Take-home Exam for Physics 623, Spring 2010, O.W. Greenberg

Please write each problem on a separate set of pages, stapled separately, so that the problems can be graded separately. Of course your name must be written in large legible letters on each problem. You must sign that you have done this work yourself, without talking with other people. You can use books, your own notes, etc.

1. (60 points) A system has two degenerate states $|l_1\rangle, |l_2\rangle$ of energy 0 and a state $|l_3\rangle$ of high energy M. The degenerate states are coupled to each other with coupling a and to the high energy state with couplings b_1 and b_2, labeled by the corresponding zero energy state.
(a) Write the Hamiltonian for this system.
(b) Use degenerate perturbation theory to find the first-order eigenvalues for this system.
(c) Use degenerate perturbation theory to find the first-order eigenstates for this system. Hint: the zero energy eigenstates get corrections both in the high-energy sector and in the degenerate sector.
(d) Use degenerate perturbation theory to find the second-order eigenvalues for this system.
(e) For the special choice $b_1 = b_2 = b$ find a relation between a and b that makes the eigenvalue equation quadratic (and thus easily solvable).
(f) Solve the case of (e) exactly and compare with your results in (b) and (c).
2. (20 points) A spin 1/2 particle with magnetic moment μ is in a constant magnetic field $\mathbf{B_0}$. A time-dependent magnetic field $\mathbf{B_1} exp(-i\omega t)$ is turned on for a time T. The field $\mathbf{B_1}$ is in the plane orthogonal to $\mathbf{B_0}$. If the magnetic moment is in its ground state initially, estimate the spin-flip probability at the end of time T.
3. (20 points) Use first-order time-dependent perturbation theory to calculate the probability that an harmonic oscillator in its ground state in the infinite past ends
(a) in its first excited state in the infinite future if it is excited by a time-dependent *force* (not potential)

$$F(t) = \frac{F_0 \tau/\omega}{\tau^2 + t^2}. \tag{1}$$

(b) in its second excited state over the same time interval.

(c) What is the lowest order of time-dependent perturbation theory for which the oscillator will end in its nth excited state?