

FIELDS: GRAVITATIONAL, COULOMB \vec{E}, \vec{B}

\vec{G} : A mass m located in a Gravitational field feels a force $\vec{F}_G = m\vec{G}_F$

Measure \vec{F}_G , map out \vec{G}_F .

A mass M located at the origin creates a \vec{G}_F

$$\vec{G}_F = -\frac{GM}{r^2} \hat{r}$$

consequently, $\sum_c \vec{G}_F \cdot \Delta \vec{A} = -4\pi G \Sigma M_i$

FLUX OF \vec{G}_F Through a closed surface is determined solely by the masses enclosed by the surface.

\vec{E} : A charge q located in an \vec{E} -field feels a force $\vec{F}_E = q\vec{E}$.

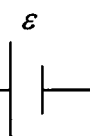
Measure \vec{F}_E , map out \vec{E} .

A stationary charge Q located at $r=0$ generates a coulomb \vec{E} -field

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}$$

[$+Q$ (source), $-Q$ (sink)] consequently, $\sum_c \vec{E} \cdot \Delta \vec{A} = \frac{1}{\epsilon_0} \Sigma Qi$

FLUX OF \vec{E} THROUGH A CLOSED SURFACE IS DETERMINED SOLELY BY THE CHARGES ENCLOSED BY THE SURFACE.

The Devices resulting from this are: (i) Battery 

(ii) Capacitor $C = \frac{Q}{V}$ which leads to energy density $\eta_E = \frac{1}{2} \epsilon_0 E^2$

That is the energy contained in $1m^3$ vol. of \vec{E} -field

(iii) Resistor $R = \frac{V}{I}$ which leads to $\vec{J} = \sigma \vec{E}$, because $I = \vec{J} \cdot \vec{A}$

That is, if you apply \vec{E} to a Conductor it responds by setting up a current density \vec{J} whose magnitude is determined by the electrical conductivity σ .

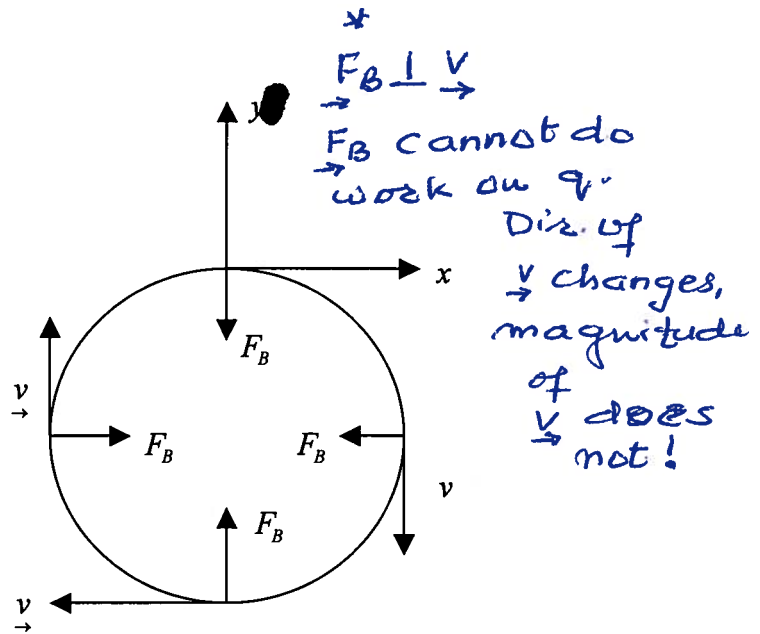
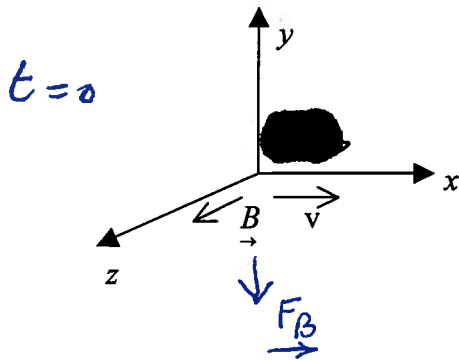
Magnetic (\vec{B}): If a \vec{B} -field is present, a charge q moving with velocity \vec{v} will experience a

force $\vec{F}_B = q[\vec{v} \times \vec{B}]$

Magnitude of $F_B = qvB \sin(\vec{v}, \vec{B})$ [Equivalently, if a moving charge experiences a force which is always perpendicular to its velocity, and there is no visible agency applying the force, then the charge must be moving in a \vec{B} -field].

