QUANTUM INTERFERENCE FLUCTUATIONS IN DISORDERED METALS

Phase coherence over thousands of lattice spacings in disordered metals can produce quantum interference effects in electrical resistance measured in very small devices.

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In statistical physics one is trained to think about the properties of large ensembles of particles, and to calculate bulk properties by averaging over many microscopic configurations. Although the quantum mechanical properties of the individual constituents of a macroscopic object are important over some length scale (typically a few lattice spacings), they are usually not correlated across the whole object. We are, however, becoming acquainted with more and more disordered systems for which this effective length scale, at low temperatures, can be 100–10 000 times the characteristic microscopic scale; the correlation can involve more than 10^{11} particles. Such phenomena occur in an intermediate "mesoscopic" regime that lies between the microscopic world of atomic and molecular orbitals and the thoroughly macroscopic world where averages tell all. The wealth of novel quantum coherence phenomena recently observed in this intermediate size regime is the subject of this article.

We are mainly interested here in disordered systems in which the motion of the electrons is diffusive. In practice this means that the mean free path l (typically about 100 Å) is much smaller than the sample length, or, more properly, than the separation L between the probes that measure electrical resistance. In the theory, and in many experiments, l is also smaller than the width or thickness of the wires that form the sample. The mean free path, in turn, is much larger than the Fermi wavelength λ_F (about 1 Å) of the electrons—the de Broglie wavelength corresponding to the energy of the Fermi surface. Therefore, treating the motion of the carriers semiclassically is a good approximation to the physics of the electrical resistance. The electrons move through the sample as nearly plane waves, colliding elastically with the impurities, which sit at random places in the lattice. The resulting motion is a random walk among the impurity sites. (See the article by Boris Al'tshuler and Patrick Lee on page 36.) Thermal fluctuations, phonon collisions and other such inelastic mechanisms alter the quantum state of an electron enough to rob it of its phase memory. As the temperature approaches zero, these processes are diminished; inelastic scattering becomes rare.

Our IBM colleague Rolf Landauer has emphasized that elastic collisions with the impurities do not destroy the phase information contained in the electron's wavefunction; they merely shift the phase by some fixed amount.[1] All electrons incident in the same state acquire the same phase shift, and a time-reversed collision would restore the original phase to the wavefunction. Inelastic collisions, on the other hand, destroy the phase memory of the electrons irrevocably.

Thus the average diffusion length between inelastic collisions is effectively the phase coherence length L_ϕ. In typical metals L_ϕ can be several microns at temperatures below 1 K. Therefore, with the aid of modern circuit lithography one can nowadays make resistance measurements at length scales where the electron retains its phase memory—where its wavefunction matters. With such measurements one sees direct evidence of the interference of the electron wavefunctions. Each electron can be described schematically by a wavefunction $Ae^{i\phi}$, where the plane-wave phase $\phi(t)$ is boosted or retarded deterministically by elastic collisions with impurities. The electron has a specific probability amplitude for every possible path as it diffuses through the sample.

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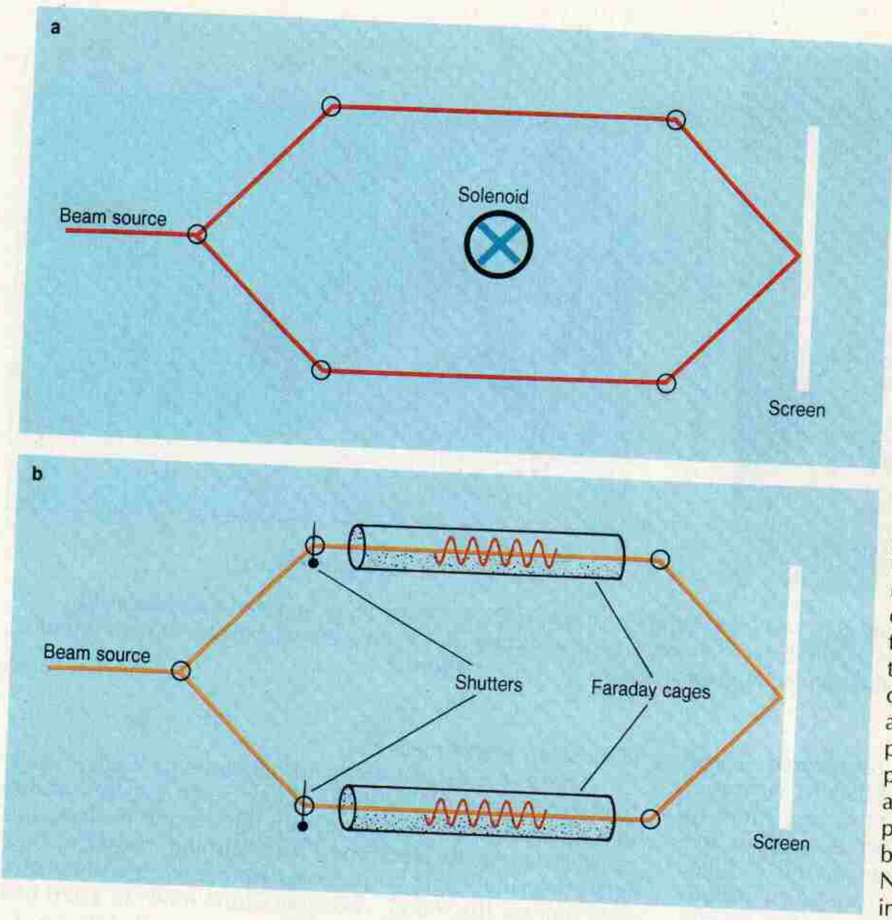


