Chapter 7: Student Difficulties with Classical Pre-requisites

Introduction

Student initial states are always important factors affecting their learning of new material. The students that come into our classes are not blank slates. They bring in with them their understandings of the physical world. A good understanding of student initial states is the starting point for good understanding of student difficulties and for the development of effective instructions.

Although quantum mechanics is different from classical physics, a good understanding of classical concepts is often crucial to the learning of quantum. A quantum system is usually constructed with various elements from classical physics. Therefore, for students coming into a quantum class, their understanding on these classical concepts is an important initial state that need to be carefully studied.

Student Difficulties Indicated from General Observations

Since most quantum systems are represented with a potential energy diagram, a correct interpretation of the energy diagram is crucial to understand the physical system. According to our observations, students' difficulties in understanding quantum systems are often the results of their failure to understand the potential energy diagram. Even for classical examples, many students are not able to make a correct connection between a potential energy diagram and the physical system it represents. Therefore, in order to help the students with their difficulties in understanding quantum systems, we need to carefully study the students' understandings of potential energy diagram and find ways to help them.

Another common difficulty is that students are often unable to understand the physical meaning of elements in the solution obtained from a quantum problem. Some students can work through the mathematics and get a result. But it is far more difficult for them to understand the meaning of the physical elements in the result such as quantum wavefunction. If there is a classical counterpart for a quantum variable, students often stick with the classical interpretations. On the other hand, if there aren't any similar classical concepts, students usually don't attempt to build a physical understanding of the concept beyond mathematics. When inquired, some of the students can even creatively use certain classical ideas that they already have to construct a classical interpretation for quantum concepts. Since all quantum phenomena need to be interpreted in a probabilistic manner, the understanding of probability can significantly affect the student understanding of quantum concepts. As observed in research and teaching, many students are found to have poor background on issues related to probability. To help students understand quantum concepts, we need to improve their understanding of probability.

Therefore, in this research, we begin with investigations on two classical issues, *Potential Energy Diagram* and *Probability*.

Probing Student Difficulties

Research Context

For this part of research, we studied the students from Physics 263 classes. This is the last course in the introductory physics series for science and engineering majors. We conducted research in two classes at the University of Maryland in the fall of 1994 and the spring of 1996. The class of fall 1994 was traditionally lectured. In the spring 1996 class, we implemented three quantum tutorials. Two of the tutorials were developed based on our research results on student difficulties with the classical pre-requisites obtained from simple concept tests and classroom observations.

Research Instruments

In research, we used three types of probing instruments including conceptual quizzes, exam questions, and problem-solving interviews. The following is a detailed discussion of the individual tools.

Conceptual Quizzes

The conceptual quiz is designed to be a 10-minute test containing 3-4 simple conceptual questions and is given in the beginning of a lecture. Figure 7-1 shows a question that was used in a quiz in Physics 263. This question is designed to probe a number of issues. First it tests whether the students understand the different shapes of wavefunction for bound states. The second issue is to see if the students can make the link between the amplitude of the wavefunction and the probability density of a particle being in certain region. It also gives the information on student understandings of the potential well.

• Homework and Exam Questions

Figure 7-2 shows an exam question used in the final exam of the Physics 263 class in fall 1996. This question is designed to probe student understanding of probability both classically and quantum mechanically.

In the spring 96 class, we developed and delivered a new tutorial on the potential energy diagram. After tutorial, a homework question was given to students to test their ability in constructing potential energy diagrams based on descriptions of various physical systems (see Appendix E for details of the tutorials and Appendix D for the homework questions).

