Statistical Mechanics of Money, Income, and Wealth

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Outline

- Boltzmann-Gibbs distribution in physics
- Probability distribution of money (theory)
- The roles of debt and time-reversal symmetry
- Probability distributions of income and wealth (data)
- Conclusions

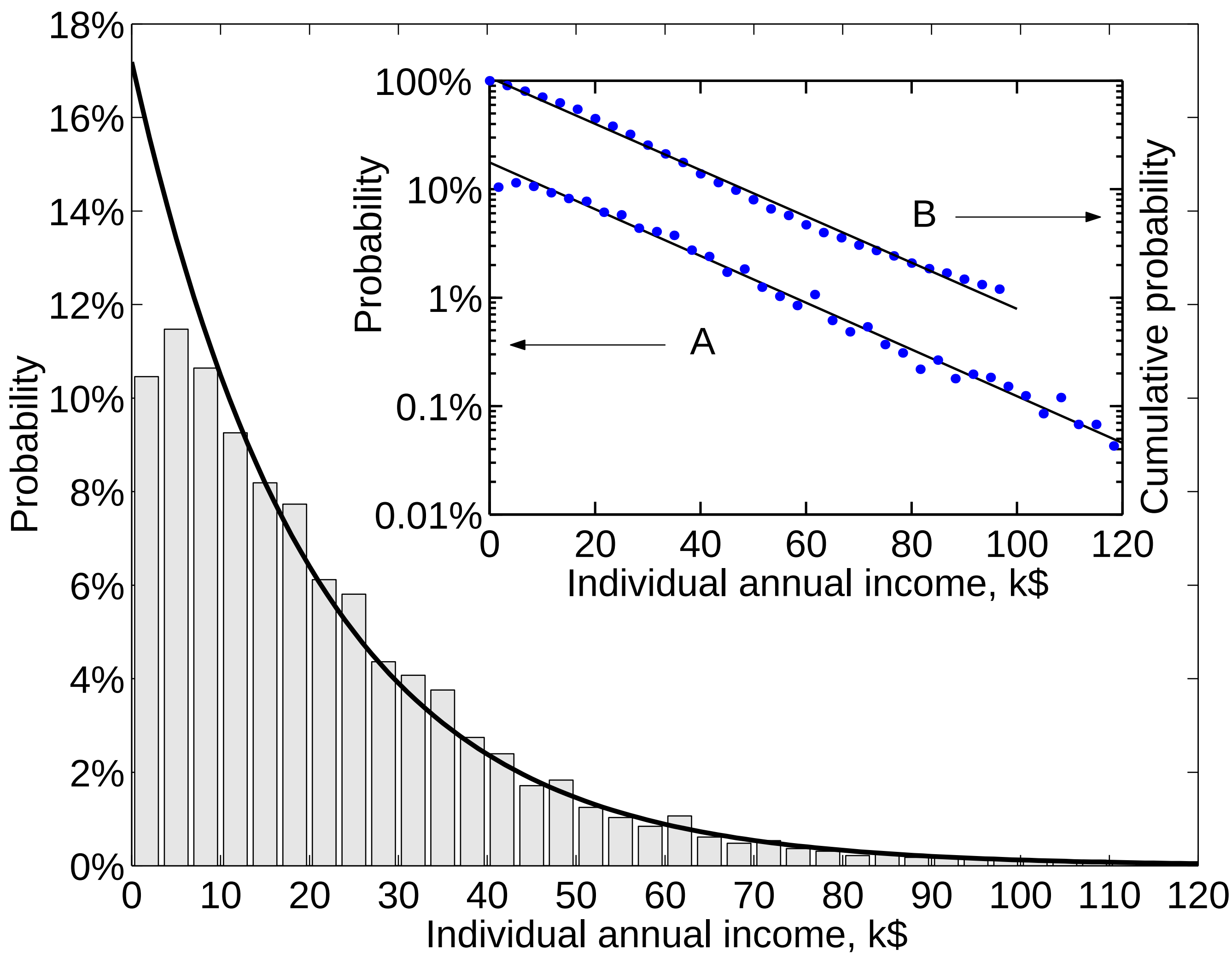
Publications

- European Physical Journal B **17**, 723 (2000), cond-mat/0001432
- European Physical Journal B **20**, 585 (2001), cond-mat/0008305
- Physica A **299**, 213 (2001), cond-mat/0103544

