Solution of Homework #1

1.4. $\vec{b}\cdot\vec{c} = |\vec{b}||\vec{c}|\cos\theta$

$\Rightarrow \cos\theta = \dfrac{\vec{b}\cdot\vec{c}}{|\vec{b}|\cdot|\vec{c}|} = \dfrac{12}{21} \quad \Rightarrow \theta = \arccos\frac{12}{21} = \arccos\frac{4}{7}$

1.7. ① If $\vec{r}\cdot\vec{s} = rs\cos\theta$ is correct,

We have $\vec{r}\cdot\vec{s} = (r_x\vec{e}_x + r_y\vec{e}_y + r_z\vec{e}_z)\cdot(s_x\vec{e}_x + s_y\vec{e}_y + s_z\vec{e}_z)$

$= \sum_i r_i\vec{e}_i \cdot \sum_j s_j\vec{e}_j$

Because $\vec{e}_i\cdot\vec{e}_j = \cos\theta_{ij} = \delta_{ij}$ ($\delta_{ij} = 0$ when $i \neq j$, $\delta_{ij} = 1$ when $i = j$)

So, $\vec{r}\cdot\vec{s} = \sum_{ij} r_i s_j \delta_{ij} = \sum_i r_i s_i$

② If $\vec{r}\cdot\vec{s} = \sum_i r_i s_i$ is correct,

We choose the axis so that $\hat{e}_x = \hat{e}_r$. So $\vec{r} = r\hat{e}_x$, $\vec{s} = \sum_i s_i\vec{e}_i$

$\vec{r}\cdot\vec{s} = \sum_i r_i s_i = r_x s_x = rs\cos\theta$, where θ is the angle between $\vec{r}$ and $\vec{s}$.

Because θ, r, s don't change when we change the choice of axis,

and $\vec{r}\cdot\vec{s}$, which is equal to $(|\vec{r}|^2 + |\vec{s}|^2 - |\vec{r}-\vec{s}|^2)/2$, also maintains its value.

So, we will have $\vec{r}\cdot\vec{s} = rs\cos\theta$ for any choice of axis.

According to ① and ②, we could conclude that $\vec{r}\cdot\vec{s} = rs\cos\theta$ and $\vec{r}\cdot\vec{s} = \sum_i r_i s_i$ are equa

1.19. $\frac{d}{dt}[\vec{a}\cdot(\vec{v}\times\vec{r})] = \dot{\vec{a}}\cdot(\vec{v}\times\vec{r}) + \vec{a}\cdot(\dot{\vec{v}}\times\vec{r}) + \vec{a}\cdot(\vec{v}\times\dot{\vec{r}})$

$= \dot{\vec{a}}\cdot(\vec{v}\times\vec{r}) + \vec{r}\cdot(\vec{a}\times\vec{a}) + \vec{a}\cdot(\vec{v}\times\vec{v})$

$= \dot{\vec{a}}\cdot(\vec{v}\times\vec{r})$

1.25. $df/dt = -3f \Rightarrow \dfrac{df}{f} = -3dt \Rightarrow \ln f = -3t + C_1 \Rightarrow f = C_2\cdot e^{-3t}$

C_1, C_2 are both constants.

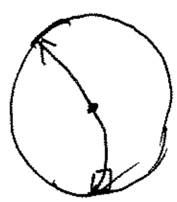
1.27. Suppose the turntable is rotating counterclockwise. If we take the turntable at rest, in other words, take turntable's frame, the velocity vector of puck would be the sum of $\vec{v}_0$ and minus velocity vector of the point on turntable contacting with the puck, which is seen in the old frame:

You would ask who supply the force to support a motion containing something like rotation. It's the transform from inertial frame to noninertial frame, for the new frame is noninertial. So, if you want to sketch the quality pic of the motion of puck in the new frame you just combine $\vec{v}_0$ and $\vec{v}'$ to get $\vec{v}$ and then you will find the path. I have to emphasize two points;

① the path must pass the origin once and only once

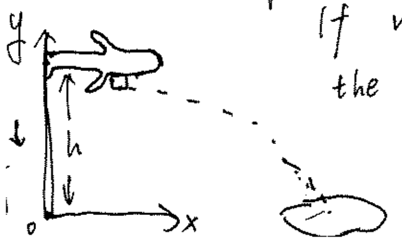
② Any other points on the turntable could be passed once or twice, depending on the relation between v_0 and w; and it also could be left unpassed. But the initial and end point on the board should be on the boarder.

The most easy case is, when $v_0 \gg w$, we have:



Other pics are also valid as long as drawn by the law of sum of velocity vector and agree with the two conditions.

1.36. (a) The position of the bundle depends on the coordinates you choose.



If we choose the axes like the pic on the left, and origin to be the position of the plane to drop the bundle, projected on the ground.