Aharonov–Bohm magnetic (**a**) and electrostatic (**b**) effects are counterintuitive consequences of quantum theory. An electron beam, split into two alternative paths, can exhibit interference effects when the beams recombine. Surprisingly, the relative phase between the two paths depends on potentials whose derivative force fields are completely shielded from the electrons. In **a** the phase difference is determined by the magnetic flux Φ through a long solenoid threaded between the paths. Changing Φ alters the interference pattern at the screen. In **b** the two paths are subjected to different electrostatic potentials, but only while the chopped beams are shielded within Faraday cages. Nonetheless, the potentials effect the interference. **Figure 1**

The phases accumulated along different paths are rarely the same; partial waves from the different paths interfere with randomly displaced phases. Assume for the moment that the electron has only two possible paths through the conductor. Then the superposition of the two wavefunctions yields the probability distribution

$$|A_1 + A_2|^2 = A_1{}^2 + A_2{}^2 + A_1 A_2 \cos(\phi_1 - \phi_2)$$

In samples where measurements are made over distances much larger than the phase-coherence length, the last term vanishes; its averaged value is zero. On the other hand, at low-temperature in small samples with dimensions on the order of the phase coherence length, the cross term has a specific, nonvanishing value that varies randomly throughout the sample. Along the two alternative paths of such a small, cold conductor, an electron would acquire different phases from the different impurities encountered on the way to a superposition measurement where the paths come together again. A pair of voltage probes placed at the two ends could, in principle, measure the resulting interference probabilities. Another sample of the same size and impurity concentration will have a resistance that differs from the first sample's by at most $\pm|A_1A_2|$. The averaged resistance from a large ensemble of such samples will of course yield the classical value, because the third term in the superposition will have averaged to zero.

The Aharonov–Bohm effect

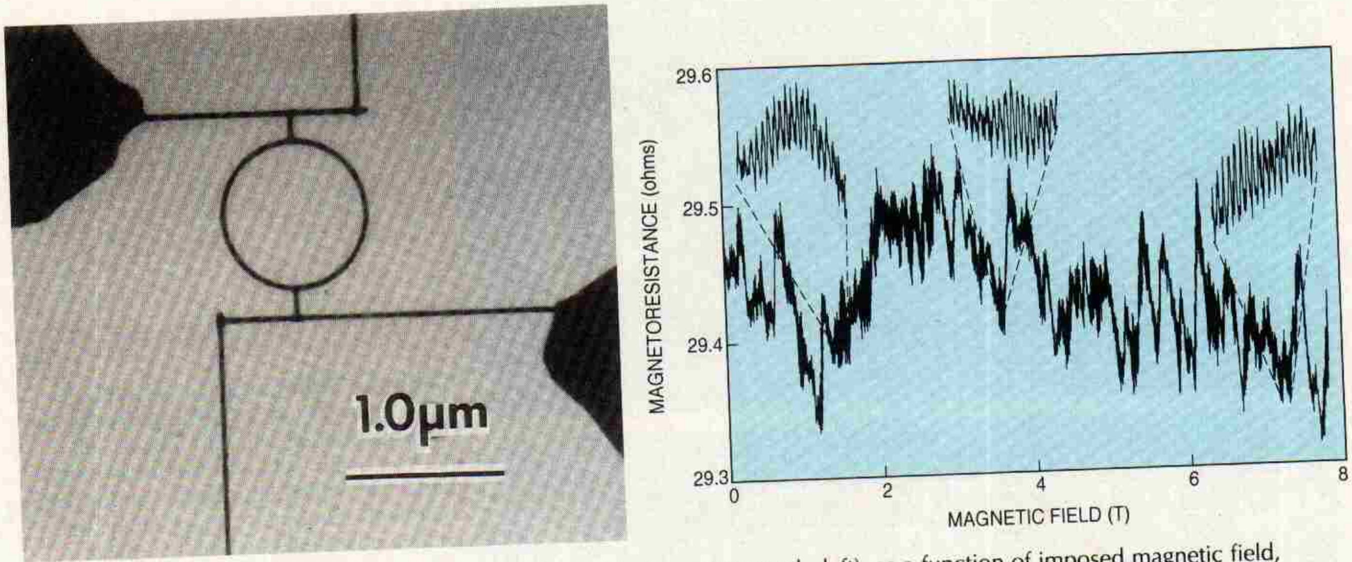
In 1959 Yakir Aharonov and David Bohm formulated a *gedanken* experiment that called attention to a surprising, counterintuitive consequence of quantum mechanics. They noted that the electromagnetic vector potential will affect the phase of an electron wavefunction, with observable consequences, even when the electron's path is restricted to regions where the electric and magnetic field intensities vanish. The Aharonov–Bohm effect has been experimentally verified by several groups with electron trajectories in vacuum.[2] Its observation with electrons traveling inside small metal devices of various geometries has provided striking manifestations of quantum coherence over mesoscopic distances in disordered solids. (See PHYSICS TODAY, January 1986, page 17.)