Probability Distribution of Individual Income

U.S. Census data, 1996 (histogram and points A)

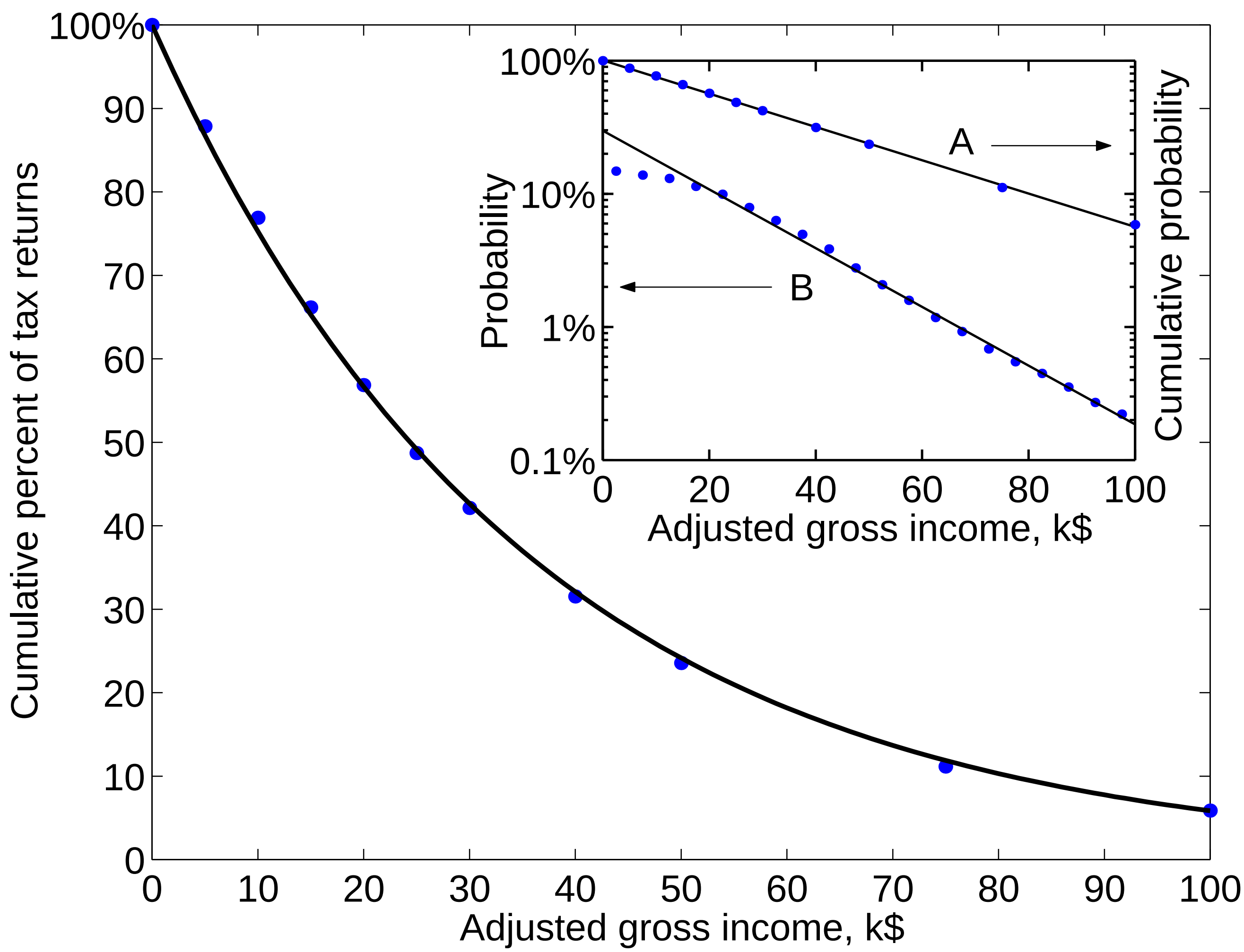
Panel Study of Income Dynamics (U. Michigan), 1992 (points B)



