Solutions – 2

FORMULAE NEEDED

Speed is a SCALAR

$$\text{speed} = \frac{\text{Distance Travelled}}{\text{Time Taken}}$$

Velocity is a Vector

Displacement is a vector

$$\underset{\rightarrow}{d} = \underset{\rightarrow}{x}(t_2) - \underset{\rightarrow}{x}(t_1)$$

$+\hat{x} \longrightarrow$

$-\hat{x} \longleftarrow$

$$= [x(t_2) - x(t_1)]\,\hat{x}$$

Average Velocity

$$\langle \underset{\rightarrow}{v} \rangle = \frac{x(t_2) - x(t_1)}{(t_2 - t_1)}\,\hat{x}$$

Instantaneous Velocity

$$\underset{\rightarrow}{v} = \lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\right)\hat{x}$$

Average Acceleration

$$\langle \underset{\rightarrow}{a} \rangle = \frac{v(t_2) - v(t_1)}{(t_2 - t_1)}\,\hat{x}$$

Instantaneous acceleration

$$\underset{\rightarrow}{a} = \lim_{\Delta t \to 0}\left(\frac{\Delta v}{\Delta t}\right)\hat{x}$$

KINEMATIC EQNS

IF $\underset{\rightarrow}{a} = 0$, $\quad \underset{\rightarrow}{v} = v\hat{x}$, $\quad \underset{\rightarrow}{x} = (x_0 + vt)\,\hat{x}$

IF $\underset{\rightarrow}{a} = a\hat{x}$, $\quad \underset{\rightarrow}{v} = (v_0 + at)\,\hat{x}$, $\quad \underset{\rightarrow}{x} = (x_0 + v_0 t + \frac{1}{2}at^2)\,\hat{x}$

$$V^2 = v_0^2 + 2a(x - x_0)$$

<u>FREE FALL</u> Near Earth all <u>unsupported</u> objects have a <u>constant</u> acceleration

$$\underset{\rightarrow}{a} = -9.8 m/s^2 \hat{y}$$

$+\hat{y} \uparrow$

Hence $\underset{\rightarrow}{v} = (v_0 + 9.8t)\hat{y}$

$-\hat{y} \downarrow$

$$\underset{\rightarrow}{y} = (y_0 + v_0 t - 4.9t^2)\hat{y}$$

$$v^2 = v_0^2 - 19.6(y - y_0)$$

$\hat{x}$, $\hat{y}$ are UNIT VECTORS OF MAGNITUDE 1 (ONE)

<u>VECTOR ALGEBRA</u> $\underset{\rightarrow}{R} = \underset{\rightarrow}{A} + \underset{\rightarrow}{B}$

$$R = \sqrt{A^2 + B^2 + 2AB\cos(\underset{\rightarrow}{A}, \underset{\rightarrow}{B})}, \quad \tan\theta_R = \frac{B\sin\theta}{A + B\cos\theta}$$

<u>COMPONENT</u> OF $\underset{\rightarrow}{V}$ Along $\hat{a}$ is $V_a = V\cos(\underset{\rightarrow}{V}, \hat{a})$

$$\underset{\rightarrow}{V} = V_x\hat{x} + V_y\hat{y}$$

Many Vectors $\underset{\rightarrow}{V_i}$

$$R = \Sigma V_{ix}\hat{x} + \Sigma V_{iy}\hat{y} = R_x\hat{x} + R_y\hat{y}$$

$$R = \sqrt{(\Sigma V_{ix})^2 + (V_{iy})^2}, \quad \tan\theta_R = \frac{\Sigma V_{iy}}{\Sigma V_{ix}}$$

PHYS 121

Homework Set 2

Spring '08

Ch. 2: 20, 21, 25, 29, 32, 42, 44, 47, 51, 52

Ch. 3: 3, 7, 12, 18, 20.

