Dynamics of a skydiver's epic free fall

José M. Colino, Antonio J. Barbero, and Francisco J. Tapiador

In October 2012 Felix Baumgartner fell farther and faster than anyone before him. The forces he experienced during his flight can be readily analyzed, thanks to the GPS data collected during his jump.

José M. Colino is a faculty member in the department of applied physics at the University of Castilla-La Mancha in Toledo, Spain; **Antonio J. Barbero** is a faculty member in the department of applied physics at UCLM in Albacete; and **Francisco J. Tapiador** is a faculty member in the department of atmospheric sciences at UCLM in Toledo.

Half a century ago, US Air Force colonel Joseph Kittinger plunged toward Earth from literally stratospheric heights. The three dives he made in 1960 were part of a research effort into survival techniques and equipment appropriate for high-altitude bailout, and they set several records, including a highest jump altitude of 31.33 km. In his final dive, taken on 16 August, he attained a maximum speed of 988 km/h, below but not too far from the speed of sound. As we shall soon discuss, flight at such high speeds is unstable; Kittinger actually deployed a small stabilization device shortly after he jumped to avoid later entering into a deadly spin.

In 1962 Yevgeny Andreyev, formerly a major in the Soviet Air Force, jumped from a capsule at 25.46 km and was in free fall for 24.50 km. Andreyev was skilled enough to control his body position by skydiving techniques alone and thus set the record for the longest-duration genuine free fall—4 minutes and 30 seconds. By the mid 1960s, high-altitude jumping had moved from the realm of research into the realm of adventure.

Some 50 years after his legendary jumps, Kittinger helped mentor Austrian skydiver Felix Baumgartner for the Red Bull Stratos mission. On 14 October 2012, Baumgartner ascended to an altitude of about 39 km in a helium-filled balloon. Panel a of the figure shows the skydiver in free fall, headed toward a landing site near Roswell, New Mexico. Thanks to a GPS apparatus fastened to Baumgartner's chest pack, the position of a high-altitude stratospheric jumper could be precisely tracked.

In February 2013 the organizers of the Stratos mission released a final data set. The numbers, confirmed by the International Air Sports Federation, established that Baumgartner had set three records. First, he achieved a maximum vertical speed of 1357.6 km/h, greater than the speed of sound by 25% and the highest vertical speed ever achieved without the aid of a stabilization device. Second, he jumped from an altitude of 38.9694 km, the highest ever. Third, Baumgartner was in free fall for a record vertical distance of 36.4026 km. The data also reveal another feature of the jump that may not have grabbed headlines but that is of great interest to those who study motion through fluids: The effect of the atmosphere on Baumgartner's motion changed dramatically as he approached the speed of sound.

It's a drag

Objects falling downward toward Earth don't just feel the force of gravity. They also experience an upward-directed drag force generated by the atmosphere. For a massive object like a skydiver in free fall, the drag force is proportional to the square of the speed v and is given by $F_D = -\frac{1}{2} C_D A \rho v^2$. Here A is the cross-sectional area of the object perpendicular to the direction of motion, which we'll assume to be vertical when we turn to Baumgartner's jump; ρ is the air density; and C_D is the so-called drag coefficient.

The drag coefficient increases dramatically when an object's speed approaches about Mach 0.8—that is, about 80% of the speed of sound. At that speed, some of the airflow over the body is subsonic and some supersonic, and the body is said to have entered the transonic regime. By around Mach 1.2, maybe a bit higher, the vast majority of the airflow over the object is supersonic; it has now passed through the transonic regime to supersonic flight.

Associated with the change in drag coefficient are the severe instabilities that Kittinger knew well. They arise from the complex mix of subsonic and supersonic flows over the body. Shock waves form where subsonic and supersonic flows meet (see panel b of the figure), and the skydiver must contend with those. For transonic flight at greater than Mach 1, a large shock wave forms downstream of the skydiver; since the wave itself propagates at the speed of sound, the diver will eventually cross that disruptive shock.

A closer look at Baumgartner's jump

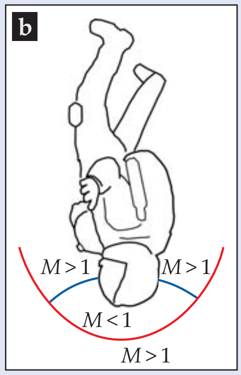
Armed with the GPS data released by the Stratos mission, we attempted to extract the dramatic change in C_D that theoretically occurred in the course of Baumgartner's jump. To begin, we applied Newton's second law to the skydiver and expressed the drag coefficient in the form $C_D A = 2m(g - a_z)/\rho v^2$. Here m is Baumgartner's 118-kg mass, a_z is the downward component of acceleration, and g is the altitude-dependent gravitational acceleration.

The mission data revealed that Baumgartner had no component of velocity parallel to the ground when he began his jump; they also included the vertical component of his velocity as a function of time and provided altitude values for selected times. By taking velocity differences and dividing by the time step, we obtained the acceleration a_z. Multiplying velocities by the time step gave altitude changes from which we could determine absolute altitudes; to check that numerical integration, we compared our calculated altitudes with the values released by the Stratos mission and found good agreement.

