Due May 2

1. In class we developed the averaging method for describing nearly harmonic motion for systems satisfying equations of the form $\ddot{x} = -\omega_0^2 x + f(x, \dot{x})$ where f is assumed to be "small". The approximate solutions were shown to be of the form $x(t) = A(t)\cos(\omega_0 t + \phi(t))$ with $\dot{A} = -\frac{\langle f \sin(\omega_0 t + \phi(t)) \rangle}{\omega_0}$ and $\dot{\phi} = -\frac{\langle f \cos(\omega_0 t + \phi(t)) \rangle}{A\omega_0}$ where $\langle \;\; \rangle$ indicates averaging over one period. In this problem I want you to use this method to get an approximate solution for a lightly damped oscillator of the form $\ddot{x} = -\omega_0^2 x - 2\beta \dot{x}$.

2. Of course we can exactly solve the damped oscillator in 1. Show that the exact solution agrees with the approximate one in the sense the frequency and damping factor agree up to first order in β.

Problems 4.3, 4.4, 4.8