2-20

Initial vel $v_i = 5.0\,m/s\,\hat{x}$

Final vel $v_f = 8.0\,m/s\,\hat{x}$

Time interval $\Delta t = 4.0\,s$

Average acceleration: $\vec{a} = \frac{v_f - v_i}{\Delta t}\hat{x}$

$= \frac{(8.0 - 5.0)\,m/s}{4.0\,s}\hat{x} = \frac{3.0}{4.0}\,m/s^2\,\hat{x}$

$$\boxed{a = 0.750\,m/s^2\,\hat{x}}$$

2-21

Average acceleration $\vec{a} = 0.60\,m/s^2\,\hat{x}$

Initial vel. $\vec{v_i} = 55\,mi/h\,\hat{x}$ [Don't forget to convert to m/s]

$= 55\,\cancel{mi/h}\,\frac{0.447\,m/s}{\cancel{mi/h}}$

$= 24.585\,m/s\,\hat{x}$

look at the inside of the front cover for conversion factor

Final speed $v_f = 60\,\cancel{mi/h} \cdot \frac{0.447\,m/s}{\cancel{mi/h}}$

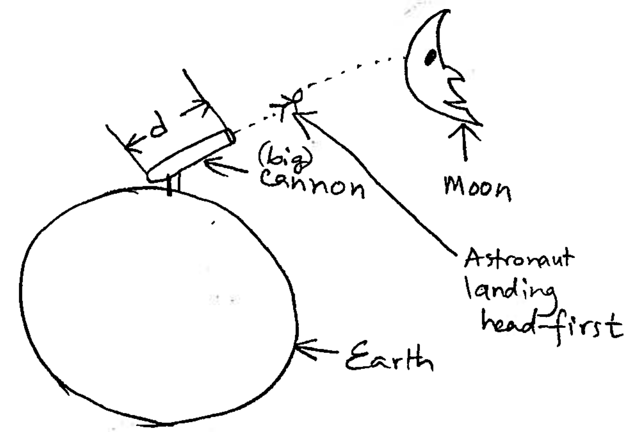
$= 26.82\,m/s\,\hat{x}$

$\vec{a} = \frac{v_f - v_i}{\Delta t}\hat{x}$ $\quad \Delta t = \frac{v_f - v_i}{a} = \frac{(26.82 - 24.585)\,\cancel{m/s}}{0.60\,\cancel{m}/s^{\cancel{2}}} = 3.725\,s = \boxed{3.73\,s = \Delta t}$

$\boxed{2\text{-}25}$ Cannon length $d = 220 m$

Final speed $v_f = 10.97 km/s$

$= 10970 m/s$



Necessary assumption

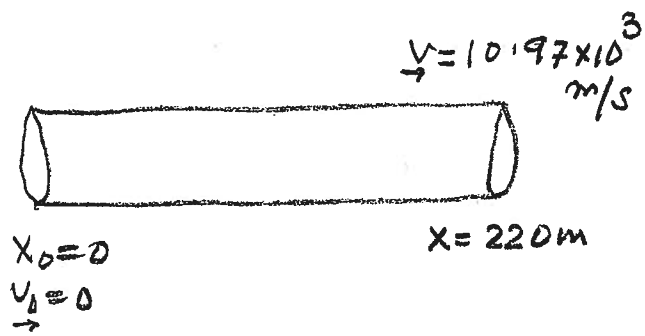
Initial speed $v_0 = 0 m/s$

$x_0 = 0$.

(Man/capsule starts from rest inside the cannon)

What is the acceleration a?

We know,



$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\Rightarrow 2a(x - x_0) = v^2 - v_0^2$$

$$\Rightarrow \vec{a} = \frac{v^2 - v_0^2}{2(x - x_0)}\hat{x}$$

$$= \frac{(10970 m/s)^2 - 0}{2(220m)}\hat{x}$$

$$= 273502.0455\hat{x} m/s^2$$

$$\Rightarrow \boxed{a = 274 km/s^2}$$

We can withstand 15 g.

$1 g = 9.8 m/s^2$

So, we get $a = \frac{274 \times 10^3 m/s^2}{9.8 m/s^2} g = 27908 g$ $\boxed{\text{Whoa!}}$

$\boxed{2\text{-}29}$ Lift-off speed = final speed : $\vec{v_f} = 120\ km/h\ \hat{x}$

$= 120\ \cancel{km/h} \cdot 0.278 \frac{m/s}{\cancel{km/h}}$

$= 33.36\ m/s\ \hat{x}$

Starting from rest : Initial speed : $v_i = 0\ m/s$

(a) Length of take-off run $(x - x_0) = 240\ m$

What is the acceleration a?