The Aharonov–Bohm mechanics provides a simple means of altering the phases of wavefunctions. These are depicted schematically in figure 1. In figure 1a the electrons are emitted from the source, and the beam is split so that it encircles a magnetic flux. (In the strictest sense, the magnetic flux Φ must be completely shielded from the electron paths,[2] but in the experiments described here, this nicety is never attempted.) The equivalent experiment for the electric potential is displayed in figure 1b, where a voltage is turned on while the electrons are inside field-free Faraday cages and the voltage is turned off again before they emerge from the cages. The prediction of quantum theory is that the phase ϕ of the wavefunction is pushed up or down by the electromagnetic potentials according to

$$\Delta\phi = (e/\hbar) \int (V\mathrm{d}t - \mathbf{A}\cdot\mathrm{d}s)$$

where ds and dt are the space and time elements of the electron's path. Simply ramping the potential causes the current impinging at some point on the screen to oscillate periodically as a function of the flux or time-integrated voltage; increasing Φ or $\int V\mathrm{d}t$ by h/e completes an Aharonov–Bohm cycle.

Classically, this interference effect cannot occur, because the electron never "sees" the fields **E** and **B**, and



Electrical resistance of a tiny gold ring (electron micrograph, left), as a function of imposed magnetic field, measured by the authors[6] at 0.06 kelvin. The fine Aharonov–Bohm oscillations (shown expanded) correspond to a flux periodicity of h/e through the area encircled by the ring. **Figure 2**

so it does not experience the classical Lorentz force. These Aharonov–Bohm effects have been studied in vacuum,[3] where the electrons suffer no collisions of any kind; in small metals whiskers, where transport is deep in the ballistic regime[4] (again, with no collisions); and in long, thin metal cylinders, where a magnetoresistance oscillation of twice the Aharonov–Bohm frequency ($\Delta\Phi = h/2e$) occurs near zero magnetic field because of the incipient localization of the electron wavefunction in the metal.[2] (See the article by Al'tshuler and Lee.) In the first two cases the absence of any scattering "guarantees" the occurrence of Aharonov–Bohm effects. The third example, first observed in 1981 by Yuri Sharvin and his son at the Institute for Physical Problems in Moscow, provided the original demonstration that these effects are not destroyed by elastic scattering. The observation of coherence effects from these relatively long cylinders is possible because the interference from time-reversed motion along paths around the cylinder does not average to zero as the cylinder length exceeds L_ϕ.

The simplest circuit analog of the Aharonov–Bohm geometry is a loop of thin wire fed by two leads. Five years ago theorists Markus Büttiker, Yoseph Imry and Landauer at IBM and Yuval Gefen and Mark Azbel at the University of Tel Aviv considered the possibility of observing oscillations of electrical resistance as a function of the flux threading a tiny ring of this geometry.[5] They concluded that in this case, as distinguished from a cylinder of macroscopic length, the "magnetoresistance" oscillations would exhibit a flux period of h/e—twice the period for cylinders.