Direction of $\vec{F}_B \rightarrow$ *rt hand rule* $\left\{ q \vec{v} \parallel \text{Thumb}, B \parallel \text{fingers}, F_B \perp \text{Palm} \right\}$

Problem I: At $t=0$, charge q is at origin and has velocity $\vec{v} = v\hat{x}$. Turn on a field $B = B\hat{z}$. Immediately, it experiences \vec{F}_B along $-\hat{y}$. Makes \vec{v} turn, but \vec{F}_B turns also. Net result is as shown in Figure. q goes around in circle. $F_B \perp \vec{v}$ always so Kinetic Energy fixed, magnitude of \vec{v} does not change.



Particle moves under influence of $\vec{F}_B = -qvB\hat{r}$ [\vec{v} & \vec{B} are \perp to one another]

Note: Plane of orbit \perp to \vec{B} field.

Note: Uniform circular motion needs a centripetal force.

$$\vec{F}_C = \frac{-Mv^2}{r} \hat{r}$$

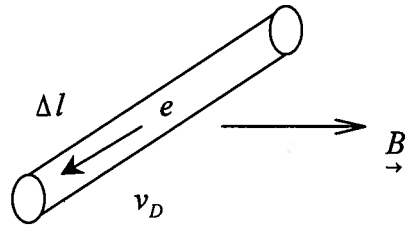
\vec{F}_B provides it.

$$\vec{F}_B = \vec{F}_C \text{ so } r = \frac{Mv}{qB}$$

angular velocity $\vec{\omega} = \frac{-qB}{m} \hat{z}$ (see picture above)

Note: ω independent of v .

Problem II: Force on Current Carrying conductor of length l ; Cross. Sec A , charge density n_e each electron feels $\vec{F}_B = (-e)[\vec{v}_D \times \vec{B}]$



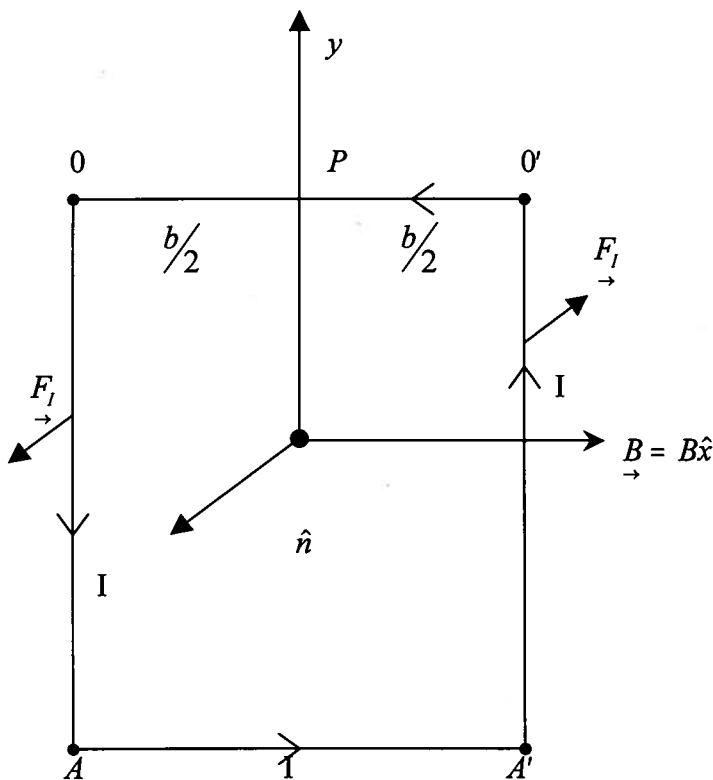
of electrons = $n_e A \Delta l$

so total force on conductor $\vec{F}_I = n_e A_e \Delta l [\vec{v}_D \times \vec{B}]$

electrons constrained to move along Δl so $\vec{F}_I = n_e A_e v_D [\Delta l \times \vec{B}] = I \Delta l \times \vec{B}$