The solid lines are fits to the exponential distribution $P_1(r) = \exp(-r/R)/R$

Probability Distribution of Individual Income

Internal Revenue Service data, 1997 (main panel and inset A) and 1993 (inset B)



The solid lines are fits to the exponential cumulative distribution $N_1(r) = \int_r^\infty P_1(r')\,dr' = \exp(-r/R)$

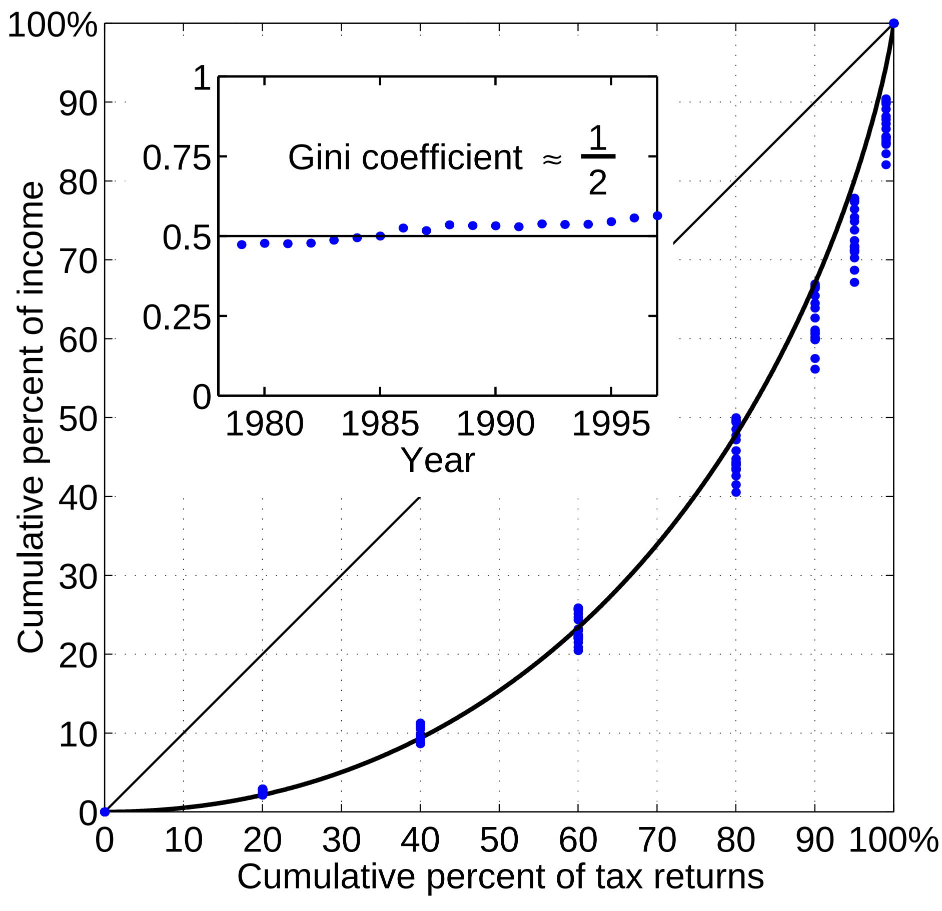
Lorenz Curve for individual income:

Horizontal axis $x(r)$ – the cumulative fraction of population with income below r: $x(r) = \int_0^r P(r')\,dr'$.

Vertical axis $y(r)$ – the fraction of income this population accounts for: $y(r) = \int_0^r r' P(r')\,dr' \,/\, \langle r \rangle$.

Gini Coefficient is the measure of inequality:

$$G = \frac{\text{Area between the diagonal and the Lorenz curve}}{\text{Area of the triangle beneath the diagonal}}$$



For the exponential distribution:

Lorenz curve: $y = x + (1-x)\ln(1-x)$, the solid line.

Gini coefficient: $G_1 = 1/2$.