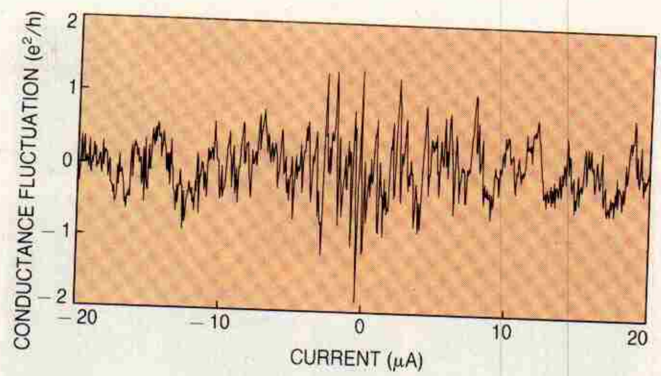
Such a loop of gold is shown in figure 2, together with the data obtained by measuring its magnetoresistance in our laboratory. After taking some care to filter noise away from the sample, we found that the resistance as a function of the magnetic flux through the ring exhibits oscillations that are periodic in the magnetic field. Our results and those of several other groups[6] have demonstrated that the period of oscillation is indeed $\Delta\Phi = h/e$, where Φ is the flux through the area enclosed by the loop—just the period Aharonov and Bohm predicted for their *gedanken* experiment *in vacuo*. We see then, in this resistance measurement on a disordered metal loop of 1-micron diameter, the direct signature of the interference of electron wavefunctions.

There is a second quantum mechanical contribution to the resistance of this loop. The periodic h/e oscillations are superimposed on a randomly fluctuating background resistance. As the electrons diffuse through the wires that form the ring, their trajectories randomly enclose a flux that pierces the metal. Douglas Stone (now at Yale) has shown that these random fluxes also contribute to Aharonov–Bohm interference.[7] This interference appears in figure 2 as random fluctuations in the resistance as a function of the imposed magnetic field. These fluctuations typically exhibit a much longer oscillation scale in magnetic-field increment than does the primary, periodic Aharonov–Bohm contribution. Naively speaking, the difference in field scale reflects the different areas enclosing flux: For the periodic Aharonov–Bohm oscillations, all of the area encircled by the loop contributes to the flux, while for the random component, only the area covered by metal contributes. Stone pointed out that the ratio of the field scales is approximately the ratio of these areas.

The total electrical resistance $R(H)$ of a small two-lead metal loop threaded by a flux Φ due to an imposed magnetic field H can be written[8]

$$R(H) = R_c + R_0 + R_1 \cos\left\{\frac{\Phi}{\hbar/e} + \alpha_1\right\} + R_2 \cos\left\{\frac{2\Phi}{\hbar/e} + \alpha_2\right\} + \cdots$$

R_c is the classical resistance, which includes a magnetoresistance term proportional to B^2. Empirically, one finds that R_n and α_n are random functions of H with oscillation scales proportional to the area of the metal, and, roughly speaking, inversely proportional to n. Al'tshuler, Lee and Stone and others[2,9] have proved that the amplitudes of the various terms are such that the conductance ($1/R$) exhibits fluctuations with rms amplitude on the order of e^2/h. This conductance fluctuation amplitude is "universal" in several loose senses of the word. For instance, the factor preceding e^2/h is weakly dependent on geometry, independent of length (so long as phase coherence is maintained) and independent of the average resistance. Any metal sample exhibits conductance fluctuations of about this



Failure of Ohm's law. For a sufficiently small, cold wire, the resistance depends on the voltage drop and is not symmetrical under reversal of the current direction. Conductance fluctuations, deviating from Ohm's law, were measured[17] as a function of current in a 0.6-μm length of Sb wire. Note the universal e^2/h conductance fluctuation and the asymmetry under current reversal. **Figure 3**

amplitude. The reasons for this universality are quite subtle and still under debate,[10,9] but it has been thoroughly verified in many experiments.[2,11] The article by Al'tshuler and Lee on page 36 discusses the theory of the universal conductance fluctuation in some detail.

In the equation on page 47 for the interference amplitude, the absolute phase change does not enter—only the relative phases of the two paths. This in turn implies that the interference is not necessarily destroyed as the magnetic field increases to rather strong values, where a significant flux pierces the actual metal. The increasing magnetic field intensity does not alter the character of the oscillations until the field reaches classically strong values where Landau cyclotron orbits form, destroying the diffusive motion of the electrons.[12] Figure 2 shows that the oscillations persist to rather large magnetic field without any attenuation whatsoever. They have, in fact, been observed at field intensities above 15 tesla without attenuation. Landau cyclotron orbits will not form in our gold rings until one gets to about 200 tesla.

The quantum domain

How does a quantum system evolve into the more familiar classical system? Experiments on these small phase-coherent systems have already provided a partial answer. Temperature, sample size, diffusion coefficient and details of how the measurement is made play a crucial role in determining whether or not quantum interference effects contribute significantly to the electrical resistance in a disordered system. Phase randomization due to elastic scattering and thermal effects have historically been thought to be the most devastating killers of coherence. But the gold-loop and cylinder experiments have demonstrated that elastic scattering is not destructive.