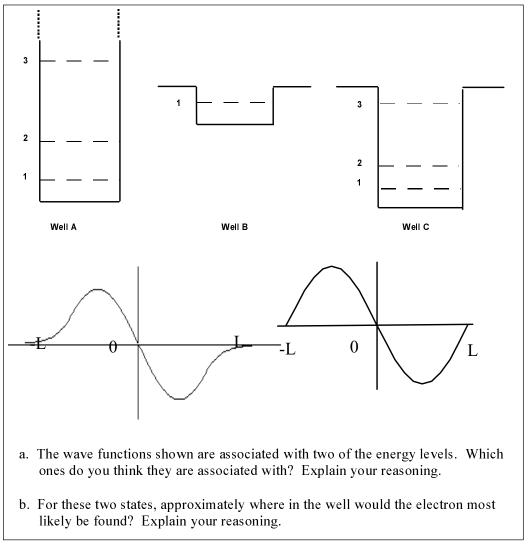
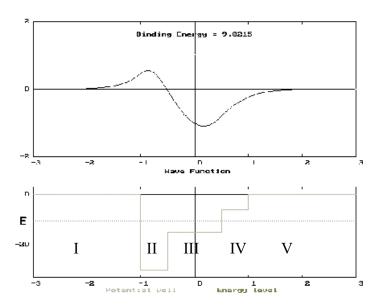


Figure 7-1. Conceptual quiz question on potential well given in Physics 263 class at UMd in fall 1994 and spring 1996

In the figure below is shown a plot of a one-dimensional potential energy function U[x] (lower plot) and the wavefunction (upper plot) of an eigenstate for an electron in the influence of that potential. The energy, E, associated with that state is also shown on the energy plot. Distances are measured in nanometers and energies in eV.



The potential energy U[x] is defined so that it has four values in five regions of the x axis:

 $\begin{array}{lll} \textbf{Region I:} & x < -1 & U[x] = 0 \\ \textbf{Region II:} & -1 < x < -0.5 & U[x] = -30 \\ \textbf{Region III:} & -0.5 < x < +0.5 & U[x] = -15 \\ \textbf{Region IV:} & +0.5 < x < +1 & U[x] = -6 \\ \textbf{Region V:} & 1 < x & U[x] = 0 \end{array}$

- 1.1 If the particle were moving classically (i.e., its motion were described by Newton's laws) in the potential U[x], and it had an energy E, which of the following statements would be true? List all that apply.
- (a) It would move the fastest when it was in region II.
- (b) It would move the fastest when it was in region III.
- (c) If we took a photograph of the particle at a random time, we would never find it in region IV.
- (d) If we took a photograph of the particle at a random time, we would be most likely to find it in region II.
- (e) If we took a photograph of the particle at a random time, we would be most likely to find it in region III.
- 1.2 If the particle were moving quantum mechanically (i.e., its motion were described by the Schrödinger equation) in the potential U[x], and it had an energy E, which of the following statements would be true? List all that apply.
- (a) If we measured the position of the particle at a random time, we would never find it in region I.
- (b) If we measured the position of the particle at a random time, we would never find it in region IV.
- (c) If we measured the position of the particle at a random time, we would most likely find it in region II.
- (d) If we measured the position of the particle at a random time, we would most likely find it in region III.
- (e) The state shown represents the lowest energy state that can be found in this well.

Figure 7-2. Exam question on quantum wavefunction given in Physics 263 class at UMd in fall 1996

Interviews

Further insight into student thinking and their difficulties are obtained through problem-solving interviews with individual students. We interviewed six students from Physics 263 (UMd) of fall 1996 using the problem shown in figure 7-3.

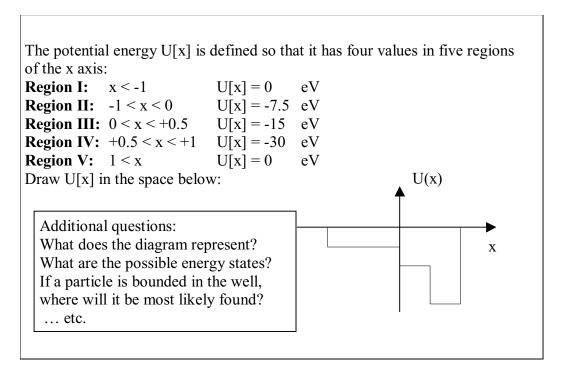


Figure 7-3. Interview question to students in Physics 263 (UMd) of fall 1996

Most students can come up with a potential energy diagram similar to the one shown in the figure and additional questions are then asked to probe their understandings on other aspects of the potential well. As we asked students to construct the potential diagram, additional issues such as the understanding of the positive/negative total energy were also studied.

