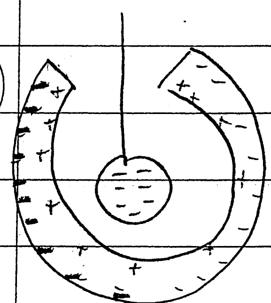
from fig 15-20 Ball initially has $5\mu C$

a)




| Inner | Outer |
|---|---|
| 0c | 0c |

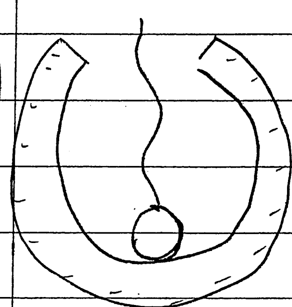
b)



| Inner | Outer |
|---|---|
| $+5\mu C$ | $-5\mu C$ |

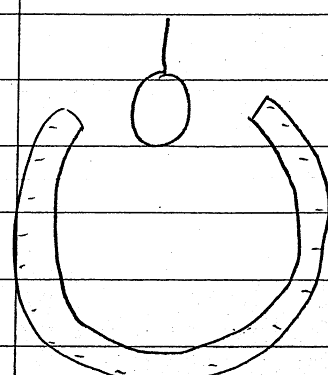
b/c $\vec{E}$ field inside conductor must be zero and charge is conserved so $+5\mu C - 5\mu C = 0$.

c)



| Inner | Outer |
|---|---|
| 0c | $-5\mu C$ |

d)



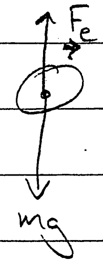
| Inner | Outer |
|---|---|
| 0c | $-5\mu C$ |

15-36

$|\vec{E}| = 3 \times 10^{4}\ N/C$

$q = 1.6 \times 10^{-19}\ C$

Estimate the radius of an oil drop of density $858\ \frac{kg}{m^3}$ for which its weight could be balanced by the electric force of this field on one electron.




$$\vec{F_e} - mg\hat{y} = 0$$

$$F_e = qE = mg$$

$$\rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi r^3}, \quad m = \rho \frac{4}{3}\pi r^3$$

so $qE = \rho \frac{4}{3}\pi r^3$

$$r = \left(\frac{3qE}{4\rho\pi}\right)^{1/3} = \left(\frac{(3)(1.6)(3)}{4(858)\pi} \times 10^{-19+4}\ m^3\right)^{1/3}$$

$$\approx \left(0.00134 \times 10^{-15}\right)^{1/3} m$$

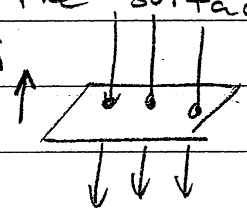
$$= 1.11 \times 10^{-6}\ m \sim 1\ \mu m$$

15-38

A flat surface of area $3.2\ m^2$ is rotated in a uniform electric field $|\vec{E}| = 6.2 \times 10^5\ N/C$. Determine the electric flux through this area a) when the field is perpendicular to the surface and b) when the field is parallel to the surface.

a) When the field is perpendicular to the surface, all the field lines penetrate the surface.

$\vec{E} \parallel \hat{n}$

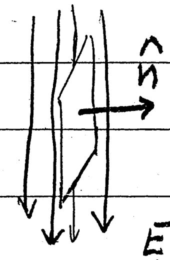


so $\Phi = EA = (6.2)(3.2) \times 10^5 \frac{Nm^2}{C}$

$$\Phi = 2 \times 10^6 \frac{Nm^2}{C}$$

$$\Phi = \vec{E} \cdot \vec{A} = EA \cos(\vec{E}, \hat{n}), \quad \cos 0 = 1$$

b) when the surface is parallel to the field, no field lines penetrate the surface.



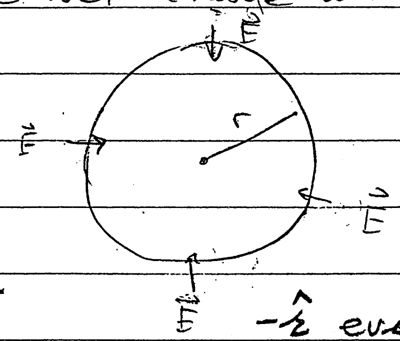
so $\Phi = 0$, $\cos \frac{\pi}{2} = 0$

$\vec{E} \perp \hat{n}$

15-40

The electric field everywhere on the surface of a thin spherical shell of radius 0.75m is measured to be equal to 8900 N/c and points radially toward the center of the sphere.

a) What is the net charge within the sphere's surface?




$\sum \vec{E} \cdot \vec{\Delta A} = \frac{1}{\varepsilon_0} \sum Q_i$

$\vec{E} = -E\hat{\varepsilon}$

$\vec{E} \cdot \vec{\Delta A} = E\Delta A \cos\pi$

Here $\vec{E}$ is parallel to $-\hat{\varepsilon}$ everywhere so $\cos\pi = -1$. Also $E$ is same everywhere so we can write

$$\varepsilon_0 \sum \vec{E} \cdot \vec{\Delta A} = -\varepsilon_0 EA = -\varepsilon_0 E\, 4\pi r^2$$

$$= -(8.85)(890) \cdot 4\pi (0.75)^2 \times 10^{-12}\, C$$

$$= -5.6 \times 10^{-8}\, C$$

b) What can you conclude about the nature and distribution of the charge inside the sphere.

As we saw above:

Since the field points toward the center of the sphere, the charge is negative. Since the field is spherically symmetric, the charge distribution must also be spherically symmetric.