ELECTRODYNAMICS
PROBLEM SET 3
due February 16^{th}, before class

Problem 1.: Angular momentum

One point that was not emphasized enough in class is that conservation laws are a consequence of symmetries of Nature. For instance, translation symmetry leads to the conservation of momentum and energy, gauge invariance leads to conservation of charge, ... Rotation invariance, being a symmetry of the electromagnetic action, should lead to the existence of a conserved quantity: angular momentum.

a) Show that the angular momentum tensor $M^{\mu\nu\lambda} = x^\mu T^{\nu\lambda} - x^\nu T^{\mu\lambda}$ satisfy $\partial_\lambda M^{\mu\nu\lambda} = 1/c(x^\mu J_\alpha F^{\alpha\nu} - x^\nu J_\alpha F^{\alpha\mu})$.

b) Write down the conserved *charge* in terms of the Poynting vector.

Problem 2.: Stress-tensor in action

Consider two parallel, infinite charged planes with (surface) charge density equal to σ and $-\sigma$.

a) Calculate the electric field generated by them.

b) Calculate the potential generated by them.

c) Using the result in b), calculate the force (per unit area) between the planes.

d) Calculate the stress tensor.

e) Using the conservation of momentum law, calculate the force between the planes by integrating the stress tensor over a surface separating the two planes.

Problem 3: Stress-tensor in action again

Consider two equal point charges q separated by a distance L. Direct application of the Coulomb law tells us what the force between them is. Here we will calculate the same thing in a convoluted way in order to build familiarity with the formalism.

a) Compute the electric field generated by the charges.

b) Compute the stress tensor on the plane equidistant to the two charges.

c) Compute the force acting on the charges using the result in b).

Problem 5: An action for the Schroedinger equation

Derive the equations of motion following from the action

$$S = \int dt d^3r \; \psi^* \left(i\frac{\partial}{\partial t} + \frac{\hbar^2 \nabla^2}{2M} - V(t, \vec{r}) \right) \psi. \qquad (1)$$

Treat ψ and ψ^* as independent quantities.

Energy density of a straight wire

Consider an infinite straight wire carrying a current I.

a) Compute the magnetic field generated by the wire.

b) Compute the energy density around the wire.

c) The total energy *per unit length* is divergent, both at short and long distances. Estimate what the energy per unit length is assuming the wire has a thickness $\sim \delta$ and that the field vanish at a distance $\sim R$ (maybe because the circuit is closed by a returning current at a distance $\sim R$ away from the first wire).

d) What is the value of the energy per unit length assuming a typical residential current (my house can take up to $20A$ of current before burning). Take R to be the size of a typical room and *delta* the thickness of a typical wire. *Note: I state the problem in MKS units because not many people know the value of typical currents, etc., in the gaussian system. The conversion to gaussian units is part of the problem. You can also simply look up the formula for the energy in MKS units somewhere and just use the result. I want the answer in J/m^3).*