We know,

$$v_f^2 = v_i^2 + 2a\,\Delta\,(x - x_0)$$

$$\Rightarrow\ 2a(x - x_0) = v_f^2 - \cancel{v_i^2}^{\,0}$$

$$\Rightarrow\ a = \frac{v_f^2}{2(x - x_0)} = \frac{(33.36\,m/s)^2}{2(240\,m)} = \boxed{2.32\ m/s^2 = a}$$

$\vec{a} = 2.32\,m/s^2\,\hat{x}$

(b) How long does the take-off run take?

i.e. $\Delta t = ?$

We know,

$$v_f = v_0 + at$$

$$\Rightarrow\ \Delta t = \frac{v_f}{a} = \frac{33.36\,\cancel{m}/\cancel{s}}{2.32\,\cancel{m}/s^{\cancel{2}}} = \boxed{14.4\,s}$$

2-32 Landing vel = initial vel $= \vec{v_0} = 100 m/s\ \hat{x}$

(Maximum) acceleration $= a = -5.00 m/s^2\ \hat{x}$

final veloc $= \vec{v_f} = 0$ m/s.

(a) Time needed for the plane to stop $= \Delta t = ?$

We know,

$$\vec{v_f} \overset{0}{=} (v_0 + a\Delta t)\hat{x}$$

$$\Rightarrow a\Delta t = -v_0 \Rightarrow \Delta t = -\frac{v_i}{a} = \frac{+\cancel{100}^{20} m/s}{+\cancel{5}.00 m/s^2}$$

$$\Rightarrow \boxed{\Delta t = 20\,s}$$

(b) Runway length $d = 0.800$ km
$= 800$ m

$\vec{v_f} = 0$; $\vec{v_0} = 100 m/s\ \hat{x}$

$\vec{a} = -5 m/s^2\ \hat{x}$

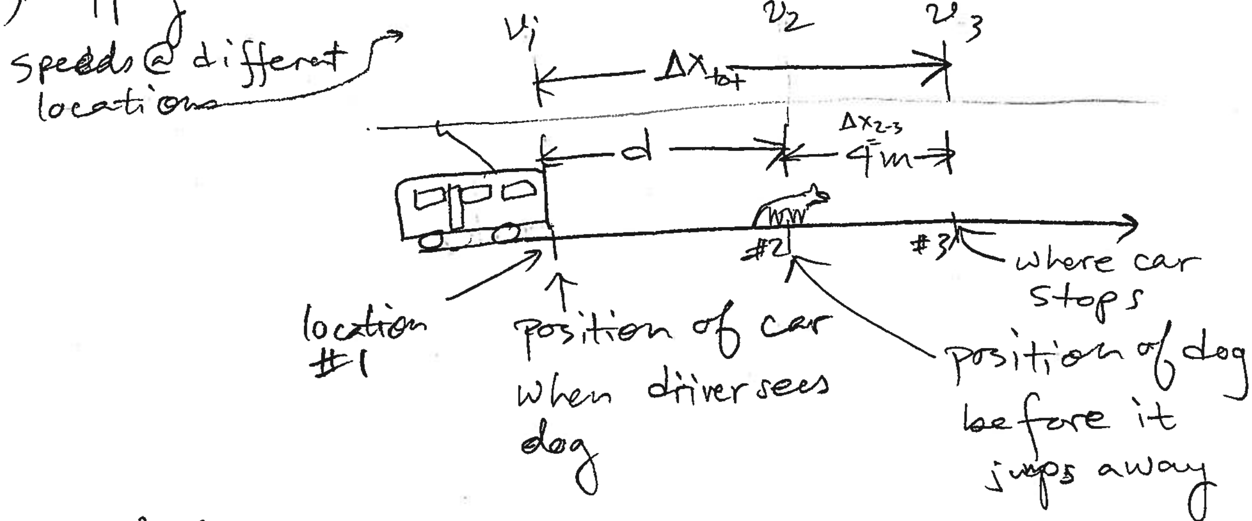
$$v^2 = v_0^2 + 2a(x - x_0)$$

So $(x - x_0) = -\frac{v_0^2}{2a} = -\frac{(100)^2}{-2\times 5} m$

$= \underline{1000 m}$

The plane will not make it!

