Physics 121 Dr. Dragt
Spring 2003

Study Sheet for Exam #3

14. Physics knowledge, like all subjects having some substance, is cumulative. You are still responsible for all material on the Study Sheets for Exams 1 and 2. Make sure that you now know how to do Exams 1 and 2 perfectly. Exams are excellent opportunities to discover and learn what you previously did not know.

15. Chapter 10: Simple Harmonic Motion and Elasticity

    (a) A spring is characterized by two parameters, its *natural* length $l_n$ and its *spring constant* $k$. Hooke discovered that if a spring is deformed (stretched or compressed) so that its actual length is $l$, the spring pulls or pushes back with a *restoring* force $F$. In particular he found the relation

    $$F = -k(l - l_n).$$

    Call the change in length $x$ by making the definition

    $$x = l - l_n.$$

    Then Hooke's law can also be written as

    $$F = -kx.$$

    Hooke described this in Latin by stating *Ut tensio sic vis*, which can be translated as "As the extension so the force". There was fierce competition among 17th century scientists over priority of discovery. To assure he would not be scooped by others before he had time to write a detailed paper, Hooke first published only the anagram *ceiiinosssttuv*, which consists of the letters in the Latin sentence above in alphabetical order.

(b) Suppose an object of mass $m$ is acted on by a Hook's law force. Newton would say the object should move according to the law

$$a = F/m.$$

Suppose we arrange the coordinate system so that $x$ is both the position of the object and the change in length as defined above. Then combining Newton's and Hooke's laws gives the result

$$a = -(k/m)x. \qquad (*)$$

We want to know what motion satisfies (*). That is, we want to find $x(t)$ and $v(t)$. We know from Newtonian Determinism that they are completely determined once we specify the initial conditions $x^i$ and $v^i$ at some initial time $t^i$.

Consider the special initial conditions

$$t^i = 0,\ x^i = A,\ v^i = 0.$$

Then the solution to the equation of motion (*) is given by the relations

$$x(t) = A\cos(\omega t), \qquad (**)$$

$$v(t) = -\omega A\sin(\omega t), \qquad (***)$$

with

$$\omega = \sqrt{k/m}. \qquad (****)$$

The motion (**) is called *simple harmonic*, and any system that obeys (*) is called a *simple harmonic oscillator*. The quantity $\omega$ is called the *angular frequency*, and $A$ is called the *amplitude*.

Evidently the motion is periodic, and has period $T$ given by

$$T = 2\pi/\omega.$$

Finally, the ordinary frequency $f$ is defined by the relations

$$f = 1/T = \omega/(2\pi).$$

(c) To verify that (**) and (***) do in fact describe the motion we first observe that they satisfy the initial conditions:

$$x(0) = A = x^i,$$

$$v(0) = 0 = v^i.$$

Next, we perform a gedanken experiment. Consider, as in item 8 of the Study Sheet for Exam 2, uniform circular motion. Then, upon making the substitution $R = A$, we find that the $x$ component of uniform circular motion satisfies (**) and (***). Also, from the result $\boldsymbol{a} = -w^2\boldsymbol{r}$ for the acceleration vector in uniform circular motion, we see that taking the $x$ component gives the relation

$$a_x = -\omega^2 x.$$

Upon comparing this result and (*), while making the mental substitution $a_x = a$, we see that the equation of motion (*) is also satisfied provided the relation (****) holds. You should understand this proof, and be able to make graphs of $x(t)$, $v(t)$, and $a(t)$. Moreover, you should be able to verify that these graphs are related by the usual "slope of tangents" procedure.

(d) Section 10.1, The Ideal Spring and Simple Harmonic Motion; Section 10.2, Simple Harmonic Motion and the Reference Circle. These sections cover the same material as items $a$ through $c$ above. You should understand all concepts, figures, and examples in these sections.

(e) By considering the work done by a Hooke's law force when there is a displacement $x$, we found in class that the potential energy $V(x)$ associated with a deformed spring is given by the relation

$$V(x) = (1/2)kx^2.$$

Next we considered a mass and spring combination. As before, the kinetic energy $K$ of this system is given by the relation

$$K = (1/2)mv^2.$$

Then we considered the total energy $E$ of a mass and spring combination given by the usual relation

$$E = K + V.$$

Upon using the relations (**) and (***) we found the result

$$E = (1/2)kA^2.$$

Note that $E$ is independent of the time. That is, energy is conserved, as also expected. You should be verify this result starting from (**) and (***).