Results

The interviews were conducted when the course is finished. The data reveals subtle insights on student understandings of potential energy diagram and probability.

Tendency to Interpret 1-D Potential Well as 2-D Gravitational Well

When presented with a square well potential diagram, most of the students cannot associate an appropriate physical system with it. Many go by their intuition of a gravitational well and interpret the graph as a picture of a 2-D system. For the class (Physics 263) in fall 94, we gave the students the quiz question, shown in figure 7-1, after

two weeks of instruction on quantum mechanics. At the time of the quiz, the students had been given lectures on solving problems using quantum potential wells.

The student responses reveal difficulties in understanding energy diagrams and probability. Only one out of a total of 94 students answered correctly. About half of the students tried to describe the position of the electron using the energy levels shown in the graph. Some typical student responses are:

"The electron will most likely be found at level 2."

"It will be most likely found in the dashed lines."

"It will be most likely at bottom of the well."

When we discussed this issue with students during office hours and after lectures, some students explicitly said that the electron is "bumping back and forth in the well and will climb up the wall at the end of the well".

These student responses imply that these students are taking the potential energy diagram as a picture of a 2-D system. Since most students are familiar with a real gravitational well where the y-axis in the vertical direction describes the position and is also associated with potential energy, it is natural for students to match this 2-D gravitational well with the potential energy diagram.

This result is also reflected in student responses on interviews. In the interview, we asked the students to construct a potential energy diagram and many students were able to do it successfully (as shown in figure 7-3). We then asked the students to explain their graph. Many of them used the term "border" to describe the vertical lines in the potential energy diagram. Since the border is a concept for a physical boundary, we suspected that the students were interpreting the diagram as a two-dimensional physical system. To probe more details, we asked the students to tell the story about the graph. A student, Newt (code name), gave a typical response:

"Well. If we move along the x-axis we fall into an energy well, it just means we have fallen into the well. Something happened at x = -1 that made us fall into instantaneously, into a lower potential energy. And something happens again at x=1 pulls us back all the way up to zero."

We saw a lot of similar stories where the terms like "fall into" and "climb up" are frequently used. These terms have clear meaning of motion in the vertical direction. We think the students were in a very confusing stage or at a transition point in a learning process towards the correct understanding. Although they didn't explicitly spell out a physical two dimensional well, which is encouraging, all of the them were uncertain about what "fall into" meant and persisted in using the term "fall into", which implies that at least they were still using the intuition or metaphor of vertical motion.

Insufficient background knowledge about probability

For the class of fall 94 (Physics 263), when the students encountered the question in the conceptual quiz of where in the well will the electron be most likely found, most of them seemed to have almost no idea on how to answer it, despite the fact that the quiz was given after instruction. Nearly half of the students left it blank. About 40% used an incorrect dimension for position. Only one student came up with a somewhat correct answer. For all the students, only 10% of them provided some kind of reasoning.

We also discussed this issue with students in office hours and lectures. When we asked them if they had any previous knowledge about probability, most of them answered "no". Even for the few students who answered "yes", they only had the impression of doing some math in a math class and did not remember any details of the mathematics. None of them recalled any experience in using probability to describe a physical event.

These results clearly indicate that for most students in our introductory physics classes, the concept of probabilistic interpretation of a physical system is weak or missing.

New Instructions to Help Students on Classical Pre-requisites

The results from our investigations reveal that the student understanding on the two classical pre-requisites is very poor. Since both issues are crucial elements for developing a good understanding of quantum mechanics, and since traditional instruction does not provide effective assistance on learning these concepts, we developed a tutorial and used it in the spring 96 class (see Appendix D for the tutorial).¹

Potential Energy Diagrams

The potential energy diagram tutorial is designed to help student understand potential energy diagrams and classical probability. We provide students an example of a one-dimensional classical system with a cart moving along a track constrained by springs or bouncing off springy blocks at the end of the track.² The motion is recorded in real time using a computer interface (ULI) and an ultrasonic motion detector. The mechanical force experienced by the cart is measured simultaneously with a force probe mounted on the cart.³ Students use the computer to graph the force as a function of position of the cart and use this data to construct the potential energy diagram by integration. In the tutorial, before students do any experiment, they are always asked to make predictions based on their current knowledge and to discuss their predictions with the group. This environment can give students opportunities to compare their various ideas and also encourages them to do qualitative reasoning. Figure 7-4 is a schematic diagram for the setup of the experiment.

