Name: ______________________________

S.S.N.: ______________________________

Exam I,
Physics 122-Summer 2003, Thu. 7/24/2003
Instructor: Dr. S. Liberati

General Instructions

- Do all the problems by writing on the exam book (continue to work on the back of each page if you run out of room).
- Write your name (in capital letters) on every page of the exam.
- Purely numerical answers will not be accepted. Explain with symbols or words your line of reasoning. Corrected formulae count more than corrected numbers.
- Use a calculator

Hints to do well

- Read the problem carefully before you start computing.
- Do problems with symbols first (introduce them if you have to). Only put in numbers at the end.
- Check your answers for dimensional correctness.
- If you are not absolutely sure about a problem, please write down what you understand so that partial credit can be given.

Honor Pledge: Please sign at the end of the statement below confirming that you will abide by the University of Maryland Honor Pledge

"I pledge on my honor that I have not given or received any unauthorized assistance on this assignment/examination."

Signature:______________________________

Short Answers

Q.1) ________________________

Q.1) ________________________

Q.1) ________________________

Q.1) ________________________

Short Exercises

E.1) ________________________

E.1) ________________________

Problems

P.1 ________________________

P.1 ________________________

P.1 ________________________

Total Score

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Useful Constants & Formulae

Constants:

Fundamental unit of charge: $|e| = 1.6 \times 10^{-19}\ C$

Coulomb constant: $k_e = 8.99 \times 10^{9}\ N \cdot m^2/C^2$,

Permittivity of vacuum: $\varepsilon_0 = 1/(4\pi k_e) = 8.85 \times 10^{-12}\ C^2/(N \cdot m^2)$

Coulomb Law: $F_e = k_e \dfrac{q_1 q_2}{r^2}$

Electric field:

$E = F/q_0$ For a point charge Q: $E = k_e Q/r^2$ For a parallel plate capacitor: $E = q/(\varepsilon_0 A) = \sigma/\varepsilon_0$

Gauss' Law: $\sum (E\cos\theta)\Delta A = Q/\varepsilon_0$

Electric potential energy: $W_{AB} = -\Delta(EPE) = EPE_A - EPE_B$

Electric potential:

$V = EPE/q_0$ $W_{AB} = -q_0 \Delta V = q_0 V_A - q_0 V_B$

For a point charge: $V = k_e Q/r$

Relation between electric field and electric potential: $E = -\dfrac{\Delta V}{\Delta s}$

Dielectric constant: $\kappa = E_0/E$

Capacitors:

$C = q/V$ For a parallel plate capacitor: $C = \kappa\varepsilon_0 A/d$

Energy stored in a capacitor $EPE_{capac} = \dfrac{1}{2}CV^2$

Ohm's law: $V = RI$ For a wire: $R = \rho\dfrac{L}{A}$

Electric power:

DC generator: $P = I \cdot V = I^2 R = V^2/R$

AC generator: $\overline{P} = I_{rms} \cdot V_{rms} = I_{rms}^2 R = V_{rms}^2/R$

Series wiring: Resistors: $R_s = R_1 + R_2 + \ldots + R_n$ Capacitors: $\dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots + \dfrac{1}{C_n}$

Parallel wiring: Resistors: $\dfrac{1}{R_p} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots + \dfrac{1}{R_n}$ Capacitors: $C_p = C_1 + C_2 + \ldots + C_n$

Part I: Short Answers (5 points each)

Question 1

Concerning charge, which of the following statements are true or false:

a) Charge is quantized: Q=ne, where $e=1.6\times10^{-19}$ C and n=0,±1, ±2, ±3,…
a) Like charges repel, unlike charges attract.
a) A negatively charged object is regarded as having a deficiency of electrons
a) Ordinary matter is electrically neutral.
a) $Q_{electron}=+1e$, $Q_{proton}=-1e$, $Q_{neutron}=+2e$

a) True, b) True, c) False, d) True e) False

Question 2

State the SI units and dimensions of each of the following physical quantities:

Quantity	**S.I. Unit**	**Dimension** (in terms of [M],[L], and [T])
Electric Force	N	$[ML/T^2]$
Electric Field	N/C	$[ML/QT^2]$
Electric Potential Energy	J	$[ML^2/T^2]$
Electric Potential	J/C=V	$[ML^2/QT^2]$
Electric Current	C/s=A	$[Q/T]$

Question 3

Four conductors A, B, C, D, are electrically charged. You observe that

a) A and B mutually repel
a) A and C mutually attract
a) A and D mutually attract

Which of the following predictions is supported by the empirical evidence?

i) C will attract D; C and D will each repel B
i) C will attract D; C and D will each attract B
i) C will repel D; C and D will each repel B
i) C will repel D; C and D will each attract B
i) C and D will each attract B but we cannot predict which the mutual effect of C and D will be.