Then for $0 < t < \sqrt{\frac{2h}{g}}$, $x(t) = v_0 t$, $y(t) = h - \frac{1}{2} g t^2$

$t \geq \sqrt{\frac{2h}{g}}$, $x(t) = v_0 \sqrt{\frac{2h}{g}}$, $y(t) = 0$.

(b) The time for the bundle to drop to the ground is:

$$\cancel{t} = \sqrt{\frac{2h}{g}}$$

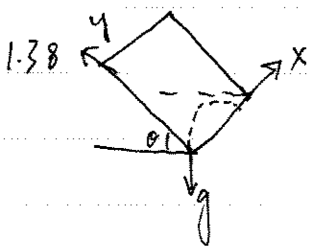
So the distance it travels in the x axis is:

$$x = v_0 t = v_0\sqrt{\frac{2h}{g}} \doteq 22\cancel{4}\ m$$

Which is the quantity requested.

(c) $\Delta t = \frac{\Delta x}{v_0} = \frac{10m}{50m/s} = 0.2s$

1.38



$$\vec{F} = m\vec{a} = -mg\sin\theta\,\vec{e}_y$$

$$\Rightarrow \begin{cases} x_{(t)} = v_{0x}\cdot t & -① \\ y_{(t)} = v_{0y}t - \frac{1}{2}g\sin\theta\, t^2 & -② \end{cases}$$

with the initial time $t=0$, and the expression is valid when the puck is on the board.

According to ②, the time cost by the puck to return the floor level is:

$$t = \frac{2v_{0y}}{g\cdot\sin\theta}$$

And, by using ①, we have $x_{(t)} = \frac{2v_{0x}v_{0y}}{g\cdot\sin\theta}$.

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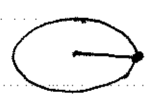
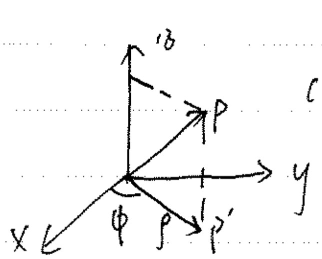


$$\vec{a} = (\ddot{r} - r\dot{\phi}^2)\hat{r} + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\hat{\phi} = -r\dot{\phi}^2\hat{r}$$

$$\vec{F} = m\vec{a} = -m\omega^2 r\hat{r}$$

$$F = |\vec{F}| = m\omega^2 R$$

.47



(a) $\rho = \sqrt{x^2+y^2}$, the perpendicular distance from p to z axis.

$\phi = \arctan(\frac{y}{x})$

$z = z$

If we choose r to express ρ, it is easy to confuse us that what does r mean: $r = \rho = \sqrt{x^2+y^2}$ or $r = |\hat{r}| = \sqrt{x^2+y^2+z^2}$.

(b) The unit $\hat{\rho}$ points directly away from z axis; $\hat{\phi}$ is tangent to a horizontal cirle through the center at z axis (positive direction is counterclockwise); $\hat{z}$ is parallel to z axis.

$$\hat{r} = \rho\hat{\rho} + z\hat{z}$$

(You could dedue it by drawing pic, or make some transformation:

$$\hat{r} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z;$$

$$x = \rho\cos\phi, \quad y = \rho\sin\phi;$$

$$\hat{e}_x = \hat{\rho}\cdot\cos\phi - \hat{\phi}\cdot\sin\phi, \quad \hat{e}_y = \hat{\rho}\cdot\sin\phi + \hat{\phi}\cdot\cos\phi \;)$$

c)

$$\left.\begin{aligned} \dot{\hat{r}} &= \rho\dot{\hat{\rho}} + \dot{\rho}\hat{\rho} + \dot{z}\hat{z} \\ \dot{\hat{\rho}} &= \dot{\phi}\hat{\phi}, \quad \dot{\hat{\phi}} = -\dot{\phi}\hat{\rho} \end{aligned}\right\}$$

$$\Rightarrow \ddot{\hat{r}} = (\ddot{\rho} - \rho\dot{\phi}^2)\hat{r} + (\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi})\hat{\phi} + \ddot{z}\hat{z}$$

1.48.

$$\left\{\begin{aligned} \hat{\rho} &= \hat{e}_x\cdot\cos\phi + \hat{e}_y\sin\phi \\ \hat{\phi} &= -\hat{e}_x\cdot\sin\phi + \hat{e}_y\cdot\cos\phi \\ \hat{z} &= \hat{z} \end{aligned}\right.$$

$$\Rightarrow \left\{\begin{aligned} \dot{\hat{\rho}} &= \dot{\phi}\,(-\hat{e}_x\cdot\sin\phi + \hat{e}_y\cos\phi) = \hat{\phi}\dot{\phi} \\ \dot{\hat{\phi}} &= \dot{\phi}\,(-\hat{e}_x\cdot\cos\phi - \hat{e}_y\sin\phi) = -\dot{\phi}\hat{\rho} \\ \dot{\hat{z}} &= 0 \end{aligned}\right.$$