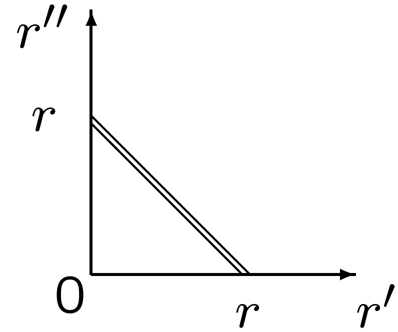
Income distribution for two-earners families

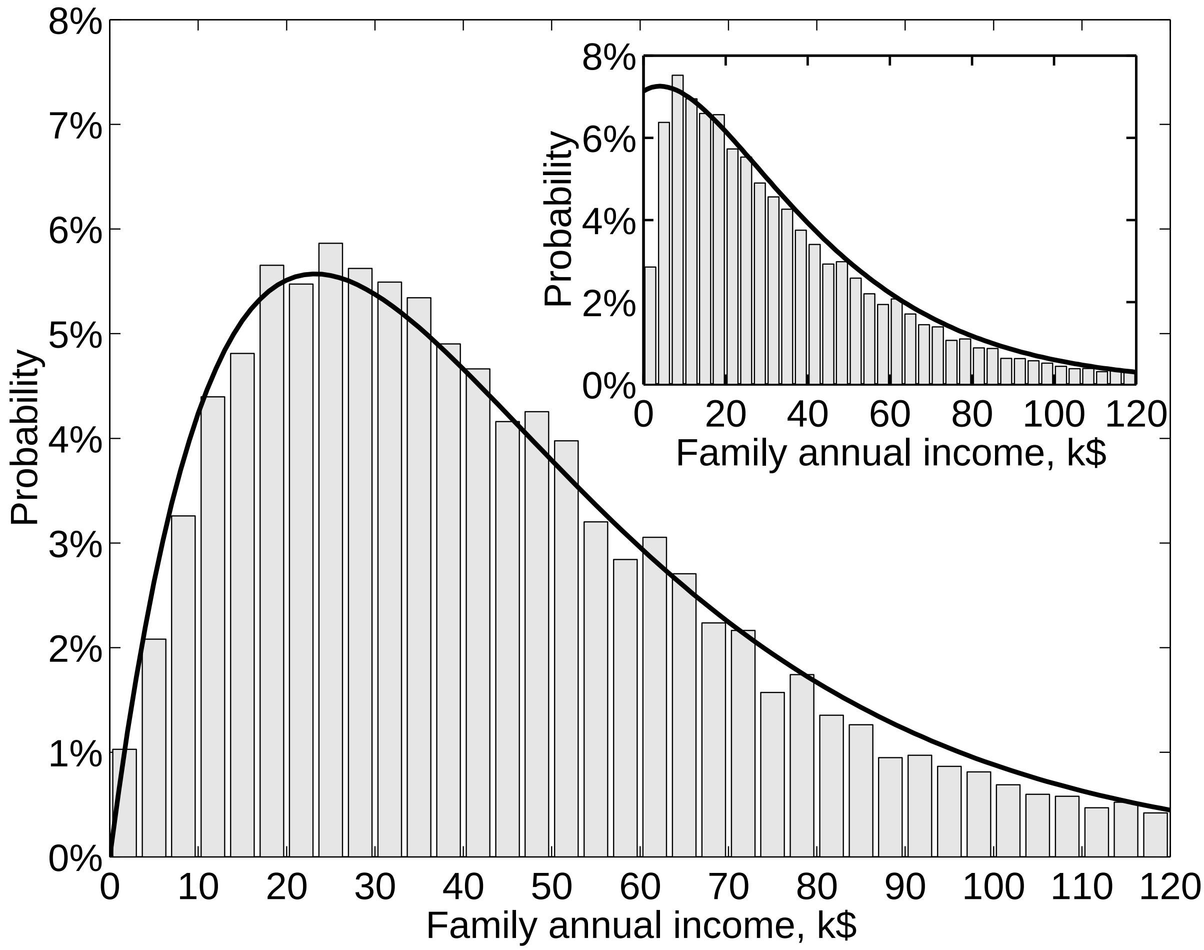
$$r = r' + r''$$

$$P_2(r) = \int dr' \int dr'' \, P_1(r') \, P_1(r'') \, \delta(r' + r'' - r)$$

