SOLUTIONS HW 2

13-26 $x = (0.3m) \cos\left(\frac{\pi t}{3}\right)$

The general Equ. of an Oscillator is

$$x = A \cos(\omega t + \theta_0)$$

where A = Amplitude

ω = angular frequency

θ_0 = Phase Angle.

a) $x(0) = 0.3m \cos 0 = 0.3m$

$$x(0.6s) = 0.35 \cos\left(\frac{\pi \times 0.6}{3}\right) = 0.283m$$

b) Amplitude

$A = 0.3m$

c) Frequency

$$f = \frac{\omega}{2\pi} = \frac{\pi}{3 \times 2\pi} = \frac{1}{6}\ Hz.$$

d) Period $T = \frac{1}{f} = 6$ sec.

[13-38] Since a bat can detect an insect whose size is about one wavelength of the sound it emits, it is neccessary to find the wavelength of that wave. We are given

$$\begin{aligned} f &= 60.0 \text{ kHz} \\ &= 60.0 \times 10^3 \text{ Hz} \\ c_S &= 340 \text{ m/s} \end{aligned}$$

where f is the frequency and c_S is the speed of sound in air. Now

$$\lambda f = c_S$$

with λ the wavelength. Hence

$$\begin{aligned} \lambda &= \frac{c_S}{f} \\ &= \frac{340 \text{ m/s}}{6.00 \times 10^4 \text{ Hz}} \\ &= 56.7 \times 10^{-4} \text{ m} \\ &= 5.67 \text{ mm} \end{aligned}$$

[13-41] The generator that creates the wave is going to determine the frequency of that wave. We are given that this oscillator makes 40.0 vibrations in 30.0 s. So, the frequency of the wave must be

$$\begin{aligned} f &= \frac{40.0 \text{ vibrations}}{30.0 \text{ s}} \\ &= 1.33 \text{ Hz} \end{aligned}$$

We are also given the distance that a maximum travels, and the time that it takes to traverse this distance; from this, we can get the speed of the wave:

$$v = \frac{425 \text{ cm}}{10.0 s}$$

$$
\begin{aligned}
&= \frac{4.25 \text{ m}}{10.0s} \\
&= 0.425 \text{ m/s}
\end{aligned}
$$

We now have the speed of the wave and its frequency so we can get λ

$$
\begin{aligned}
\lambda &= \frac{v}{f} \\
&= \frac{0.425 \text{ m/s}}{1.33 \text{ Hz}} \\
&= 0.320 \text{ m}
\end{aligned}
$$

[13-42] Let's first define a coordinate system; we'll choose $\hat{\mathbf{x}}$ to be East and $\hat{\mathbf{y}}$ to be North. With this definition, we have

$$
\begin{aligned}
\mathbf{v}_{wave}^{shore} &= 4.0 \text{ m/s } \hat{\mathbf{x}} \\
\mathbf{v}_{boat}^{shore} &= -1.0 \text{ m/s } \hat{\mathbf{x}}
\end{aligned}
$$

Where the notation is that the subscript informs us of the object that is moving and the superscript states to what object that value is relative.

Now, since the problem asks us for two velocities of the boat, we'll solve the general case for any velocity of the boat. What is important here is the velocity of the ocean waves with respect to the boat. This is given by

$$\mathbf{v}_{wave}^{boat} = \mathbf{v}_{wave}^{shore} + \mathbf{v}_{shore}^{boat}$$

with

$$\mathbf{v}_{shore}^{boat} = -\mathbf{v}_{boat}^{shore}$$

Of course, we're actually only concerned about the speed:

$$v_{wave}^{boat} = \left| \mathbf{v}_{wave}^{shore} + \mathbf{v}_{shore}^{boat} \right|$$

with the speed, and the wavelength, $\lambda = 20$ m, given by the problem, we can get the frequency:

$$f = \frac{v_{wave}^{boat}}{\lambda}$$

$$
\begin{aligned}
&= \frac{\left|\mathbf{v}_{wave}^{shore} + \mathbf{v}_{shore}^{boat}\right|}{\lambda} \\
&= \frac{\left|\mathbf{v}_{wave}^{shore} - \mathbf{v}_{boat}^{shore}\right|}{\lambda}
\end{aligned}
$$

Armed with that, we can now answer the question:

(a) $\mathbf{v}_{shore}^{boat} = 0$, so

$$
\begin{aligned}
f &= \frac{\left|\mathbf{v}_{wave}^{shore}\right|}{\lambda} \\
&= \frac{4.0\ \mathrm{m/s}}{20\ \mathrm{m}} \\
&= 0.2\ \mathrm{Hz}
\end{aligned}
$$