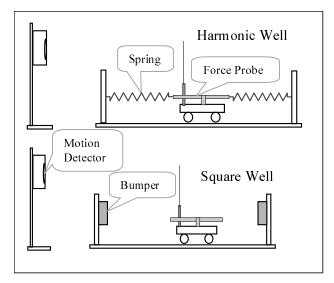


Figure 7-4. Experiment setup for the tutorial of potential energy diagram

In the experiment, students construct two potential wells:

- 1. Harmonic Well: The cart constrained by two springs (Figure 7-4) is set oscillating on the track and the force is recorded as a function of displacement. The potential energy is then computed as the area under the graph and plotted as a function of displacement to construct the harmonic well.
- 2. Square Well: The cart is set rolling freely along the track bouncing off springy blocks fixed at the two ends of the track. The force is recorded as a function of position and the students again use this graph to plot the corresponding potential energy diagram.

In each case, the students qualitatively describe the motion to indicate where the cart is moving relatively fast, relatively slowly and where, if any place, the velocity is zero. They are also asked to discuss the forces that are acting on the cart, draw free body diagrams, sketch the kinetic, potential and total energy along the path, predict the F-x and U[x]-x graphs before they actually proceed to do the experiment. This allows the students to actually think about the problem by themselves and test their hypothesis. This tutorial is to provide the students an opportunity to personally construct a potential diagram with a real hands-on physical experiment. We believe that students are more actively engaged in such instructional settings and can often construct a better understanding of the concept.

Probability Function

Since most students don't have previous experience with probability, we need extra effort to help students start on this issue. First we designed a metaphor, the *random picture model*, to introduce the probability by asking the students to imagine taking a series of flash photographs of a classically moving object at random times and to predict where the object is most likely to be found.

Based on this notion, we designed additional questions in the tutorial asking the students to use the random picture metaphor on the cart and to predict in which segment of the path will the cart more likely be found. This is followed by leading questions about where the cart spends relatively more time and less time on the track. The students then write an expression for the time Δt spent in a segment Δx centered around the position x and relate this to the probability of locating the cart in that particular segment. The mathematics can be shown as the followings.

For periodic motions,

$$\Delta t = 2 \left| \frac{\Delta x}{v(x)} \right|$$

The probability P(x) is found to be

$$P(x, \Delta x) = 2 \frac{\Delta t(x, \Delta x)}{T} = \frac{2}{T} \left| \frac{\Delta x}{v(x)} \right|$$

where x is taken as the mid-point of Δx region. Since $P(x, \Delta x) = \rho(x) \Delta x$, the probability density can be calculated with

$$\rho(x) = \lim_{\Delta x \to 0} \frac{P(x, \Delta x)}{|\Delta x|} = \frac{2}{T|v(x)|}$$

The normalization condition can be verified by

$$\oint \rho(x)dx = \oint \frac{2dx}{T|v(x)|} = \int_0^T \frac{dt}{T} = 1$$

Evaluations of the New Instruction

Improvement on Understanding of Potential Well and Classical Probability

To compare with the results of the conceptual quiz given to the fall 94 class (given after instruction), the same quiz was given to the spring 96 class after the students have done the new tutorial. The results are encouraging, though not as good as what we would hope. Table 7-1 is the comparison of responses of students from classes of fall 94 and spring 96. From the data, we can see that after the tutorial, about 30% of the students start to use "x" to represent position compared to only 9% in the class without tutorial. This can be an indication of a better understanding of potential diagram.