(f) Section 10.3, Energy and Simple Harmonic Motion. This section is related to item $e$ above, and also considers other forces and their associated potential energies. With the exception of rotational motion, you should understand all concepts, figures, and examples in this section.

(g) Section 10.4, The Pendulum. In class we obtained the motion of a pendulum starting from Newton's law and considering the force of gravity and the tension in the cord. You are responsible for the relation (10.16) and being able to make graphs of the motion.

(h) In class we considered the damped and (sinusoidally) driven harmonic oscillator, and studied how the response depended on the driving frequency $\Omega$. We found the response peaked at

$$\Omega = \omega.$$

This condition is called resonance.

(i) Section 10.5, Damped Harmonic Motion; Section 1.6, Driven Harmonic Motion and Resonance. These sections cover some of the material in item $h$ above. Unfortunately they do not give a graph of the response vs $\Omega$. You should understand all concepts and figures in this section.

(j) There are two wonderful things about the harmonic oscillator:

i. We can solve for the motion exactly, and therefore can describe it in detail.

ii. An enormous variety of systems in nature act like harmonic oscillators. They include:

spring and mass system
bell
chime
vibrating string
tuning fork
quartz crystal in a quartz watch
pendulum
tuning circuit in a radio or TV
radio transmitter
light
elementary particles (electrons, quarks, etc.)

You should be aware of this broad range of applications. Note the fact that light and elementary particles act like harmonic oscillators means that mastering the harmonic oscillator is one of the keys to a fundamental understanding of Nature.

(k) Section 10.7, Elastic Deformation; Section 10.8, Stress, Strain, and Hooke's Law. For the case of (one-dimensional) stretching or compression, tensile stress and tensile strain are defined by the relations

$$\text{tensile stress } = F/A,$$

$$\text{tensile strain } = \Delta L/L.$$

In Physics and Engineering, strain is the response to stress.

Young's modulus $Y$ is defined by the ratio

$$Y = (\text{tensile stress})/(\text{tensile strain}) = (F/A)/(\Delta L/L).$$

Turning this relation around gives (10.17) of the text. The Young's modulus is independent of the dimensions of a sample, and depends only on the material of which it is made.

For the case of shear, the shear stress and shear strain are defined by the relations

$$\text{shear stress } = F/A,$$

$$\text{shear strain } = \Delta x/L.$$

The shear modulus $S$ is defined by the ratio

$$S = (\text{shear stress})/(\text{shear strain}) = (F/A)/(\Delta x/L).$$

Turning this relation around gives (10.18) of the text. The shear modulus is also independent of the dimensions of a sample, and depends only on the material of which it is made.

For the case of volume deformation, the volume stress and volume strain are defined by

$$\text{volume stress } = F/A,$$

$$\text{volume strain } = -\Delta V/V.$$

The *bulk* modulus $B$ is defined by the ratio

$$B = \text{volume stress/volume strain } = (F/A)/(-\Delta v/V).$$

The bulk modulus is also independent of the dimensions of a sample, and depends only on the material of which it is made.

You should understand all concepts, figures, and examples in these sections.

(l) Section 10.9, Concepts and Calculations. This section provides further illustrations of the concepts of the previous sections.

You should be able to work all class examples and examples in the book and all assigned homework having to do with items *a* - *l* above. Understand all figures and pictures in chapter 10.

16. Labs 5, 8, and 10.

(a) Lab 5, Conservation of Energy. You should understand the Theory for this experiment. You should understand all concepts, equations, calculations, and figures associated with this lab.

(b) Lab 8, Equilibrium of Rigid Bodies. You should understand the conditions for equilibrium and how to use them. You should understand all concepts, equations, calculations, and figures associated with this lab.

(c) 10, Simple Harmonic Motion and Hooke's Law. You should understand the Theory for this experiment. You should understand all concepts, equations, calculations, and figures associated with this lab.

You should be able to work all examples in the lab manual having to do with items *a* - *c* above. Understand all pictures and figures in these labs.