$\boxed{2-42}$ (Minimum) Stopping time $\Delta t = 10s$



Stopping distance $\Delta x_{tot} = ?$

Car goes distance $\underline{d}$ in time $\Delta t_{1-2} = 8.0s$

<u>Find d.</u>

Final speed of car $v_3 = 0$ m/s

Unknown values:

$\because$ car goes d distance in $\Delta t_{1-2} = 8$ s,

we can deduce that to go 4 m after that, it takes

$$\Delta t_{2\to3} = 10s - \Delta t_{1-2} = (10-8)s$$

$$\Rightarrow \Delta t_{2-3} = 2s$$

The acceleration is the same throughout the time. Let's call it a. We'll need to find what a & v_1 are in order to find d.

$\leftarrow$

We know,

$$v_f = v_i + a\Delta t \quad \text{——— ①}$$

$$\Delta x = v_i\Delta t + \frac{1}{2} a \Delta t^2 \quad \text{——— ②}$$

and $$v_f^2 = v_i^2 + 2a\Delta x \quad \text{——— ③}$$

Consider the 2nd part :

final speed : v_3
initial speed : v_2

$$\cancel{v_3}^{0} = v_2 + a\Delta t_{2-3}$$

$$\Rightarrow v_2 = a\Delta t_{2-3} \quad \text{——— ④}$$

Plug this into ③ —

$\Delta x_{2\to3} = 4\,m$

$$\cancel{v_3^2}^{0} = v_2^2 + 2a\Delta x_{2-3}$$

$$\Rightarrow -2\cancel{a}\Delta x_{2-3} = a^{\cancel{2}}\Delta t_{2-3}^2$$

$$\Rightarrow a = -\frac{2\Delta x_{2-3}}{\Delta t_{2-3}^2} = -\frac{2(4m)}{(2s)^2} = -\frac{\cancel{8}^{2}\,m}{\cancel{4}\,s^2}$$

$$\boxed{a = -2\,m/s^2}$$

Let us now use this to find Δx_{tot}:

Now
initial speed $= v_1$
final speed $= v_3$

$$\cancel{v_3}^{0} = v_1 + a\Delta t$$

$$\Rightarrow v_1 = -a\Delta t$$

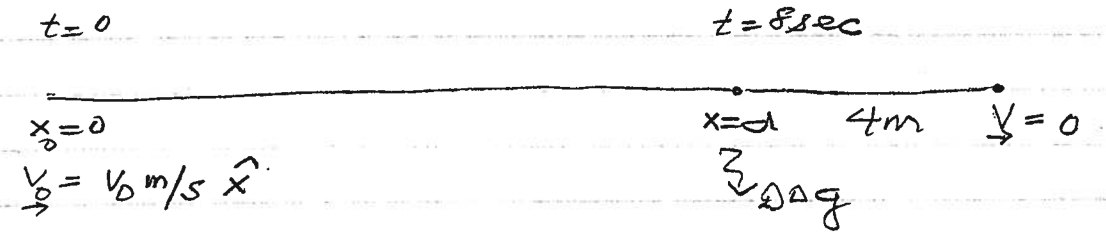
Plug this into ② —

$$\Delta x_{tot} = v_1\Delta t + \frac{1}{2} a\Delta t^2$$

$$= -a(\Delta t)^2 + \frac{1}{2} a\Delta t^2$$

$$= -\frac{1}{2} a(\Delta t)^2 = +\frac{1}{\cancel{2}}(+\cancel{2}m/\cancel{s^2})\,10^2\cancel{s^2} = \boxed{100\,m = \Delta x_{tot}}$$

2-42 Alternate Solution:



1. CLAIRE CAN STOP IN 10 secs.

$$\vec{V} = (V_0 + at)\hat{x}$$

$$0 = (V_0 + 10a)\hat{x}$$

$$\vec{a} = -\frac{V_0}{10}\ \text{m/s}^2\,\hat{x} \qquad -①$$

2. CAR TRAVELS d meters in 8 secs.

$$x - x_0 = d = 8V_0 - \frac{1}{2}\cdot\frac{V_0}{10}\cdot 8^2 \quad \rightarrow ②.$$

3. at 8 secs $\vec{V} = \left(V_0 - \frac{8V_0}{10}\right)$ m/s $\hat{x} = \frac{2V_0}{10}$ m/s $\hat{x}$

3. CAR TRAVELS 4m and stops

$$v^2 = \left(\frac{2V_0}{10}\right)^2 - 2\cdot\frac{V_0}{10}\cdot 4 = 0$$

$$\frac{2V_0}{10} - 4 = 0 \qquad \vec{V}_0 = 20\ \text{m/s}$$

From ① $\quad \vec{a} = -\frac{20}{10}\ \text{m/s}^2\,\hat{x}$

From ② $\quad d = 8 \times 20 - \frac{1}{2} \times 2 \times 64 = \underline{96\text{m}}$

$\therefore \; d = \Delta X_{tot} - \Delta X_{2\text{-}3}$

$= (100 - 4)\,m$

$\boxed{d = 96\,m}$ finally! Hurrah!