The question of thermal averaging arises because even a device as small as the gold ring in figure 2 contains 10^8 atoms; the average separation ΔE between its electron energy levels is about 10^{-8} eV, which corresponds to a temperature of 10^{-4} K. When the thermal energy exceeds some characteristic energy of the system, the delicate quantum effects begin to disappear. One might naively expect that the oscillations will decay like $\exp(-\Delta E/kT)$. The surprising discoveries from the resistance fluctuations are that the spacing between energy levels is *not* the characteristic energy scale, and that the decay is much slower. As Imry and others have pointed out, the energy scale that governs the physics is related to the Thouless energy,[2,7,8]

$$E_c = \frac{\hbar D}{L_\phi^2}$$

determined from the coherence length and D, the diffusion coefficient for elastic scattering. Once kT exceeds E_c, the magnitude of the quantum corrections to the resistance decays slowly,[3,7,8] like

$$\sqrt{\frac{E_c}{k_B T}}\,.$$

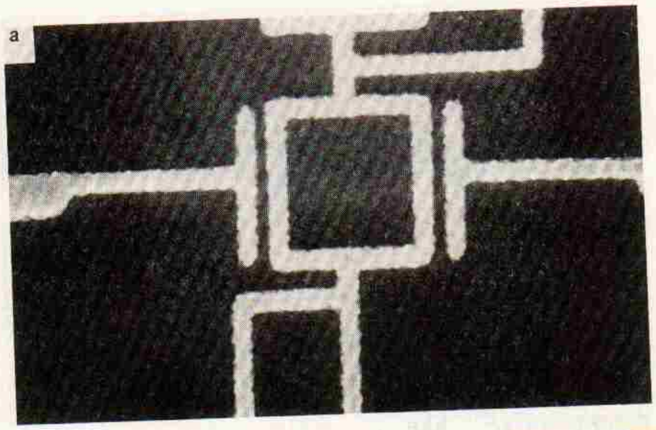
For our gold ring, E_c/k is only about 0.03 K, but because of the weak algebraic averaging, the quantum interference effects are readily observable above 1 K. In semiconductors, where E_c/k can be as high as 10 K, quantum interference effects have been reported above 100 K.

This weak algebraic averaging of the quantum interference also appears in the size dependence, whose characteristic length scale is L_ϕ. When the length of the sample is increased beyond L_ϕ, the resistance fluctuations grow like $(L/L_\phi)^{1/2}$, but the average resistance grows linearly with L. Thus the *relative* magnitude of the quantum interference contributions decay as the square-root of the number of uncorrelated length segments. For a single ring larger than L_ϕ, the loss of phase coherence reduces the oscillations severely; the oscillations decrease exponentially with increasing path length L. The reason is simply[14] that the probability for an electron to reach the terminus with its phase coherence intact decreases like $\exp(-L/L_\phi)$.

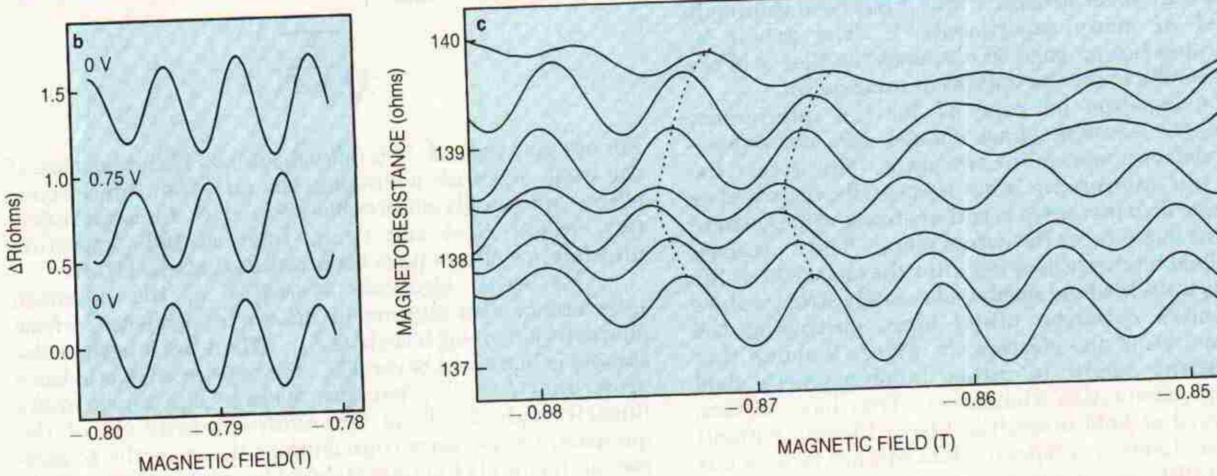
Because all of these averaging mechanisms depend upon the coherence length, the temperature dependence of L_ϕ determines the observability of quantum interference effects. As the temperature increases, the probability that an electron will absorb and re-emit a phonon increases. For narrow wires at low temperature, it turns out that L_ϕ has only a weak power-law (nonexponential) dependence on temperature.