The data also shows that a significant fraction, 27%, of the students can relate the probability of finding the electron in certain regions with the velocity of the electron. Although it is still a classical argument, compared to the situation of fall 94 where nobody

could come up with any reasoning on probability, the results in spring 96 is encouraging. A significant fraction of students seem to have been affected by the tutorial.

Table 7-1. Comparison of student responses on conceptual quizzes

Student Responses	Fall 94	Spring 96
Use energy levels/ states (vertical dimension) to describe the position of an electron in a potential well (wrong)	36%	27%
Use <i>x</i> (horizontal dimension) to describe the position of an electron in a potential well (correct)	9%	30%
Others	15%	14%
Blank	40%	29%
Student Reasoning	Fall 94	Spring 96
Use velocity for reasoning of probability	0%	27%
Have reasoning*	11%	33%

The reasoning of fall 94 students is mostly based on irrelevant issues. The reasoning of the spring 96 students is based on energy and velocity in a classical sense.

We also find 33% of the students in the spring 96 class attempted to explain their reasoning and most of them used velocity and energy. In the fall 94, only 11% of the students attempted any kind of reasoning and few of those made any physical sense. This may imply that the tutorial can encourage students to do qualitative reasoning.

On the final exam of spring 96, we gave students the multiple-choice multiple-response question shown in figure 7-2. Student responses on this question (table 7-2) show encouraging results. As we can see, more than half of the students can answer either the quantum or the classical part of the questions with all correct choices (it is a MCMR question). Nearly half of the students can answer both parts with all correct choices. Since it is a MCMR question, the number of students giving partially correct answers are quite high – around 80%. Unfortunately, we don't have data from the fall 94 class, so we cannot make comparisons. However, the students performance on this questions can be considered very positive considering the response of the control class (fall 94) on the other questions.

Table 7-2. Student responses on the question in final exam of physics 263 in spring 1996

	Classical Part	Quantum Part
Students picked all correct choices	58%	69%
Students picked correct choices and also some incorrect choices	81%	84%
Students answer both parts with all correct answers	42%	

The data from the homework given to the spring 96 class also reveals a similar situation. About 63% of the students gave correct answers. These results suggest that the tutorial has made some positive effect on student learning of potential energy diagrams and probability.

Continuing Difficulties

To confirm these test results and to look for further details of student difficulties, we conducted 6 interviews after the final exam in the spring of 96 (All of these students earned A's in the class). The students in our interview showed difficulties similar to what have been found from the test results. However, with detailed discussion, student difficulties on more subtle issues were discovered. The issues discussed in the following sections have not yet been completely validated through systematic research. However, the results can provide valuable clues for further study.

Difficulties in Understanding Negative Total Energy

Surprisingly, almost all the students interviewed had considerable difficulties in understanding what values the total energy can have in classical case. When asked that if a particle can have negative total energy, some student responses were:

"In a classical system, we can not have negative energies so you can have anything greater than or equal to zero, but not anything less. Classically."

"Negative total energy? No it does not make sense. Just like there is no negative time. By definition, energy is how much work can be done. Now if you have negative something, what does that mean ... it does not have a ... classically that does not mean anything."

Only one student, Newt, showed some correct understanding in differentiating the total energy and kinetic energy:

"Kinetic energy - you could not have certain things for the classical system. In a classical system negative kinetic energy means nothing. Right, because it can not have negative mass, velocity can't be negative - it is squared so it is weird. Negative kinetic energy means nothing. But energy ... yeah, I thought energy you could have any thing. Except infinite. It has to be finite."

This statement provided us a lead to study this difficulty. He knew that the total energy is the sum of the potential and kinetic energies. We asked if the requirement that kinetic energy cannot be negative puts any limits on the total energy. Newt tried to figure that:

"So if kinetic energy can not be negative, are you suggesting that ... oh, yeah. I guess energy can't be below the potential energy. In a classical system, outside the well, we can't have energy being less than zero because that would mean that the kinetic energy would have to be negative in order to give us a potential energy. Anywhere. OK. Anywhere, the energy can't be less than the potential energy. That is a constraint on the energy."