The above information tells us that : C and D have the same charge which is opposite to that of A and B This implies that only proposition (iv) is true.

Question 4

We have two light bulbs, say A and B. Their filaments are made of the same material and have the same length. However the filament in A is thicker than the filament in B. On the base of this information answer the following questions:

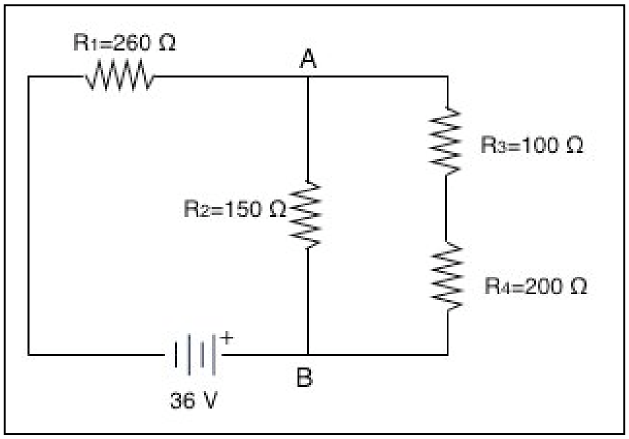
a) If we connect the two bulbs in parallel to an outlet, which of the two will be brighter? Explain your reasoning
a) If we connect them in series, which of the two will be brighter? Explain your reasoning.

Hints: 1) Remember the dependence of the resistance of a wire on geometry and material. 2) The brightness of the bulb is determined by its power.

$R = \rho L/A \quad \Rightarrow \quad R_A < R_B$
When in parallel V applied is the same and $P = V^2/R \quad \Rightarrow \quad P_A > P_B$
When in series I applied is the same and $P = I^2R \quad \Rightarrow \quad P_B > P_A$

Part II: Short Exercises (10 points each)

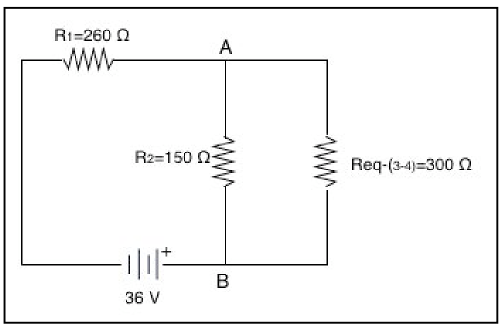
Exercise 1



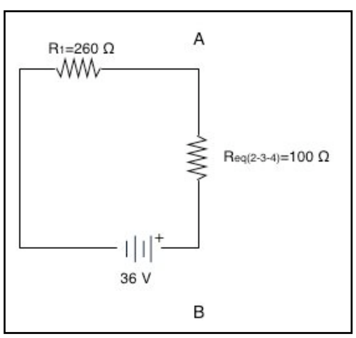
The circuit shown in the picture is composed by a battery and four resistors whose resistances are R_1=260 Ω, R_2=150 Ω, R_3=100 Ω, R_4=200 Ω. Find:

a) The <u>total current</u> supplied by the battery. [Hint: consider the equivalent resistance of the circuit]

a) The voltage between points A and B of the circuit. [Hint: consider the equivalent resistance between A & B]



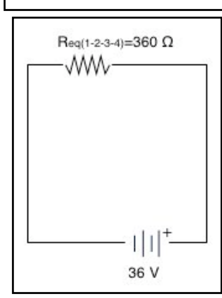
a) The total current can by found by Ohm's law once the equivalent resistance of the system is known. One can find the equivalent resistance of the circuit in few steps. As shown in this picture we can start replacing R_3=100 Ω, R_4=200 Ω. R by an equivalent resistance $R_{eq-(3-4)}$= R_3+R_4=300 Ω.



We can now find the equivalent resistance between A & B by considering that R_2 and $R_{eq-(3-4)}$ are in parallel.

$$\frac{1}{R_{eq-(2-3-4)}} = \frac{1}{R_2} + \frac{1}{R_{eq-(3-4)}} = \frac{1}{150\Omega} + \frac{1}{300\Omega} = \frac{1}{100\Omega}$$

$$R_{eq-(2-3-4)} = 100\ \Omega = R_{AB}$$



We can now determine the total equivalent resistance of the circuit by considering the series of R_1 and $R_{eq-(2-3-4)}$. Ohm's law then provides the total current provided by the battery

$$R_{eq} = R_1 + R_{eq-(2-3-4)} = 260\Omega + 100\Omega = 360\ \Omega \quad I = \frac{V}{R_{eq}} = \frac{36\ V}{360\ \Omega} = 0.10\ A$$

b) The Voltage across AB is finally found considering the equivalent resistance between AB and Ohm's law: $V_{AB} = IR_{AB} = (0.10A)(100\ \Omega) = 10\ V$

Exercise 2

In a certain region of space the electric field has a constant value of 5.0×10^{3} V/m and it is directed downward. Consider a point P at the center of this region. Find the variation in the electric potential when one moves from point P to the following points:

i) Point A located 8.0×10^{-3} m directly above P.
i) Point B located 5.0×10^{-3} m directly below P.
i) Point C located at 3.0×10^{-3} m directly to the left of P.