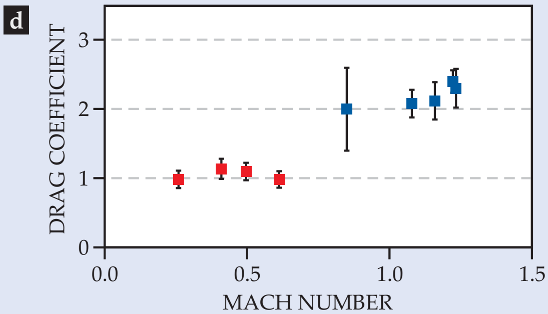
To determine the air density as accurately as possible, we

This article is copyrighted as indicated in the article. Reuse of AIP content is subject to the terms at: http://scitation.aip.org/termsconditions. Downloaded to IP: 129.2.40.52 On: Fri, 22 Aug 2014 15:20:52

A fall like no other. (a) As Felix Baumgartner commenced his record-breaking jump, he could see, nearly 40 km below, geographical features of the New Mexico landing area. (Photo by Jay Nemeth, courtesy of the Red Bull Stratos mission.)



(b) When Baumgartner's speed was near that of sound (Mach 1), some of the airflow over his body was subsonic ($M < 1$) and some was supersonic ($M > 1$). Shock waves form where the two regimes meet (blue curve). If Baumgartner's speed is a bit greater than Mach 1, as sketched here, an additional large shock wave forms downstream (red curve); the wave is detached from the skydiver, as is typical for a blunt body moving faster than sound. **(c)** Baumgartner used his diving skills to maintain a head-down, low-drag position during transonic flight. (Courtesy of the Red Bull Stratos mission.) **(d)** The drag coefficients C_D for subsonic (red data) flight and near-Mach-1 transonic (blue data) flight are dramatically different.



combined data obtained from balloon-borne radiosonde instruments with the US standard atmosphere model, which makes various assumptions about atmospheric composition and so forth. The sounding data, which extended to altitudes up to 32 km, were taken over Albuquerque, New Mexico, the day before Baumgartner's jump. With them, we calculated the air density, taking measured humidity into account, and confirmed that our values were in accord with the standard atmosphere. That agreement gave us the confidence to use the standard atmosphere to estimate air density at altitudes up to Baumgartner's jump-off point, 39 km above Earth.

In principle, the GPS data and density estimate are all we need to determine the evolution of the product $C_D A$ during the course of Baumgartner's jump, but we note that our results are not reliable for the jump's early stages, when $g - a_z$ and v are small. (Indeed, for the first 20 seconds of the jump, $g - a_z$ cannot be distinguished from zero.) In that regime, the small imprecision in our determinations of a_z and v lead to huge relative uncertainties in $C_D A$.

About 25 seconds into his jump, Baumgartner entered the transonic regime and achieved the head-low positioning, shown in panel c of the figure, that minimizes A and hence the drag force. For a significant time, his skydiving skills allowed him to maintain a constant-A, head-low stance without dramatic maneuvering, though he did rotate slightly about a vertical axis. For five short, but not too short, intervals of time, Baumgartner's motion was steady enough that we could determine A (0.25 m^2, chest pack and chute bag included) and extract C_D as a function of speed in the transonic regime. Panel d of the figure shows the results, which ranged from 2.0 to 2.4. The highest values of C_D correspond to speeds very close to Baumgartner's record maximum of Mach 1.25. It is possible that at higher transonic speeds, C_D would further increase. Indeed, many experiments on streamlined and blunt bodies have shown a maximum drag at speeds much greater than Mach 1.

When Baumgartner was falling at his fastest, he was 50 seconds into his jump. At that time, his body position still corresponded to a low drag, but he eventually lost control and turned belly-up while rotating in a flat spin. Then, about 80 seconds into his jump, he flipped over, regained control, and subsequently fell in a prone, belly-to-earth position for most of the time. We obtained four C_D data points for subsonic motion in that prone position, for which A = 0.80 m^2. Panel d shows the results, which range from 1.0 to 1.1, much lower than for transonic flight, but consistent with values determined for other subsonic dives in both head-low and prone orientations.

The simple analysis presented in this Quick Study ignores a number of effects, including the fluttering of Baumgartner's suit. Still, it is gratifying that the elementary treatment confirms the expected increase of C_D for transonic flight. A more sophisticated analysis, in particular one including horizontal velocity components, may help optimize the approach trajectories and chute opening times of atmospheric reentry vehicles or suggest cost-effective flight strategies for vehicles released from balloons conducting high-altitude research on the atmosphere.

Additional resources

▶ Engineering ToolBox, "Drag Coefficient," http://www.engineeringtoolbox.com/drag-coefficient-d_627.html.
▶ Red Bull Stratos mission, http://www.redbullstratos.com.
▶ R. A. Granger, *Fluid Mechanics*, Dover, New York (1995), chap. 15.
▶ L. N. Long, H. Weiss, "The velocity dependence of aerodynamic drag: A primer for mathematicians," *Am. Math. Mon.* **106**, 127 (1999).
▶ C. Ryan, *The Pre-Astronauts: Manned Ballooning on the Threshold of Space*, Naval Institute Press, Annapolis, MD (2003).
▶ J. M. Colino, A. J. Barbero, "Quantitative model of record stratospheric free-fall," *Eur. J. Phys.* **34**, 841 (2013).

This article is copyrighted as indicated in the article. Reuse of AIP content is subject to the terms at: http://scitation.aip.org/termsconditions. Downloaded to IP: 129.2.40.52 On: Fri, 22 Aug 2014 15:20:52