QUANTUM PHYSICS II

PROBLEM SET 2

due September 22, before class

A. Spin rotated and measured

A spin-1/2 particle is initially in the state

$$\psi = \frac{\psi_+ + 2i\psi_-}{\sqrt{5}}, \tag{1}$$

where $\psi_\pm$ are the spin up and down states (along the z-axis). By means of magnetic fields the spin of the particle is rotated around the z-axis by an angle of π. At this point the x-component of the spin is measured. What are the possible outcomes and with which probabilities?

B. A spinor pointing towards an arbitrary direction

The operator corresponding to the spin projection along an arbitrary direction in space (parametrized by the unit vector $\mathbf{n}$) is given by $\hat{S}_\mathbf{n} = \hat{\mathbf{S}}.\mathbf{n} = \hat{S}_x n_x + \hat{S}_y n_y + \hat{S}_z n_z$. Let $\mathbf{n} = \sin\theta\cos\phi\ \mathbf{e}_x + \sin\theta\sin\phi\ \mathbf{e}_y + \cos\theta\ \mathbf{e}_z$. Find the eigenvectors of $\hat{S}_\mathbf{n}$ with eigenvalue $+\hbar/2$ as a function go the angles θ, ϕ. Hint: you may want to start with the known eigenvector of $\hat{S}_z$ and apply rotations to align it in the θ, ϕ direction. If you do it this way, make sure you verify at the end that the ket you obtained is indeed an eigenvector of $\hat{S}_\mathbf{n}$.

C. Spin acrobatics

An electron is at rest in an oscillating magnetic field

$$\mathbf{B} = B_0 \cos(\omega t)\mathbf{e}_z, \tag{2}$$

where B_0 and ω are constants.

(a) Construct the hamiltonian matrix (in the basis ψ_+, ψ_- we have been using) for this system.

(b) The electron starts out (at $t = 0$) in the spin-up state with respect to the x-axis. Determine the state of the electron spin at any subsequent time. *Beware* this is a time-dependent hamiltonian, so you cannot find $\psi(t)$ in the usual way from stationary states. Fortunately, in this case you can solve the time-dependent Schroedinger equation (and feel free to use computers).

(c) Find the probability of getting $-\hbar/2$ if you measure $\hat{S}_x$.

(d) What is the minimum field (B_0) required to force a complete flip in $\hat{S}_x$?