There are other ways to destroy the phase coherence of electron wave functions; the dominant one at low temperatures is paramagnetic scattering.[15] Because the phase shift acquired by scattering from an impurity with a fluctuating spin depends on the instantaneous direction of that spin, the electron will in general acquire a time-dependent phase shift, and this will destroy the interference effects. In experiments where a low concentration of paramagnetic impurities is adsorbed onto the surface of a gold ring, the h/e oscillations disappear at low fields. But as the field is increased beyond some critical value, the full amplitude of the resistance fluctuations is recovered. The reason for this striking recovery is that when the magnetic field energy becomes larger than the thermal energy, the paramagnetic spins align with the field; classically they cease to fluctuate. Above this field intensity, every electron will acquire the same phase shift from the impurity, which now acts like a stationary elastic scatterer.

One obvious way to destroy quantum interference effects in a disordered conductor is to pass too much current through it. Joule heating will raise the tempera-



Electrostatic Aharonov–Bohm effect measured at IBM with an Sb loop 0.8 μm on a side, with capacitor gates along two arms. (See micrograph **a**.) Plot of resistance change vs magnetic field (**b**) shows that gate voltage (labeled) can shift oscillation phases reproducibly. Wider field sweeps (**c**) show gradual change of magnetoresistance pattern with increasing capacitor voltage. Each trace is measured at 0.2 volts higher than the one below. **Figure 4**



ture. Perhaps not so apparent is the observation that currents too small to cause significant Joule heating can nonetheless average away the interference terms. If the voltage drop across a phase-coherent segment exceeds E_c/e, there will be an averaging over uncorrelated energy regions, and the magnitude of the quantum interference effects will be reduced by $(E_c/eV)^{1/2}$.

The current–voltage curve

Ohm's law fails when transport is phase coherent. Transmission of current across a random potential depends erratically on the average (net) slope of the potential.[16] Ohm's law, which prescribes a linear current–voltage curve, describes only the average behavior of the wire. Suppose a small conductor with a random impurity potential is biased between two reservoirs of differing chemical potential—the terminals of a battery. This will simply tilt the random potential distribution. Not only is the effective resistance of such a biased impurity potential a random function of the bias voltage; one cannot even assume that $R(I)$, the resistance as a function of current, is equal to $R(-I)$. The resistance is not symmetric under exchange of the battery terminals because the potential has no mirror symmetry; a realistic three-dimensional wire is not symmetric under inversion.

This nonlinearity in the I–V curve has been measured.[17] A representative example of non-ohmic behavior found in an antimony wire at low temperatures, is shown in figure 3, where the deviation from Ohm's law is plotted. One sees that quantum interference causes the conductance to fluctuate randomly (and reproducibly) throughout the range of current studied. Near zero current, the fluctuations are sharp, with rms amplitude e^2/h. As the drive current increases, the fluctuation amplitude decreases slowly, and the current scale for a fluctuation grows. The dependence of the amplitude and current scale on the drive depends in a complicated way on certain averaging processes and heating of the electrons by the current, but the fluctuations are qualitatively in reasonable agreement with the theory.[16] The startling prediction that $R(I)$ does not equal $R(-I)$ is also borne out; there is no correlation between the fluctuations at positive and negative currents.

As the drive current tends to zero, ohmic behavior should be recovered asymptotically—but only for currents yielding voltage drops much smaller than kT/e. At a temperature of 0.01 K, one reaches the ohmic regime only when the voltage drop across the sample is much less than a microvolt.

Electrostatic Aharonov–Bohm effects

The electrical circuit analog of figure 1b has been studied to test the effects of a scalar potential on quantum interference in disordered systems. The experimental device, shown in figure 4, is a loop with capacitor probes along its arms. A static voltage applied to the capacitor probes will contribute an accumulated phase increment of $e/\hbar\int V dt$ to the electrons in the arms of the loop. Figure 4b shows that the phase of the h/e oscillations from this loop can be shifted by about π when the voltage applied to the capacitor plates is changed by 0.75 V; but the amplitude of the oscillations is unaffected. Thus, at any constant magnetic field, one can reproducibly change the output voltage from the ring by applying a potential to the capacitor probes. One can, in fact, use the voltage to shift the position of the oscillation smoothly along the magnetic field axis.[18] This is shown in figure 4c, which contains several magnetoresistance traces recorded with different voltages on the probes. Each trace is recorded at a voltage 0.2 V higher than the trace below it. As one begins to increase the probe voltage, the oscillations shift towards negative field; but then, after the fourth trace, they shift back toward positive field.