Problem III

Rectangular loop of wire suspended in a \vec{B} -field with current in loop as shown, start with loop in xy -plane, at $t=0$.



$$\vec{F}_I = I l B \hat{z} \text{ on } 0A$$

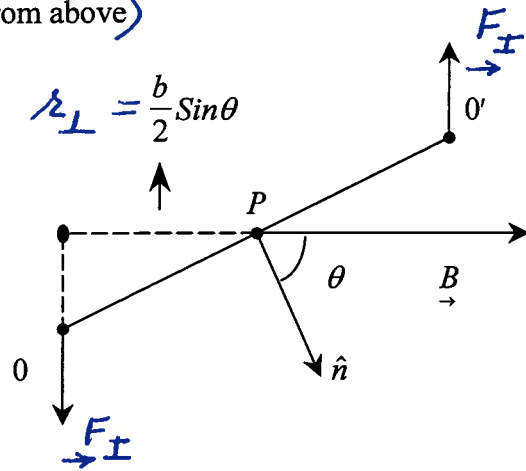
$$\vec{F}_I = -I l B \hat{z} \text{ on } 0'A'$$

$$\vec{F}_I = -I l B \hat{z} \text{ on } 0'A'$$

Net force is zero. *However, torque is given by*

Torque $\vec{\tau} = I l B \frac{b}{2} \hat{y} + I l B \frac{b}{2} \hat{y}$
 $= I l B b \hat{y}$ (1)

Rotate (look from above)

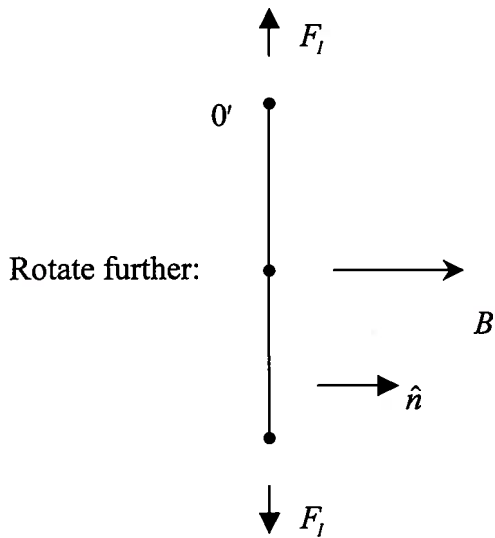


$$\vec{\tau} = [\vec{r} \times \vec{F}]$$

$$\tau = r_{\perp} F$$

Note I and B still at right angles to one another, F_{\perp} does not change but now $r_{\perp} = \frac{b}{2} \sin \theta$.

[Direction of \hat{n} also fixed by right hand rule] $\vec{\tau} = I l B b \sin \theta \hat{y}$ (2)



$$\vec{\tau} = 0$$
 (3)

$$\begin{aligned} \hat{x} \times \hat{y} &= \hat{z} \\ \hat{y} \times \hat{z} &= \hat{x} \\ \hat{z} \times \hat{x} &= \hat{y} \end{aligned}$$

Equations (1), (2), (3) combine to give $\vec{\tau} = I l b \hat{n} \times \vec{B}$

Define Magnetic (Dipole) moment $\vec{\mu} = I \vec{b} \hat{n} = IA \hat{n}$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

If the coil has N turns $\vec{\mu} = IAN \hat{n}$

Note: The top ($00'$) and bottom (AA') wires have equal and opposite Forces. They will make the coil out of shape but have no other effect.

Generation of \vec{B} -field

We have seen that a stationary charge experiences a force in an \vec{E} -field and a stationary charge creates a (coulomb) \vec{E} -field. Now we know that a moving charge experiences a force in a \vec{B} -field so it is natural to expect that a moving charge will generate a \vec{B} -field. This is indeed the content of the so-called Biot Savart Law.

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \vec{r}}{r^3}$$

where $\mu_0 = 4\pi \times 10^{-7} \frac{T \cdot m}{A}$ is universal constant. This equation has the immediate consequence that for a current I in a conductor of length Δl .

