PHYS 402 Homework---Due Monday March 28

1. Consider two identical spin 0 particles of mass *m*. The particles do not interact with each other (in the sense that there is no interaction term in the Hamiltonian). The particles sit in a three dimensional harmonic oscillator potential: $\hat{H} = \frac{\hat{p}_{1x}^2 + \hat{p}_{1y}^2 + \hat{p}_{1z}^2}{2m} + \frac{\hat{p}_{2x}^2 + \hat{p}_{2y}^2 + \hat{p}_{2z}^2}{2m} + \frac{1}{2} m\omega^2 (x_1^2 + y_1^2 + z_1^2 + x_2^2 + y_2^2 + z_2^2)$. Enumerate explicitly all states with an energy of less than or equal to $5\hbar\omega$. Note you must take into account the symmetry properties under exchange of the particles. There should be a total of 16 such states: one with an energy of $3\hbar\omega$, three with an energy of $4\hbar\omega$ and 12 with an energy of $5\hbar\omega$.

2. Consider two identical spin ½ particles of mass *m*. The particles do not interact with each other (in the sense that there is no interaction term in the Hamiltonian). The particles sit in a three dimensional harmonic oscillator potential: $\hat{H} = \frac{\hat{p}_{1x}^2 + \hat{p}_{1y}^2 + \hat{p}_{1z}^2}{2m} + \frac{\hat{p}_{2x}^2 + \hat{p}_{2y}^2 + \hat{p}_{2z}^2}{2m} + \frac{1}{2} m\omega^2 (x_1^2 + y_1^2 + z_1^2 + x_2^2 + y_2^2 + z_2^2)$. The ground state of this system has an energy of $4\hbar\omega$. It turns out that the ground state has a nine-fold degeneracy. Enumerate explicitly these nine states. Note you must take into account the symmetry properties under the exchange of the two particles and must do this for the combined spin and spatial parts of the state.

3. In this problem start with a traditional one dimension harmonic oscillator $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 x^2$, and ask what happens to the energies of the ground state if we add to this a perturbation of the form $\hat{H}' = \alpha x^2$.
   a. Compute the shift of the ground state energy at first order in perturbation theory.
   b. Compute the shift of the ground state energy at second order in perturbation theory.
   c. Note that this problem is exactly solvable: it is simply a Harmonic oscillator with a modified frequency. Show that the exact energy of the ground state is $E_0 = \frac{1}{2}\hbar\sqrt{\omega^2 + \frac{2\alpha}{m}}$.
   d. Verify that a Taylor expansion of the exact energy in α coincides with the pertubative expressions. Explain why.

4. It turns out from quantum field theory that if the photon has a small mass instead of being massless, the coulomb force is $V(r) = \frac{q_1 q_2}{r} e^{-m_\gamma c r/\hbar}$ where m_γ is the mass of the photon. Let us ask the question of how would the ground state energy of the hydrogen atom change if the photon had a small mass. We will work to lowest nontrivial order in m_γ. Thus it is legitimate to expand the exponential in a Taylor series and truncate at first order $V(r) \approx \frac{q_1 q_2}{r}(1 - r\, m_\gamma c/\hbar)$. Treating the second term as a perturbation, calculate the first order shift in the ground state energy of the hydrogen atom.