$\boxed{2\text{-}44}$ Free fall

Initial speed $\vec{v}_0 = 100\,m/s\;\hat{Y}$

(a) Gravitational acceleration $\vec{a} = -9.8\,m/s^2\,\hat{y}$

g acts downwards, so it slows down the arrow.

Arrow stops rising because : $\vec{v} = 0\,m/s\;\hat{x}$

Distance/height covered $= (y - y_0)$.

We know, $v^2 = v_0^2 - 19.6(y - y_0)$

$\Rightarrow \; y - y_0 = \dfrac{v_0^2}{19.6}$

$= \dfrac{100 \times 100}{19.6}\,m = \underline{\underline{510\,m}}$

$y = y_0 + v_0 t - 4.9t^2$

$y_0 = 0$
final position
$y = 0$

$0 = 0 + 100\,t - 4.9t^2$

$t = \dfrac{100}{4.9} = 20.41\,sec$

ALTERNATE SOL.

How long does the arrow stay in the air?

$\Delta t_{tot} = 2\,\Delta t_{up}$ time it takes for it to reach maximum height.

Going up

$\cancel{v_f}^{\,0} = v_i + a\,\Delta t_{up} = 0$

$\Rightarrow a\,\Delta t_{up} = -v_i$

$\Rightarrow \Delta t_{up} = -\dfrac{v_i}{a} = \dfrac{\cancel{+}v_i}{\cancel{+}g} = \dfrac{100\,m/s}{9.80\,m/s^2} = 10.2\,s$

$\therefore \Delta t_{tot} = 2(10.2\,s)$

$\boxed{\Delta t_{tot} = 20.4\,s}$

Case 1

$\boxed{2\text{-}47}$ Helicopter is descending steadily with speed $-1.5\,m/s\,\hat{y}$. So is the mail bag before being dropped has

$\vec{v}_0 = -1.5\,m/s\,\hat{y}$.

$\uparrow +\hat{y}$ $\downarrow -\hat{y}$

Elapsed time after dropping the bag: $\Delta t = 2.00\,s$

Case 2

Helicopter ascending.

$v_i = +1.50\,m/s\,\hat{y}$.

$\Delta t = 2.00\,s$ the same

(a) Final velocity of mailbag, $v_f = ?$

We know, $\vec{v} = (v_0 - 9.8t)\,\hat{y}$

$\Rightarrow$

Case 1

$\vec{v}_i = -1.50\,\text{m/s}\,\hat{y}$

$v_f = -1.50\,\text{m/s} - 9.80\,\text{m/s}^2\,(2\,\text{s})$

$\Rightarrow \boxed{\vec{v}_f = -21.1\,\text{m/s}}\,\hat{y}$

Case 2

$v_i = +1.50\,\text{m/s}\,\hat{y}$

$v_f = 1.50\,\text{m/s} - 9.80\,\text{m/s}^2\,(2\,\text{s})$

$\Rightarrow \boxed{\vec{v}_f = -18.1\,\text{m/s}\,\hat{y}}$

(b) How far below the helicopter is the mail bag?

First, find the distance the mail bag falls after being dropped:

$\vec{\Delta y} = (v_0 t - 4.9t^2)$

v_i

Case 1

$\Delta y = (-1.50\,\text{m/s})2\,\text{s} - \frac{1}{2}(9.80\,\text{m/s}^2)4\,\text{s}^2$

$\Rightarrow \vec{\Delta y} = -22.6\,\text{m}\,\hat{y}$

Case 2

$\Delta y = 1.50\,\text{m/s}\;2\,\text{s} - \frac{1}{2}\,9.80\,\text{m/s}^2\;4\,\text{s}^2$

$\Rightarrow \Delta y = -16.6\,\text{m}\,\hat{y}$

Then, find how far up/down the helicopter has moved in that time: $v_{heli} = v_i$