All the other students had difficulties seeing the contradiction and often represented the given negative value of total energy correctly on the potential energy diagram even as they resolutely claimed:

"... total energy is going to be ... has to be positive ...".

At the outset, we had not expected this mental block against a negative value of total energy. A plausible reason for the inconsistent thinking could be that the students see energy as a directly measurable physical quantity like mass or temperature and expect all these to have an absolute and positive value. Actually, in everyday life, we often use the term of *energy* differently from what is meant in physics. Many real life example can lead people to think energy as a directly measurable non-negative thing, e.g., the kilowatt of electricity used in a house, the calorie contained in food, or even gallons of gas that move our cars. People usually do not see energy as a derived quantity representing an abstract attribute that depends upon how we define the zero of potential energy.

Another possibility is that many of them are confused with the difference between the total energy and the kinetic energy and do not realize that the total energy is the sum of KE and PE. This is implied by Michael's response. When we asked him to consider the top of the table to be at zero gravitational energy and set up a ramp reaching the floor, he quickly interrupted us with:

"All right. Yeah. I guess you could have negative total energy. I see what you are saying. You define ... it depends on what you define to be, I guess, your zero. If we roll something off the table to the floor, then because it is closer to the earth, it's potential energy has decreased. So it has negative potential energy. So in this (case) you could pretend that this is the potential energy of the pen (places the pen on the table) and I bring it down here, to down here, finally set on the floor and back up so ...

It took some more prodding before he concluded:

"Well this is the floor (points to the bottom of the potential energy diagram). In that analogy. So you could not go below negative 30 eV."

Possible values of total energy in quantum case

Students also have many difficulties understanding the total energy in quantum cases. Most of the students totally ignored the quantization of energy and thought that only negative total energy is allowed in quantum situations. When we asked them what the possible allowed values of total energy could the particle have in a finite square well, only Bob ever mentioned about discrete energy levels:

"Yes, there are only certain ... let me recall it ... distinct levels that it can occupy inside this little well. Yeah. Like I said, the total energy - it could be a set of discrete values. Inside the well. Let me think for a second ... it will have discrete energy levels if the total energy is below, is negative."

The remaining five students all had similar responses such as:

"No, there is no limitation. The limitation again has to be the floor. See you can't go below the floor. Yeah, which would mean you can't go below -30. So the same limitation as in the classical case."

"It can have ... negative or positive ... well it can, any where between zero and negative thirty. But it couldn't have a positive value."

Newt had a more explicit explanation:

"In the quantum mechanical system, we can have negative kinetic energy. So whatever potential energy is ... so, yes, we can have any value of the energy"

When we asked him what made him think so, he vacillated:

"I read in a book. No, I was told that. No. I don't remember being told that kinetic energy can be negative. I remember being told that where the kinetic energy is negative, you can have a (probability) of the particle being there."

This statement provides valuable indications. Classroom instruction and texts make seemingly innocuous statements such as "... there is a some small probability of finding the particle in the region in which its total energy E is less than its potential energy U_0 . This region is called the classically forbidden region because the kinetic energy, which is E-U₀, would be negative when U₀>E." and "... negative kinetic energy has no meaning in classical physics...".⁴ This revolutionary conclusion can make a dramatic impact on the students and the two fragments of information, negative kinetic energy and classically forbidden (in other words, quantum mechanically acceptable) are appear to be memorized by students and used inappropriately their reasoning.

To these students, the concept of quantization of energy levels often failed to make a significant impact. We asked them what would happen if we changed the energy to, say, - 6 eV. Without the slightest hesitation, the students drew a new horizontal line to represent the new value of the total energy.

"So my energy line would move up. My kinetic energy would increase. Inside the well."

This indicates that they haven't been able to build up the basic intuition about quantum mechanics and still consider quantum situations in a classical sense.

Confusion on the Predictability and the Indeterministic Nature of Probability

Following our instruction in which we had introduced a number of new concepts like "random picture", "probability of finding an object in certain region", etc., we were interested in finding out how students use the terms "random", "probability" and "chance".

