Physics 601 Homework 6---Due Friday October 16

1. A spaceship fires its rocket in a fixed direction (for concreteness call it -$\hat{x}$) causing the spaceship to accelerate in the +$\hat{x}$ direction such that an observer on the rocket feels a constant artificial force gravity pulling her to the backof the spaceship with a force of mg.
   a. Show that the equation of motion for the space ship is given by $\frac{du^\mu}{d\tau} = G^{\mu\nu} u_\nu$ where $G^{\mu\nu}$ is a Lorentz tensor given by $G^{\mu\nu} = \begin{pmatrix} 0 & -g & 0 & 0 \\ g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. To do so you must show that the acceleration in the commoving frame is correct and that $\frac{du^\mu u_\mu}{d\tau} = 0$.
   b. Work in a frame where at t=0 the spaceship starts from rest at the origin: $\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} u_t \\ u_x \\ u_y \\ u_z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$. Solve the equation of motion for $u^\mu(\tau)$.
   c. From the solution of b. solve for $\vec{x}(t)$.

2. Starting with $S = \int d\tau \left(-m + A^\mu u_\mu\right)$ derive the equation of motion $\frac{d}{d\tau}\left(mu^\nu\right) = (\partial^\nu A^\mu - \partial^\mu A^\nu) u_\mu$ where $\partial^\nu \equiv \frac{\partial}{\partial x_\nu} = g^{\nu\mu} \frac{\partial}{\partial x^\mu}$.

3. In electro-magnetism, one can write the scalar and vector potentials in a form that looks like a 4-vector: $A^\mu = \begin{pmatrix} \Phi \\ A_x \\ A_y \\ A_z \end{pmatrix}$. Because one can make arbitrary gauge transformations this need not transform like a 4-vector.
   a. Show that if one makes a transformation to a so-called Lorentz gauge with $\partial_\mu A^\mu = 0$ then A^μ is a 4-vector.
   b. Suppose the scalar and vector potentials corresponding to a constant electric field are given by $\Phi = 0, \quad A_x = -E_0 t, \quad A_y = 0, \quad A_z = 0$ (in some frame). Show that A^μ is a 4-vector in this gauge.

4. The relativistic equation of motion for a particle couple to a general four vector potential has the form $\frac{du^\mu}{d\tau} = qF^{\mu\nu}u_\nu$ where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, where q is a constant of proportionality---in this case identified as the electric charge.
   a. Find the equation of motion for the potential in problem 3.
   b. Suppose a particle of mass *m* and charge *q* starts form rest at the origin and experiences the electromagnetic forces associated with the potential in *b*. Find $\vec{x}(t)$. Hint: How is does this equation of motion compare to the one in problem 1.

5. In the non-relativistic regime (v<<c) the equation of motion in problem 2 reduced to the one standard one for a non-relativistic particle in an electric field. There is another relativistic equation of motion which reduces to the same non-relativistic equation of motion: $\frac{d(m+s)u^\mu}{d\tau} = \partial^\mu s$ where s is a Lorentz scalar given by $s = -qE_0 x$.
   a. Show that in the non-relativistic regime of (*v<<1,s<<m*) this does reduce to the correct equation of motion.
   b. Suppose a particle of mass, *m,* particle sarts from rest at the origin. Find $\vec{x}(t)$; you may do this as a Taylor series and work to order t^4.
   c. Does this agree with your result in 4. If so explain why. If not, which is the correct one for an electric force?

6.

Relativistic kinematics. Consider the elastic scattering of 2 particles, one of mass m_1 and the other of mass m_2. Suppose initially particle 1 is at rest and particle 2 has a velocity *v* along the z direction. After the scattering particle 1 goes off making an angle θ_1 relative to the z axis. The purpose of this problem is to find expressions forthe final velocity of particles 1 and 2 and the angle made by particle 2 in terms of m_1, m_1, *v* and θ_1. Do this by first going to the center of mass frame, assuming back-to-back scattering in this frame and then boosting to the final frame.