(b) $\mathbf{v}_{shore}^{boat} = -1.0\ \mathrm{m/s}$

$$
\begin{aligned}
f &= \frac{\left|\mathbf{v}_{wave}^{shore} - \mathbf{v}_{boat}^{shore}\right|}{\lambda} \\
&= \frac{\left|4.0\ \mathrm{m/s}\ \hat{\mathbf{x}} - (-1.0\ \mathrm{m/s})\ \hat{\mathbf{x}}\right|}{20\ \mathrm{m}} \\
&= \frac{5.0\ \mathrm{m/s}}{20\ \mathrm{m}}
\end{aligned}
$$

[13-46] The situation is depicted in Figure 2. The object is assumed at rest, and we have the Free Body Diagram for it shown in Figure 3 We then use Newton's second law, in the y-direction:

$$T - m_{obj} g_{moon} = 0$$

Again, the object is at rest so the acceleration is zero. This gives us g_{moon} in terms of the tension, T.

$$0 = T - m_{obj} g_{moon}$$

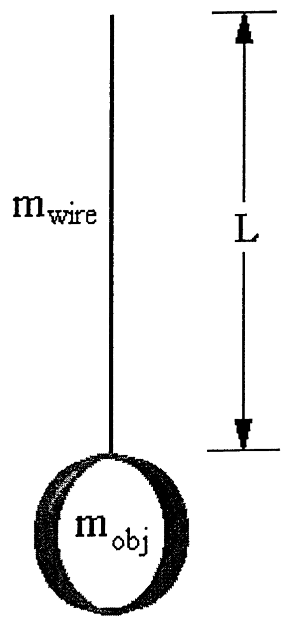


Figure 2: Object Hanging from Wire

$$
\begin{aligned}
T &= m_{obj} g_{moon} \\
g_{moon} &= \frac{T}{m_{obj}}
\end{aligned}
$$

If only we could find the tension... Oh, right. We know the speed of the wave; or rather, we can find it since we know the length of the wire ($L = 1.60$ m), and the time it takes traverse the wire ($t = 36.1$ ms $= 36.1 \times 10^{-3}$ s $=$ 3.61×10^{-2} s), hence

$$
\begin{aligned}
v &= \frac{L}{t} \\
&= \frac{1.60 \text{ m}}{3.61 \times 10^{-2} \text{ s}} \\
&= 44.3 \text{ m/s}
\end{aligned}
$$

and we can also find μ:

$$
\begin{aligned}
\mu &= \frac{m_{wire}}{L} \\
&= \frac{4.00 \times 10^{-3} kg}{1.60 \text{ m}}
\end{aligned}
$$

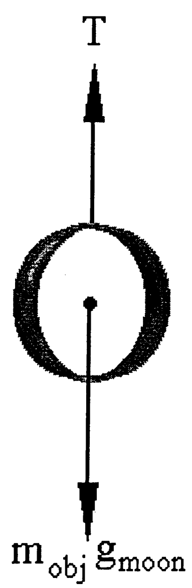


Figure 3: Free Body Diagram of Object

$$= 2.5 \times 10^{-3}\ \text{kg/m}$$

From the last problem, we found

$$T = v^2\mu$$

So that

$$\begin{aligned} g_{moon} &= \frac{T}{m_{obj}} \\ &= \frac{v^2\mu}{m_{obj}} \\ &= \frac{(44.3\ \text{m/s})^2\,(2.5\times10^{-3}\ \text{kg/m})}{3.00\ \text{kg}} \\ &= 1.64\ \text{m/s}^2 \end{aligned}$$

$\boxed{13-48}$ the only difference between the two strings is the total mass and hence the linear mass density of each

$\mu_1 = \frac{M_1}{L}$ first string

$\mu_2 = \frac{M_2}{L} = \frac{M_1/2}{L} = \frac{1}{2}\left(\frac{M_1}{L}\right) = \frac{1}{2}\mu_1$ second string

speed is given by $v = \sqrt{\frac{T}{\mu}}$

$v_1 = \sqrt{\frac{T}{\mu_1}}$ $\quad v_2 = \sqrt{\frac{T}{\mu_2}} = \sqrt{\frac{T}{\mu_1/2}} = \sqrt{2}\left(\sqrt{\frac{T}{\mu_1}}\right)$