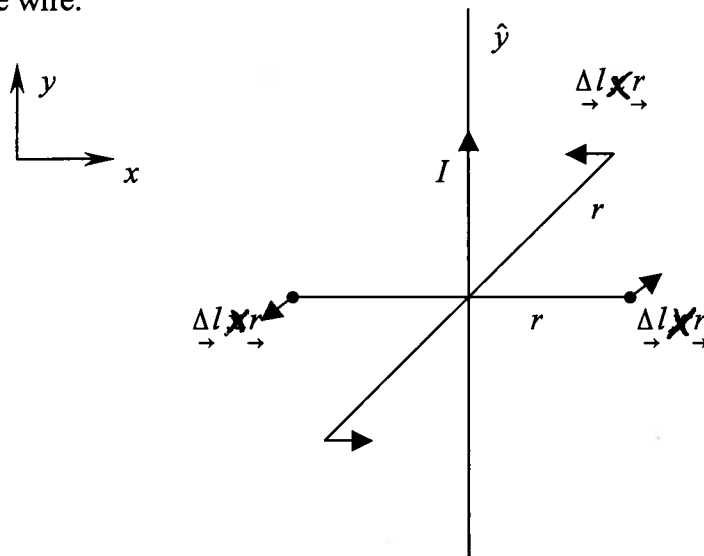
$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \frac{\Delta \vec{l} \times \vec{r}}{r^3}$$

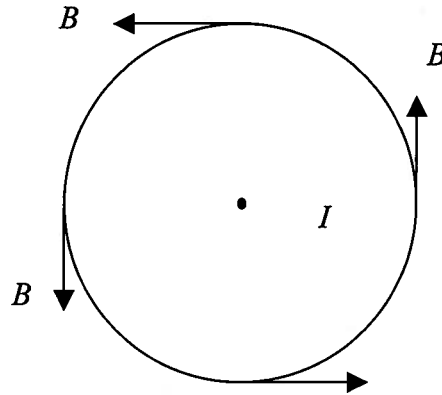
We will not use these equations in detail.

CASES OF SPECIAL INTEREST.

Single current – I in a long wire: What can we say about \vec{B} -field at a distance r from the wire?

Notice that $\Delta \vec{l} \parallel \hat{y}$. And the vector $\Delta \vec{l} \times \vec{r}$ is perpendicular to \vec{r} so we can say that \vec{B} must curl around the wire.





[I out of page]

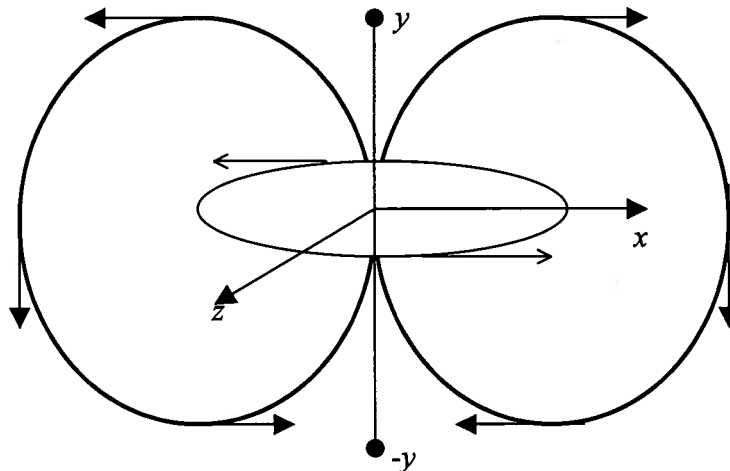
Looking end on (see picture) we have cylindrical symmetry so \vec{B} can be a function of r only. It

turns out that $B = \frac{\mu_0 I}{2\pi r}$

so, $B = \frac{\mu_0 I}{2\pi r} \hat{\phi}$

Thus, \vec{B} is said to be Azimuthal, $\hat{\phi}$ is the direction which curls around I. [check with the sheet on right hand rules].

Next, take the wire and make a circular loop out of it, put it in xz -pl. with center at the origin. What is the \vec{B} -field at y or $-y$?