$$= \int_0^r P_1(r') \, P_1(r - r') \, dr'$$

$$= (r/R^2) \, \exp(-r/R)$$

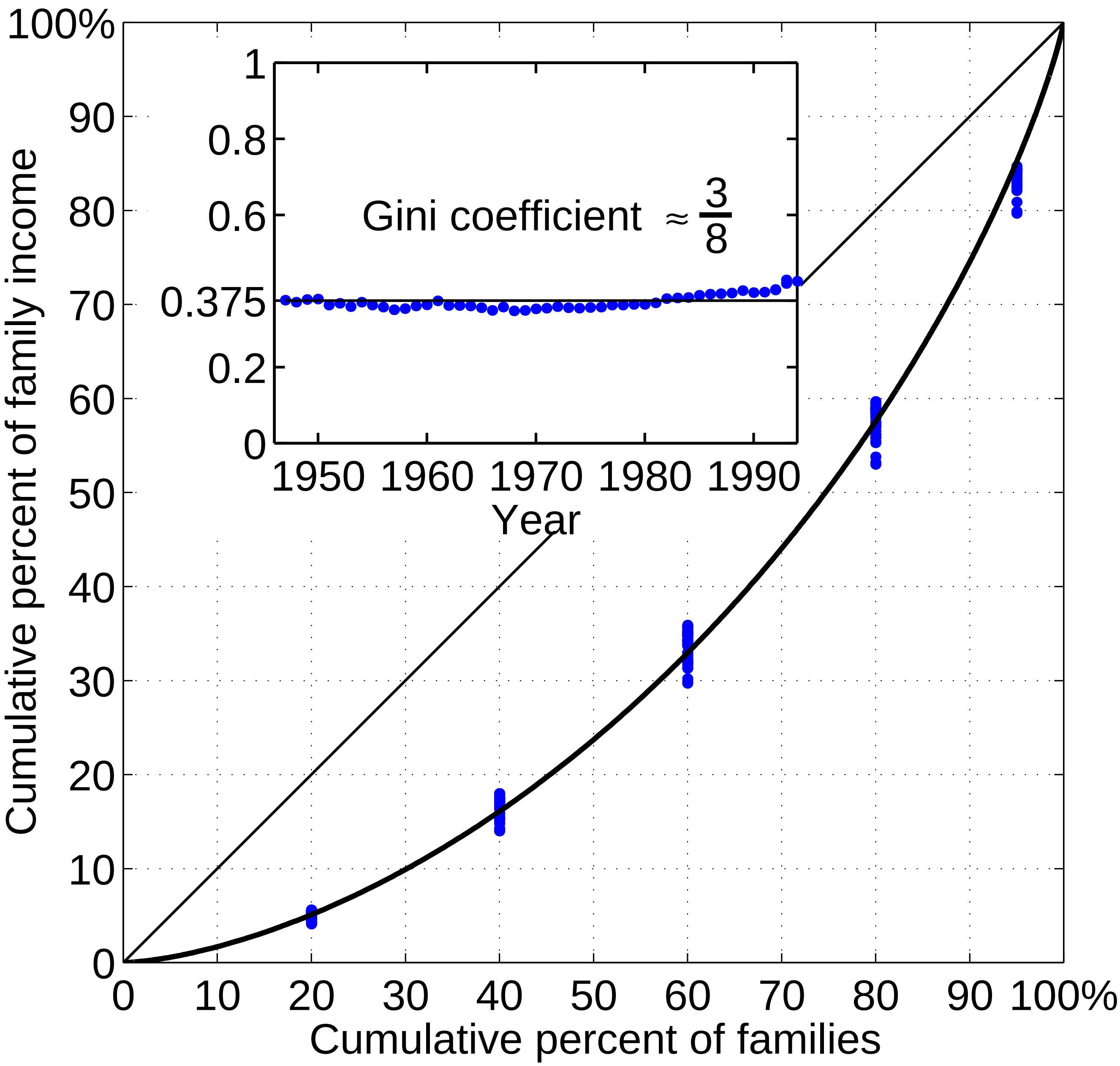




Probability distribution of income for families with two adults, U.S. Census data, 1996.