$= 5.00 \text{ m/s}$ $\quad = \sqrt{2}\, v_1 = 7.07 \text{ m/s}$

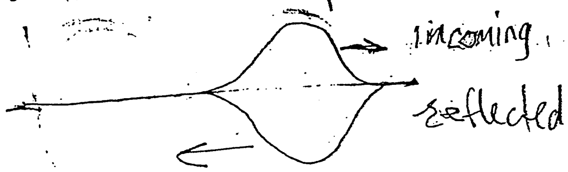
$\boxed{13-51}$ in this case we are dealing with the same undeformed string so the only quantity which will change is the tension

starting with $v = \sqrt{\frac{T}{\mu}}$, solve for tension $T = \mu v^2$

hence we have $T_1 = \mu v_1^2$ and $T_2 = \mu v_2^2$ where the only unknowns are T_2 and μ; a trick to eliminate μ is to divide the second equation by the first

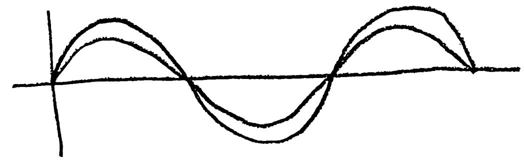
$$\frac{T_2}{T_1} = \frac{\cancel{\mu} v_2^2}{\cancel{\mu} v_1^2} \Rightarrow T_2 = T_1 \frac{v_2^2}{v_1^2} = \frac{(30.0 \text{ m/s})^2}{(20.0 \text{ m/s})^2}(6.00\text{ N}) = 13.5\text{ N}$$

$\boxed{13-52}$ a if the string is rigidly attached to the post, the pulse inverts when it reflects; hence when the forward and reflected pulses meet, they cancel and the amplitude is zero



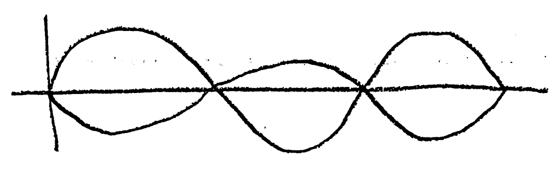
b if the string is freely attached to the post, the pulse remains the same when it reflects; hence when the forward and reflected pulses meet, the amplitude is doubled $A' = 2A = 2(0.15\,m) = 0.30\,m$

$\boxed{13-53}$ a the largest amplitude occurs when there is constructive interference and the amplitudes add



$$A_{max} = A_1 + A_2 = 0.30 + 0.20 = 0.50\,m$$

b the smallest amplitude occurs when there is destructive interference and the amplitudes subtract



$$A_{min} = A_1 - A_2 = 0.30 - 0.20 = 0.10\,m$$

$\boxed{13-60}$ a energy conservation can be used to find the kinetic energy and hence the speed of the pellet

$$E_i = E_f \Rightarrow \tfrac{1}{2}kx^2 = \tfrac{1}{2}mv^2$$

solve for the speed $v = x\sqrt{\frac{k}{m}} = (0.200m)\sqrt{\frac{9.80\,N/m}{1.00\times10^{-3}\,kg}} = 19.8\,m/s$

~~the horizontal~~ range will be limited by the travel time until it hits the floor the time needed to fall 1.00 m is given by

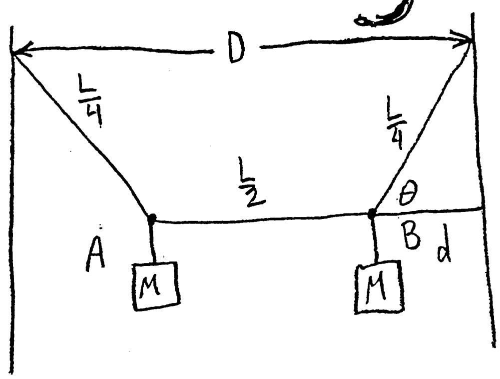
$$\Delta y = \cancel{v_{oy}t}^{\,0} + \tfrac{1}{2}a_y t^2 \Rightarrow -1.00 = \tfrac{1}{2}(-9.80)t^2$$

$$t = \sqrt{\frac{2(-1.00)}{-9.80}} = 0.452\,s$$

the horizontal travel distance is

$$\Delta x = v_{ox}t + \cancel{\tfrac{1}{2}a_x t^2}^{\,0} = (19.8\,m/s)(0.452\,s) = 8.94\,m$$

13-64



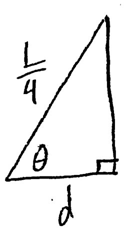
$D = 2.00\,m$
$L = 3.00\,m$
$m = 10.0\,g$
$M = 2.00\,kg$

to find the travel time, we need the speed; to find the speed we need the linear mass and the tension; to find the tension we need to solve the free body diagram of the string