This is precisely what one expects if a global phase change is added to all of the electrons in an arm of the loop. We expect that the time integral accumulating phase from the electric potential is cut off by the mean time between inelastic collisions. Using this time scale, and accounting naively for the screening of the potential by the metal in the loop, we find that the expected voltage required to shift the oscillations by π is on the order of one or two volts, in reasonable agreement with the observation.

A voltage on the capacitor plates affects the random fluctuations in a similiar way. The net result of phase changes caused by the capacitor voltage is to make R_n and α_n in the general equation for $R(H)$ random functions of this voltage as well as of the magnetic field. In figure 4 we see that the background resistance changes shape slightly each time voltage changes. The characteristic voltage required to cause fluctuations is about twice that needed to shift the oscillations. This is reminiscent of the situation for the corresponding magnetic-field scales, which also differ by a factor of about 2.

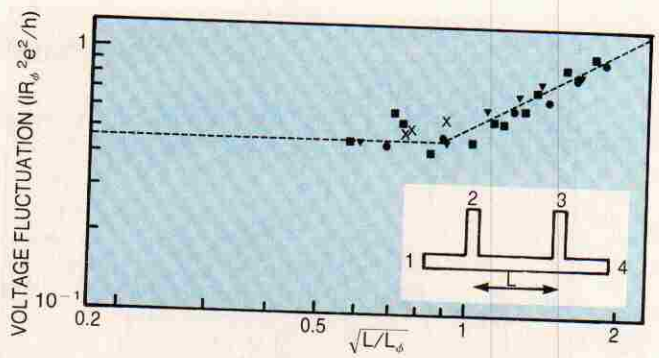
We are not studying strictly orthodox Aharonov–Bohm effects, because we make no effort to shield the current from the electric or magnetic field. This means that there are also classical Lorentz forces at work on the electrons in the loop. The electric field tends to push the electrons toward or away from the edge of the wire. (These same electrons screen the field.) Different paths among the impurities experience different potentials, causing the interference pattern to change. Just how gradually it changes with capacitor voltage remains an open question. Shifting so much as a single impurity atom[19] by more than a Fermi wavelength will substantially affect the conductance: it will cause a change of order e^2/h. In this spirit we believe that changing the electron paths within a screening length of the wire surface by as little as a Fermi wavelength might also substantially alter the resistance fluctuation pattern.

Nonlocal quantum interference

Long-range phase coherence in disordered systems has had several unexpected experimental consequences. One of the most surprising is that the rms amplitude of the voltage fluctuations measured as a function of magnetic field in a four-terminal sample becomes independent of length[20] when the separation L between the voltage probes is less than L_ϕ. The inset to figure 5 schematically displays a four-wire sample configuration. Current is injected at lead 1 and removed by lead 4, and the voltage difference is measured between leads 2 and 3. Classically, the average resistance V_{23}/I_{14} depends linearly on L. The average voltage difference between any two points along the sample is given by L times the electric field. The quantum-interference contributions to the voltage fluctuations with changing magnetic field, however, behave very differently. Figure 5 shows the results of a recent experiment on an antimony wire, which has eight probes with separations L ranging from 0.2 to 3.6 microns. The rms values of the voltage fluctuations are plotted as a function of L. The length scale has been normalized to the 1.1 μm correlation length, L_ϕ.

The striking feature of these data is that voltage fluctuations do not vanish as L goes to zero. The quantum interference contributions to the measured voltages remain constant for all probe separations up to the coherence length. For separations larger than a micron, the voltage fluctuations grow like $(L/L_\phi)^{1/2}$, as expected.

This length independence comes about because the voltage probes do not define the length scale for the measurement.[21] The effective length of the sample is really determined by the properties of electrons themselves. The electron is a wave whose phase coherence extends over a distance L_ϕ. Therefore the electron exists in classically forbidden regions. In particular, the waves can propagate coherently over a distance L_ϕ into the current and voltage probes. The application of a magnetic field alters the interference over the entire phase-coherent path.

