HW#6 —Phys374—Spring 2008 Prof. Ted Jacobson
Due before class, Friday, March 28, 2008 Room 4115, (301)405-6020
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1. If a function g satisfies Laplace's equation $\nabla^2 g = 0$ in a compact volume $\mathcal{V}$, and if $g = 0$ on the closed boundary surface $\partial\mathcal{V}$, then $g = 0$ everywhere in $\mathcal{V}$. Prove this two ways: (i) Use what you know about maxima and minima of harmonic functions. (ii) Show using integration by parts that the integral $\int_{\mathcal{V}} \nabla g \cdot \nabla g \, dV$ vanishes. Since $\nabla g \cdot \nabla g \geq 0$, this implies $\nabla g \cdot \nabla g = 0$, which implies $\nabla g = 0$, which implies g is constant, which implies $g = 0$ everywhere in $\mathcal{V}$, since $g = 0$ on the boundary $\partial\mathcal{V}$.
[5+5=10 pts.]

   Uniqueness of solutions to Laplace's equation: The previous result implies that solutions to Laplace's equation in $\mathcal{V}$ are uniquely determined their values on $\partial\mathcal{V}$. For suppose f_1 and f_2 are solutions that agree on $\partial\mathcal{V}$. Then their difference $g = f_2 - f_1$ is a solution that vanishes on $\partial\mathcal{V}$, so the above result implies that $g = 0$ everywhere in $\mathcal{V}$, i.e. $f_2 = f_1$.

2. The Laplacian in spherical coordinates takes the form

$$\nabla^2 F = r^{-2}\partial_r(r^2\partial_r F) + \text{angular part} \tag{1}$$

   where F is a scalar field. A useful "trick" is to write F in the form f/r where $f = rF$. Show that the action of the Laplacian then takes the simpler form [5 pts.]

$$\nabla^2(f/r) = r^{-1}\partial_r^2 f + \text{angular part} \tag{2}$$

3. Problems 11.4(a-k) (*Heat equation*) [1+1+2+2+3+3+2+2+2+1+1=20 pts.]
Note: There is no *a priori* guarantee that a function of the form (11.34) can be a solution, but as you show in the problem, it can.

4. Problems 11.5(a-i) (*Explosion of a nuclear bomb*) Add parts (a′) Find the dimensions of the diffusion constant κ and the neutron production rate constant λ. (a″) If the radius of the sphere is greater than some critical radius R_c, there is a runaway chain reaction (explosion) in which the neutron density grows exponentially with time. Use dimensional analysis to find how R_c depends on κ and λ.

   Note: When Snieder says at the beginning of this section that the neutron current is given by (11.29), he means that it is given *by analogy with* this equation. *Hints*: (b) If a function of only t is equal to a function of only r, then the two functions must both be constant. The value of this common constant is what is called in the book the "separation constant". (e) The neutron density N is finite at $r = 0$, and it goes to zero at $r = R$, since once the neutrons reach the surface they quickly fly off. (i) Justify your answer. [1+3+2+3+1+2+2+2+1+2+1=20 pts.]