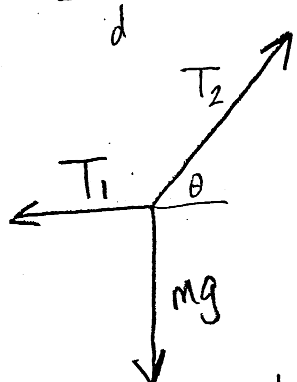
$$\mu = \frac{m}{L} = \frac{10.0\times10^{-3}\,kg}{3.00\,m} = 3.33\times10^{-3}\,\frac{kg}{m}$$

need θ first

$$d = \frac{D - 2\left(\frac{L}{4}\right)}{2} = \frac{2.00 - 2\left(\frac{3.00}{4}\right)}{2} = 0.250\,m$$



$$\cos\theta = \frac{d}{L/4} \Rightarrow \theta = \cos^{-1}\left(\frac{d}{L/4}\right) = \cos^{-1}\left(\frac{0.250}{0.750}\right) = 70.5^\circ$$



free body diagram
since it is in equilibrium, sum of the forces is zero

$$\sum F_x = -T_1 + T_2\cos\theta = 0$$

$$\sum F_y = -mg + T_2\sin\theta = 0$$

two equations, two unknowns; solving for $T_1 = 6.93\,N$

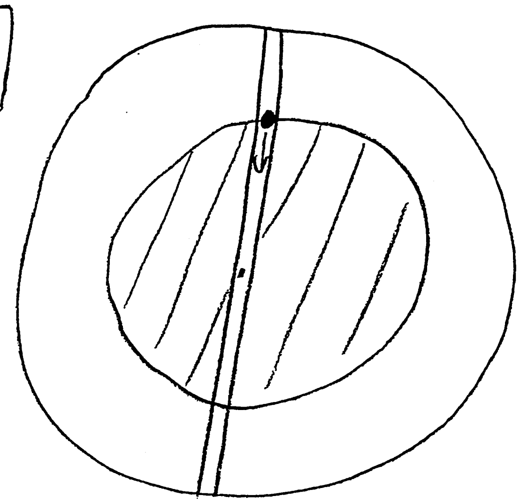
now we know enough to find the speed

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T_1}{\mu}} = \sqrt{\frac{6.93}{3.33\times10^{-3}}} = 45.6\,m/s$$

and the time is given by

$$t = \frac{d}{v} = \frac{L/2}{v} = \frac{\frac{3.00}{2}\,m}{45.6\,m/s} = 32.9\times10^{-3}\,s = 32.9\,ms$$

$\boxed{13-65}$



as stated in the problem, the falling mass is only affected by the material in the shaded region

Newton's law of gravitation states that

$$\vec{F} = -G\frac{Mm}{r^2}\hat{r}$$

hence we need the mass of the shaded region, since the density is constant

$\rho = \frac{M}{V} \Rightarrow M = \rho V$ the volume is just the volume of the sphere

$V = \frac{4}{3}\pi r^3$

PLEASE LOOK AT NOTES FROM 121 TO UNDERSTAND THE PROOF.

hence $M = \rho\frac{4}{3}\pi r^3$

plugging into Newton's gravitation equation gives

$$\vec{F} = -G\frac{(\rho\frac{4}{3}\pi r^{\cancel{3}})m}{\cancel{r^2}}\hat{r} = -(G\rho\tfrac{4}{3}\pi m)r\hat{r}$$

since $G\rho\frac{4}{3}\pi m$ is just a constant, this has the form of Hooke's law $\vec{F} = -kr\hat{r}$ where $k = G\rho\frac{4}{3}\pi m$

$\boxed{3-68}$ a use the work-energy theorem (energy conservation + work done by friction) to find the energy lost

chose the two instants to be right before and right after it hits the spring

$$E_i = E_f + W \Rightarrow \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2 + W$$

$$W = \tfrac{1}{2}mv_i^2 - \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}(8.00)(4.00)^2 - \tfrac{1}{2}(8.00)(3.00)^2$$

$$= 28.0\ \text{J}$$

b. the energy lost is used to overcome friction
the work done by friction is given by
$W_f = F \cdot d = \mu mg(2x)$ where $2x$ is twice the compression of the spring because work is done both when the spring is compressing and expanding

hence $W = 2\mu mgx \Rightarrow x = \frac{W}{2\mu mg} = \frac{28.0}{2(0.400)(8.00)(9.80)} = 0.446\text{m}$