Case 1

$\Delta y_h = v_{hei} \Delta t$

$= (-1.5\, m/s)\, 2s\, \hat{y}$

$\vec{\Delta y}_{heli} = -3m\, \hat{y}$

Case 2

$\Delta y_{heli} = +(1.50 m/s)(2s)\hat{y}$

$\vec{\Delta y}_{heli} = +3m\, \hat{y}$

Finally, combine the two distances:

$\Delta y_{tot} = \Delta y - \Delta y_{heli}$

Case 1

$\Delta y_{tot} = (-22.6 + 3)\, m\hat{y}$

$\boxed{\vec{\Delta y}_{tot} = -19.6m}\,\hat{y}$

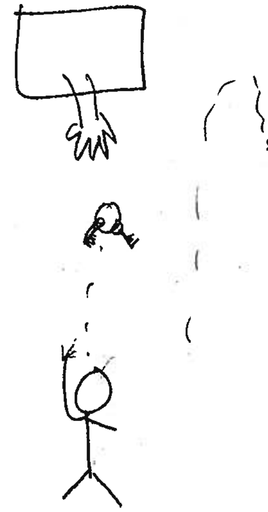
Case 2

$\Delta y_{tot} = (+16.6 - 3)\, m\hat{y}$

$\boxed{\Delta y_{tot} = -19.6m}\,\hat{y}$

$\boxed{2\text{-}51}$ $\Delta y = 4\,m$

$\Delta t = 1.5\,s$



ⓐ $v_i = ?$

$$\Delta y = v_i \Delta t - 4.9t^2$$

$$\Rightarrow v_i \Delta t = \Delta y + \frac{1}{2} g \Delta t^2$$

$$\Rightarrow v_i = \frac{\Delta y}{\Delta t} + \frac{1}{2} g \Delta t$$

$$= \frac{4m}{1.5s} + \frac{1}{2}\, 9.80\,m/s^2\; 1.5\,s$$

$$\boxed{v_i = 10.0\,m/s}\ \hat{y}$$

ⓑ $v_f = v_i + at$

$$= v_i - gt$$

$$= 10\,m/s - 9.8\,m/s^2\,(1.5\,s)$$

$$\boxed{\vec{v}_f = -4.7\,m/s}\ \hat{y}$$

The keys were caught while they were dropping.

2-52 FROG HOPPER HAS ENORMOUS Acc.

$\underset{\rightarrow}{a} = 4000\,m/s^2\,\hat{x}$

Horizontal case: Velocity after jump of 2mm

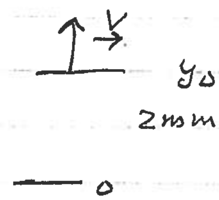
$$v^2 = v_0^2 + 2a(x - x_0)$$
$$= (0 + 2\times4000\times2\times10^{-3})\,m^2/sec^2$$

$$\underset{\rightarrow}{V} = 4\,m/s\,\hat{x}$$

VERTICAL CASE

1st part HOPPER JUMPS acquiring vel.

$$\underset{\rightarrow}{v}^2 = \left[0 + (2\times4000\times10^{-3})\right]$$



$$\underset{\rightarrow}{V} = 4\,m/s\,\hat{y}$$

In 2nd part it is "free fall"

Now $\underset{\rightarrow}{v_0} = 4\,m/s\,\hat{y}$

$\underset{\rightarrow}{V} = 0.$ at top of jump.