Hints: ij) Remember the relation between the electric field and the electric potential.
k) What is the work done if one moves a charge orthogonally to the electric field?

Let start assuming conventially that upward directions are positive.

Then E < 0 because it points downward. So $E = -5 \times 10^{3}\ V/m$

The potential at different points can be determined using the formula $E = -\frac{\Delta V}{\Delta s}$

$i)\ \ A:\ (V_A - V_P) = -E(x_A - x_P) \quad \Rightarrow \quad V_A = V_P - E(x_A - x_P)$

$V_A - V_P = 5 \times 10^{3}\ V/m \times 8 \times 10^{-3}\ m = 40\ V$

$j)\ \ B:\ (V_B - V_P) = -E(x_B - x_P) \quad \Rightarrow \quad V_B = V_P - E(x_B - x_P)$

$V_B - V_P = 5 \times 10^{3}\ V/m \times (-5 \times 10^{-3}\ m) = -25\ V$

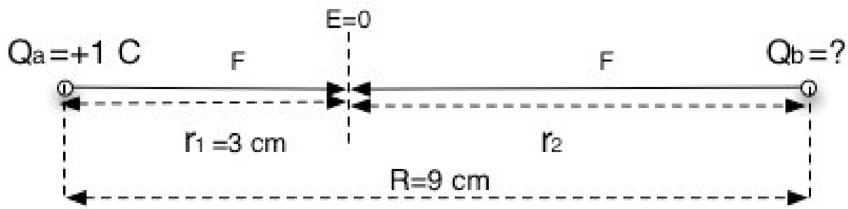
$k)$ Moving a charge orthogonally with respect to E does not involve a work

(F = qE, W = F · s = 0 is s perpendicular to F, i.e. to E)

So $\Delta V_{PC} = qW = 0$ so $V_P = V_C$

Part III: Problems (20 points each)

Problem 1



Two positive charges Q_a=+1 C and Q_b are separated by a distance R=9×10^{-2} m.

1) What is the value of Q_b if the electric field is zero at a point between the charges, 3×10^{-2} m from charge Q_a? (See picture)

$$E = k_c\left(\frac{Q_a}{r^2{}_1} - \frac{Q_b}{r^2{}_2}\right) = 0 \quad \Rightarrow \quad \frac{Q_a}{r_1^2} = \frac{Q_b}{r_2^2} \quad \Rightarrow \quad \frac{Q_a}{r_1^2}(R - r_1)^2 = Q_b$$
$$Q_b = 1C\frac{6^2}{9} = +4C$$

1) With this configuration of charges (i.e. same positions and Q_b as obtained above), what is the electric field value midway between the two charges? ($k_e=8.99\times10^9$ Nm^2/C^2)

$$E = k_e\left(\frac{Q_a}{r^2} - \frac{Q_b}{r^2}\right) = -8.99\cdot10^9 \;\; Nm^2/C^2 \cdot \frac{3\;\;C}{(0.045\;\;m)^2} \approx -\frac{27}{2025}10^{15}\;\;N/C = -13\cdot10^{12}\;\;N/C$$

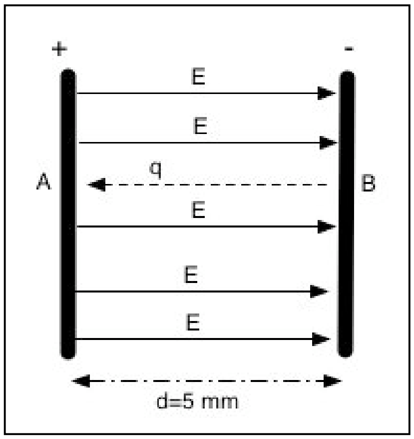
The minus sign implies that the total field has the same direction of that generated just by Q_B, so it points away from Q_B (Q_B is positive)

1) What is the net force acting on a -2 mC (N.B. mC=milliCoulomb=10^{-3} C) charge in this point equidistant from Q_a and Q_b?

$$F = E\cdot q = -13\cdot10^{12}\;\;N/C \cdot 2\cdot10^{-3}\;\;C = -26\cdot10^9\;\;N$$