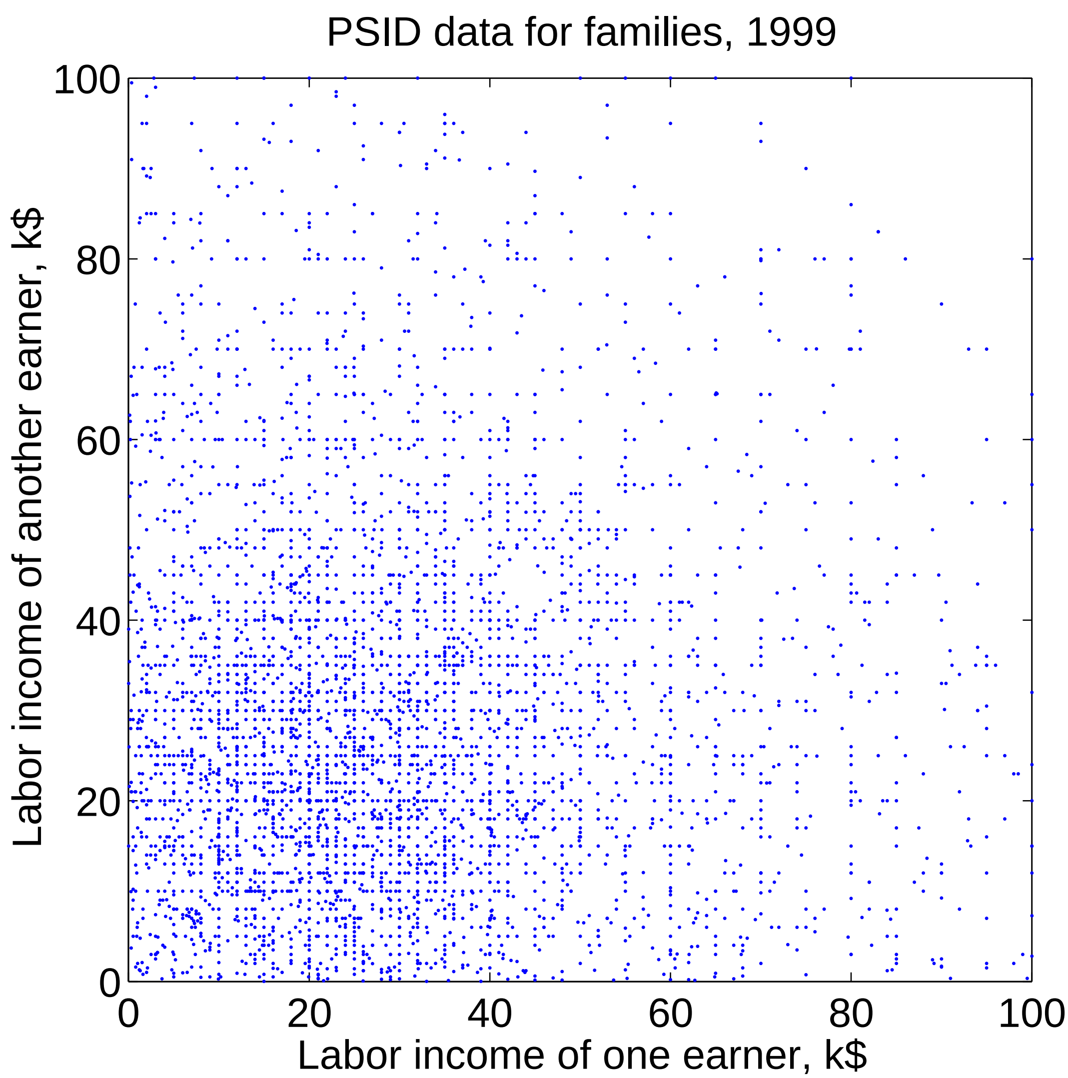
Lorenz curve and Gini coefficient for families

U.S. Census data, 1947–1994



The solid curve is for $P_2(r) = (r/R^2)\exp(-r/R)$.
The calculated Gini coefficient: $G_2 = 3/8 = 0.375$