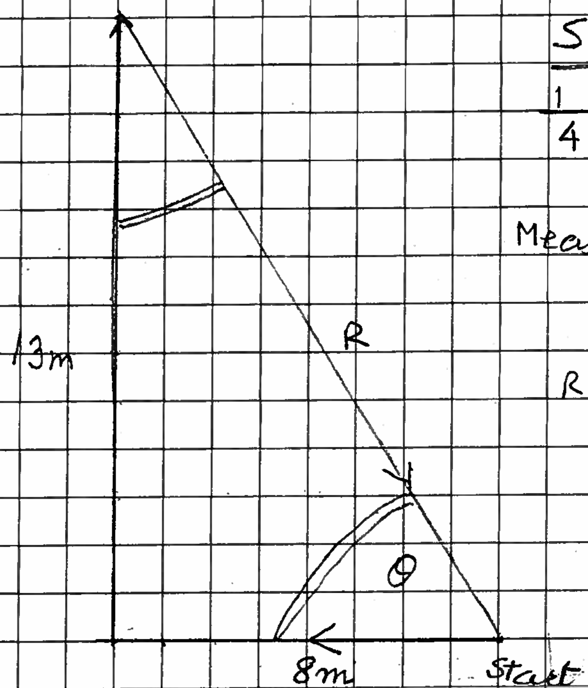
$$0 = 4^2 - 19.6(y - y_0)$$

$$y = y_0 + \frac{4^2}{19.6} = (0.002 + 0.816)\,m$$
$$= \underline{0.818\,m}$$

This neglects air resistance, apparently the actual jump is still an impressive <u>0.7m</u>

CHAPTER 3

3-3 GEOMETRICAL METHOD FOR ADDING VECTORS



Scale

$\frac{1}{4}'' = 1\,m$

Measure diagonal

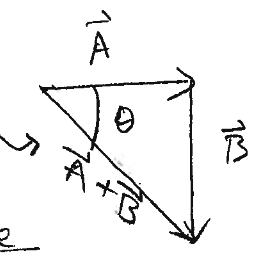
$3'' + \frac{13}{16}''$

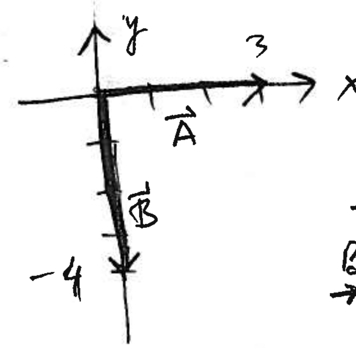
$R = 15.25\,m$

$\theta = 59^\circ$

3-7

$(\vec{A}+\vec{B})$





$\vec{A} = 3\hat{x}$

$\vec{B} = -4\hat{y}$

Magnitude

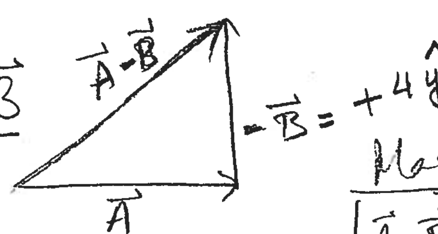
$$|\vec{A}+\vec{B}| = \sqrt{3^2+4^2} = \sqrt{9+16}$$

$$= \sqrt{25} = \boxed{5 \text{ units}}$$

Direction:

$$\tan\theta = \frac{-4}{3} \Rightarrow \theta = \tan^{-1}\left(-\frac{4}{3}\right) \Rightarrow \boxed{\theta = -53.1^\circ}$$

(b) $\vec{A}-\vec{B}$



$\vec{A}-\vec{B} = \vec{A}+(-\vec{B})$

Magnitude

$$|\vec{A}-\vec{B}| = \sqrt{3^2+4^2} = 5 \text{ units}$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ = \theta$$

3-12

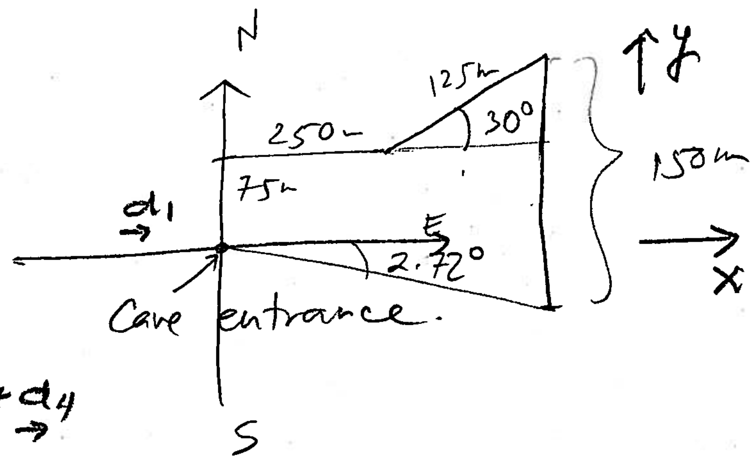
$\vec{d}_1 = 75.0\ \text{m}\ \hat{y}$

$\vec{d}_2 = +250\ \text{m}\ \hat{x}$

$\vec{d}_3 = 125\ \text{m}\ (\cos 30\,\hat{x} + \sin 30\,\hat{y})$

$\vec{d}_4 = -150\ \text{m}\ \hat{y}$

Needed $\vec{R} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 + \vec{d}_4$



