HW#9 —Phys374—Fall 2013 Prof. Alessandra Buonanno
Due before class, Tuesday, Nov. 26, 2013 Room 4212, (301)405-1440
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1. Evaluate $\int_a^b \delta(x^2-3)dx$ for (i) $[a,b]=[-1,1]$, (ii) $[a,b]=[0,2]$, (iii) $[a,b]=[-2,0]$, (ii) $[a,b]=[-2,2]$. (See Chapter 14 for Dirac delta functions. Section 14.3 discusses delta function of a function, which was also explained in class.) [10 pts.]

2. Consider the integral

$$I = \int\int f(x,y)\delta(x^2+y^2-R^2)\delta((x-a)^2+y^2-R^2)\,dxdy,$$

taken over the entire xy plane.

   (a) Make sketches in the (x,y) plane showing geometrically where the two delta functions in in the integrand are non-zero, for $a/R = 0, 1, 2, 3$.

   (b) Evaluate I. (*Suggestion*: First do the y integral, using the first delta function to identify the relevant y values.)

   (c) Explain the qualitative behavior the dependence of I on a/R in terms of your sketch in part 2a. In particular explain why it diverges where it diverges, and where it is zero. (*Guidance*: Imagine the delta functions as having a small width, before taking the limit as the width goes to zero and the height to infinity, so each of their regions of nonzero support forms a ring. Consider how the area of the region in which both delta functions are non-zero depends on a/R. The idea behind this was explained in class.) [10 pts.]

3. The relation between the real Fourier coefficients for the sine and cosine terms can be obtained with the help of the following identities:

$$\int_{-\pi}^{\pi} \cos(m\theta)\,\cos(n\theta)\,d\theta = \pi\delta_{mn} \quad (1)$$

$$\int_{-\pi}^{\pi} \sin(m\theta)\,\sin(n\theta)\,d\theta = \pi\delta_{mn} \quad (2)$$

$$\int_{-\pi}^{\pi} \cos(m\theta)\,\sin(n\theta)\,d\theta = 0, \quad (3)$$

where m and n are assumed to be positive integers. (These are equivalent to eqns (15.3-6) in the textbook.) Prove these identities by expressing the cosine and sine in terms of complex exponentials, and using $\int_{-\pi}^{\pi} e^{ik\theta}\,d\theta = 2\pi\delta_{k0}$. (Here δ_{kl} is the Kronecker delta, equal to 1 if the integers k and l are equal, and zero otherwise). [10 pts.]