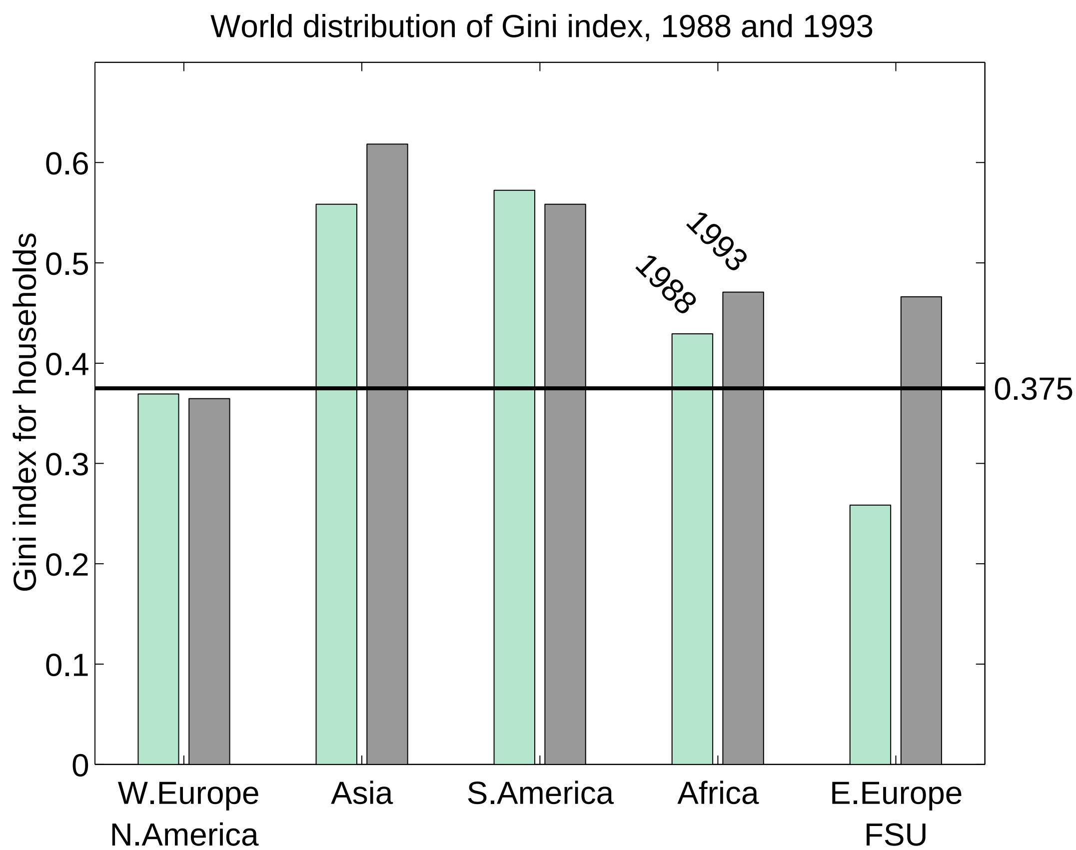
Scattering plot of two incomes for families



Every family is represented by two points (r_1, r_2) and (r_2, r_1). The absence of significant clustering of points (along the diagonal) indicates that the two incomes are approximately uncorrelated.

World distribution of Gini index

World Bank data, 1988 and 1993



In Western Europe and North America, the Gini index is close to 3/8=0.375, in agreement with our theory.

Other regions have higher Gini coefficients, i.e. higher inequlity.

A sharp increase in the Gini coefficient is observed in Eastern Europe and former Soviet Union (FSU) after the collapse of communism.

Conclusions

- Conservation law of money leads to the exponential, Boltzmann-Gibbs probability distribution of money: $P(m) \propto \exp(-m/T)$.
- Deviations from the Gibbs law exist in the models where the time-reversal symmetry is broken.
- Tax and census data for USA and UK show that distributions of individual income and wealth of more than 95% of population are exponential.
- The high-end tails have power-law distributions and account for the top 1% of population and 16% of income and wealth ("Bose condensate").
- Income distributions in UK and different states of USA collapse to a single curve, when rescaled using their individual temperatures. Temperature varies by ±25% within USA. The US temperature is twice higher than the UK one.
- A thermal machine can extract profit from the temperature difference.
- Income of families with two earners has the gamma distribution. The Lorenz curve and the Gini coefficient 3/8=37.5% agree well with the US census (1947–1994) and World Bank data.