The \vec{B} -field lines are shown schematically, it turns out that

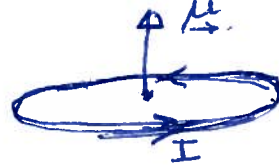
$$\vec{B}(y) = \vec{B}(-y) = \frac{\mu_0}{4\pi} \frac{2I\pi a^2}{(a^2 + y^2)^{3/2}} \hat{y}$$

Once again, we encounter $I\hat{a}\hat{n}$ so we can write using magnetic (dipole) moment

$$\vec{B}(y) = \frac{\mu_0}{4\pi} \frac{2\vec{\mu}}{(a^2 + y^2)^{3/2}}$$

Far away from $\mu, y \gg a$

$$\vec{B}(y) = \frac{\mu_0}{4\pi} \cdot \frac{2\vec{\mu}}{y^3} \leftarrow \text{Magnetic Dipole}$$

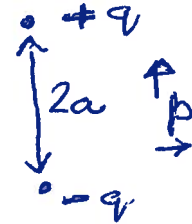


Recall that for an Electric Dipole

$$\vec{p} = 2qa\hat{y}$$

and the \vec{E} -field at y is

$$\vec{E}(y) = \frac{1}{4\pi\epsilon_0} \frac{4qay}{(y^2 - a^2)^2} \hat{y}$$

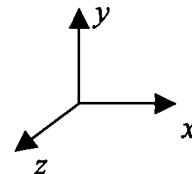
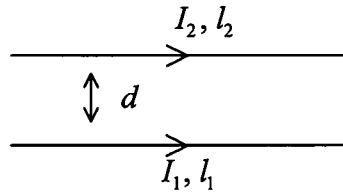


so that at $y \gg a$

$$\vec{E}(y) = \frac{1}{4\pi\epsilon_0} \frac{2p}{y^3} \leftarrow \text{Electric Dipole}$$

However, there is a major difference here: along y the magnetic dipole has no "size" while electric dipole has length ($2a$). You can split the latter but not the former. This has the extremely important consequence that whereas electric-field lines start at $+q$ and end at $-q$, magnetic field lines close on themselves there is no beginning and no end.

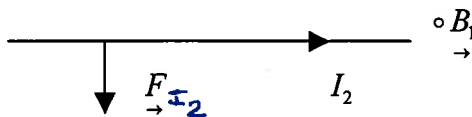
Current-Current force



Two wires of lengths l_1, l_2 carry currents I_1, I_2 . Separation d along y , wires parallel to x . Force on I_2 due to I_1 . To calculate this first. Write B_1 at location of I_2

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi d} \hat{z}$$

So I_2 is located in $B_1 = \vec{F}_{I_2}$ looks like

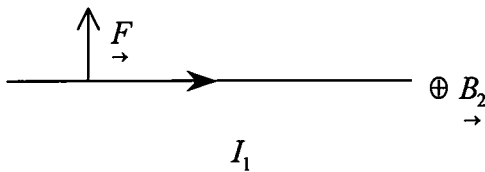


and is given by

$$\begin{aligned} \vec{F}_{I_2 I_1} &= I_2 \Delta l_2 \times \vec{B}_1 \\ &= -\frac{\mu_0 I_1 I_2 l_2}{2\pi d} \hat{y} \end{aligned}$$

Force is attractive
Force on I_1 due to I_2

$$\begin{aligned} \vec{B}_2 &= \frac{-\mu_0 I_2}{2\pi d} \hat{z} \\ \vec{F}_{I_1 I_2} &= I_1 \Delta l_1 \times \vec{B}_2 \\ &= \frac{\mu_0 I_1 I_2 l_1}{2\pi d} \hat{y} \end{aligned}$$



Force is attractive. If $l_1 = l_2 = 1 \text{ meter}$ the forces/meter $\vec{F} = \frac{-\mu_0 I_1 I_2}{2\pi d} \hat{d}$ are an action-reaction pair.

The $-$ sign with \hat{d} ensures force is attractive if I_1, I_2 parallel \rightarrow and repulsive when they are anti-parallel \rightarrow \leftarrow . *[You will do an expt. to check this equation]*

Incidentally, this is a very fundamental equation as it is used to define the unit of current- The Ampere. That is,

of

$$I_1 = I_2 = 1 \text{ amp}$$

and $d = 1 \text{ meter}$

Force per meter is $2 \times 10^{-7} N$ $\left(\frac{\mu_0}{2\pi} \right)$

And the claim is that I should be regarded as a "DIMENSION" in place of Q .

So we can write $L, T, M, \theta, [IT]$

rather than

L, T, M, θ, Q