Determine

$R_x = d_{1x} + d_{2x} + d_{3x} + d_{4x}$

$= 250\ \text{m} + 125\ \text{m}\ \cos 30^\circ = 263\ \text{m}$

$R_y = 75\text{m} + 125\text{m}\ \sin 30^\circ - 150\text{m}$

$= -12.5\text{m}$

$\vec{R} = (263\,\hat{x} - 12.5\,\hat{y})\ \text{m}$

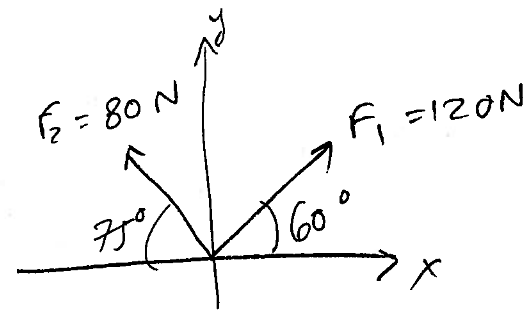
$R = \sqrt{R_x^2 + R_y^2}, \quad \tan\theta_R = \frac{R_y}{R_x}$

$R = \sqrt{(263\text{m})^2 + (-12.5\text{m})^2} = 263\ \text{m}$

$\theta_R = \tan^{-1}\left(\frac{-12.5}{263}\right) = -2.72^\circ = \theta$

3-18

(a)



Resultant force:

$\vec{F} = \vec{F_1} + \vec{F_2}$

x-dir: $F_{x,net} = F_1 \cos 60° - F_2 \cos 75°$

$= 80\text{ N} \cdot \cos 60° - 120\text{ N} \cos 75°$

$= 8.94$ N

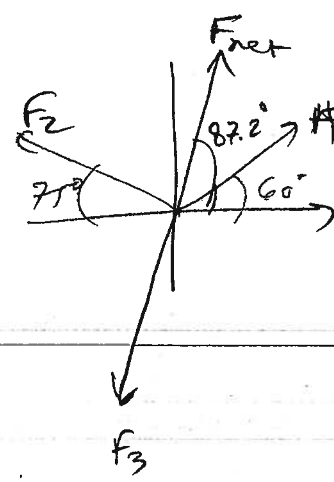
y-dir: $F_{y,net} = 80\text{ N} \sin 60° + 120\text{ N} \sin 75°$

$= 185$ N

Magnitude of the resultant force $= \sqrt{185^2 + 8.94^2}$ N

$= 185$ N

$\theta = \tan^{-1}\left(\frac{185}{8.94}\right) = 87.2° = \theta$



(b) $|\vec{F_3}| = 185$ N

$\vec{F_3} = -\vec{F}_{net}$

$\theta_3 = (87.2 + 180)°$

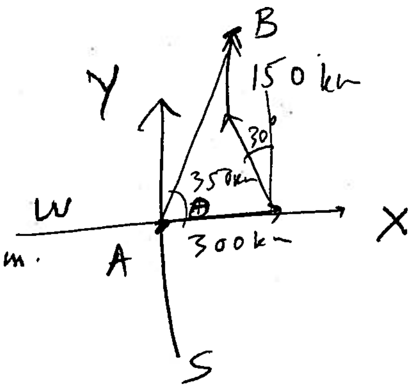
$\theta_3 = 267.2°$

$\boxed{3\text{-}20}$ $\vec{d}_{AB} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3$

$\vec{d}_1 = 300 \text{ km } \hat{x}$

$\vec{d}_2 = (-\cancel{150}^{\,35} \cos 60\, \hat{x} + 150 \cos 30^\circ \hat{y}) \text{ km}$

$\vec{d}_3 = 150 \text{ km } \hat{y}$



(a) Distance b/w A & B:

X-dir: $(d_{AB})_x = (300 - 350 \sin 30^\circ) \text{ km}$

$= 125 \text{ km}$

y-dir: $(d_{AB})_y = (350 \cos 30^\circ + 150) \text{ km}$

$= 453 \text{ km}$

Direction:

$\theta = \tan^{-1}\left(\frac{453}{125}\right) = 74.6^\circ$

(b) Magnitude $= \sqrt{453^2 + 125^2} = 470 \text{ km}$.