Some students developed a reasonably good understanding on this random picture model. Bob's description of what it means to take a photograph of the particle at a random time gives a good example.

"What does it mean? Just kind of like identify it's location. ... But it's like, taking the picture is like stopping time and saying OOPS! I found the particle here. And that's what I think of it."

"And the random part? I kind of see it as being blind folded, just leaning back and like snapping the picture."

The following is what he said about estimating the probability:

"How do I estimate? OK. I estimate the probability by the percentage of total time it spends in each region. I sort of look at the ratio of the time spent in a region over the total time as the probability"

In the interviews we also found that the students hold a deterministic empirical intuition toward probability. The description is somewhat kinematic, and some responses showed an incorrect intuition on understanding the difference between the indeterministic nature of any single observation and the somewhat determined probable distribution of the results of observations on large number of samples.

This is how the students conceive the meaning of an empirical probability distribution:

"... when you undertake an event, the likelihood that what you want will occur."

These students bring with them the belief that a small sample will replicate the probabilistic trends expected from a very large number of trials and that it is possible to predict the specific result of any single measurement on the basis of the previous sequence of outcomes. With current interview protocol, we only have limited data on student understanding on this issue. As a follow up study, we give a pretest to students in an advanced class (Phys 420) in fall 97. One of the questions is on probability with a coinflipping experiment (see Appendix D for the question). The data analysis shows that about 60% (13/22) of the students think that the result of a single coin-flipping event is dependent on the results in previous coin-flipping results. In addition, about 30% of the students think that if the coin is flipped for 100 times, there will be an exact 50/50 distribution for heads and bottoms. These results indicate that difficulties on the indeterministic nature of probability are quite common to students even for advanced level undergraduates.

Difficulties on Using Appropriate Concepts

In the interviews, we included a question similar to the final exam question shown in figure 7-2. Observations in the interviews showed that some students were not able to differentiate the difference between quantum and classical situations and were not sure when it is appropriate to apply the new quantum concepts. As a result, there was a tendency for these students to apply the quantum concepts to classical situations and create

additional difficulties. For example, when considering the classical system, some students tried use the wavefunction to describe a classical particle. Since the wavefunction, given in both the exam and interview protocol, stretched far away into \pm x direction, as a result, many students came up with the idea that the particle can exist outside the well in a classical case.

Summary of Student Difficulties on Important Classical Concepts

We have found that many students fail to correctly interpret the potential energy diagram. They often interpret the potential well that represents a 1-D physical system as a 2-D gravitational-like well. When presented with a square well potential diagram, students were unable to associate an appropriate physical system with it. Many used their simple intuition of a gravitational type of well and treated the graph as the 2-D sketch of the real system.

Students also have difficulties with total energy. Almost all the students interviewed had considerable difficulties in understanding what values the total energy can have in the classical case. Evidence from our interviews indicate that a possible reason could be that the students see energy as a directly measurable physical quantity and expect these to have an absolute and positive value. They do not see energy as a derived quantity representing an abstract attribute that depends upon how we define the zero of potential energy. It is also likely that many of them confused the total energy with the kinetic energy. When the students talk about energy or total energy, it seems what they really mean is the kinetic energy.

For the probability issues, most students had very little previous exposure to it, and almost none of them had ever used it to describe a physical system. The students' background on probability was very weak.

We also developed tutorials to help the students with their difficulties and improvement has been found in the student understanding of these issues after the modified instruction. However, even with modified instruction these two topics remain to be difficult for many students.

Reference and Endnotes:

¹ This tutorial (the original version) is a developed by P. Jolly and L. Bao. Further revisions were made by L. Bao and the PERG at UMd.

² P. Jolly et al., "Visualizing motion in potential wells," Am. J. Phys. **66** (1), 57-63 (1998).

³ The data acquisition is using the ULI interface and program MOTION available from Vernier. The transducers are the Ultrsonic motion detector (Vernier part number) and the Students Force Probe (Vernier, part number).

⁴ P. A. Tipler, *Physics for Scientists and Engineers, Volume 2*, Worth Publishers, 1991, p1195.