The minus sign says that also the forces points away from Q_B

Problem 2



A particle with charge q=-1.5 μC and mass m=2.5×10^{-6} Kg is released from rest at the negative plate of a capacitor (point A in the picture) and hits the positive plate 5×10^{-3} m away (point B in the picture) with speed v=42 m/s.

a) What is the difference in potential energy $\Delta V=V_B-V_A$ between the two plates?

b) Remember that the electric force between the plates of the parallel plates capacitor is constant and orthogonal to the plates. Taking into account that the particle moves along the line of action of the force compute the magnitude and direction of the electric field between the plates.

c) Knowing that the permittivity of vacuum is $\varepsilon_0=8.85\times10^{-12}\ C^2/(N\cdot m^2)$ compute the charge per unit area σ of the plates.

d) If the plates have an area of 5×10^{-2} m^2, compute the capacitance C of the capacitor.

$a)\ W=-q\Delta V$ but also $W=\Delta KE=\frac{1}{2}mv_B^2=2205\times10^{-6}\ J$

$\Rightarrow\ \Delta V=\frac{W}{-q}=\frac{2205\times10^{-6}\ J}{1.5\times10^{-6}\ C}=1470\ V\ ,\ \Delta V>0\ \Rightarrow\ V_B>V_A$ (Ok, the negative charge "goes uphill")

$b)\ E=F/q\ ,\ W=F\cdot d=-Fd$ (displacement opposite to the direction of E and hence of F)

$F=-W/d=-\frac{2205\times10^{-6}\ J}{5\times10^{-3}\ m}=-441\times10^{-3}\ N\ \Rightarrow\ E=F/q=\frac{-441\times10^{-3}\ N}{-1.5\times10^{-6}\ C}=294000\ \frac{N}{C}$

The field is directed from positive to negative (opposite to direction of the negative charge motion)

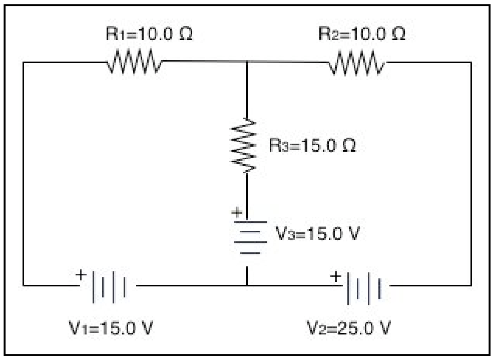
c) $E_{ppCap}=\frac{\sigma}{\varepsilon_0}\ \Rightarrow\ \sigma=\varepsilon_0E=8.85\times10^{-12}\frac{C^2}{N\cdot m^2}\times\left(294000\ \frac{N}{C}\right)=2.6\frac{\mu C}{m^2}$

$d)\ C=Q/V\ \Rightarrow\ Q=\sigma\cdot A=\left(2.6\frac{\mu C}{m^2}\right)\left(25\times10^{-2}m^2\right)=0.65\ \mu C$

$C=Q/V=\frac{0.65\ \mu C}{1470\ V}=4.4\cdot10^{-4}\ \mu F$

Alternative path to d) $C=\kappa\varepsilon_0A/d$ but $\kappa=1$ (No dielectric) $\Rightarrow\ C=\varepsilon_0A/d=4.4\cdot10^{-4}\ \mu F$

Problem 3



For the specific circuit shown in the picture calculate:

a) The current I_1 through the resistor R_1

b) The current I_2 through the resistor R_2

c) The current I_3 through the resistor R_3

Hint: use Kirchhoff's laws

Let's choose the current flows as in the picture below

Applying the loop rule to the left loop gives

$V_1 = R_1I_1 + R_3I_3 + V_3 \quad \Rightarrow \quad R_1I_1 + R_3I_3 = V_1 - V_3$

Applying the loop rule to the right loop gives

$0 = V_2 + R_2I_2 + R_3I_3 + V_3 \quad \Rightarrow \quad R_2I_2 + R_3I_3 = -V_2 - V_3$

The junction rule implies $I_1 + I_2 = I_3$

So we need to solve the system $\begin{cases} I_1 + I_2 = I_3 \\ R_2I_2 + R_3I_3 = -V_2 - V_3 \\ R_1I_1 + R_3I_3 = V_1 - V_3 \end{cases}$

$$\begin{cases} I_1 + I_2 = I_3 \\ RI_2 + R_3I_3 = -V_2 - V_3 \\ RI_1 + R_3I_3 = 0 \end{cases} \quad \Rightarrow \quad I_1 = 1.5\ \ A,\ I_2 = -2.5\ \ A,\ I_3 = -1.0\ \ A$$

Whitin the choose direction the signs for the currents imply

$I_1 = 1.5\ \ A \quad toward\ \ right, \quad I_2 = 2.5\ \ A\ \ toward\ \ right, \quad I_3 = 1.0\ \ A \quad up$