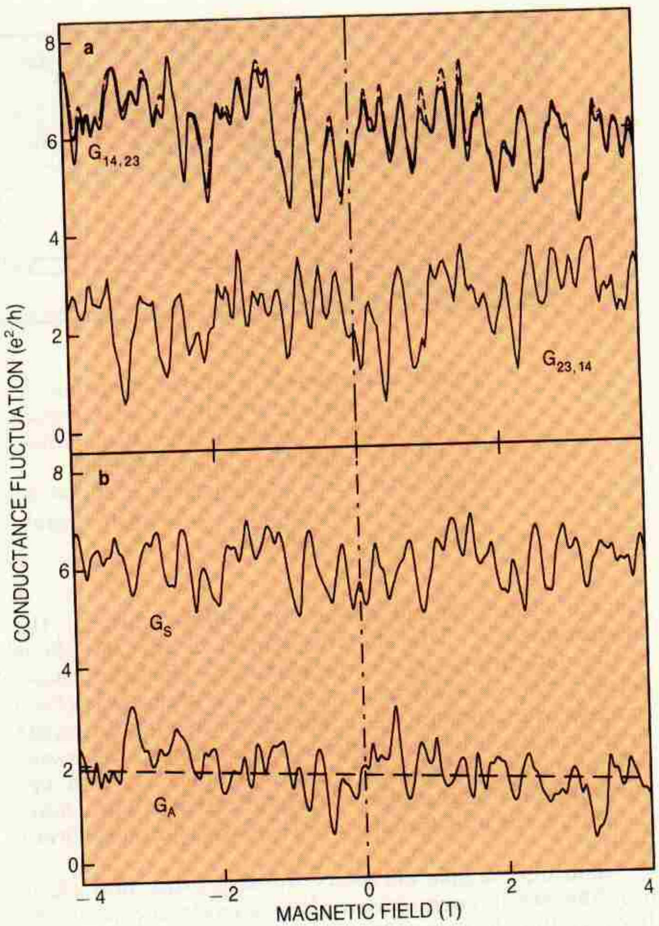


Length dependence of voltage fluctuation amplitude. Inset is a schematic of a four-lead sample. Length L between the voltage probes (2 and 3) is scaled to L_ϕ, the coherence length, and voltage is normalized to $IR_\phi 2e^2/h$, where R_ϕ is the resistance of a length L_ϕ and I is the current. **Figure 5**

The microscopic details of the scattering impurities are different in each section of the sample and its leads. Because the electrons propagate into the probes, a finite "nonlocal" voltage is measured between leads 3 and 4 when the current flows from lead 1 to lead 2—even though leads 3 and 4 are attached to the same point on the classical current path. Recent theoretical analysis[21,22] is consistent with the general shape of the data curve in figure 5. This length independence clearly demonstrates that there is a nonlocal relationship between electric field and current density in the sample due to quantum interference over a distance of about 1 micron.

Another counterintuitive discovery was that the magnetoresistance $R(H)$ measured in any four-terminal arrangement on a small wire does not remain the same when the magnetic field is reversed.[23] Classically, of course, $R(H)$ should equal $R(-H)$. Figure 6a dramatically illustrates the asymmetric behavior found in a small gold wire, which is typical of all four-probe measurements made on small wires and rings. $R(-H)$ bears little resemblance to $R(+H)$. The second trace in figure 6a is the magnetoresistance obtained by interchanging the current and voltage leads. At first glance, it seems to bear little resemblance to the upper curve. Upon closer inspection, however, one sees that the positive-field half of the second trace is very similar to the negative-field half of the first trace. All the other permutations of leads yield different, asymmetric patterns of magnetoresistance fluctuation. These patterns are amazingly constant over time. After days of (gently) switching probes and currents, one can return to the original configuration and find a pattern of fluctuations that correlates with the original pattern to better than 95% (dashed curve with first trace in figure 6a).

None of these observations violates any fundamental symmetries. In fact, as Büttiker has shown, the general properties displayed in figure 6a are consistent with the principle of reciprocity, which requires that the electrical



Asymmetry under field reversal. a: Measurements on a small gold wire[23] show that conductance fluctuations change when magnetic field is reversed. These fluctuations are reproducible after days of measurement (dashed curve at top). Decomposing these measurements into symmetric (G_S) and antisymmetric (G_A) parts, one sees (**b**) that these components do indeed exhibit the expected symmetry under field reversal. **Figure 6**

resistance in a given measurement configuration and magnetic field H be equal to the resistance at field $-H$ when the current and voltage leads are interchanged.[23]

$$R_{14,23}(H) = R_{23,14}(-H)$$

where

$$R_{ij,kl} \equiv V_{kl}/I_{ij}$$

The Onsager symmetry relations for the local conductivity tensor may be more familiar, but they describe the local relationship between current density and electric field. As we have seen, conductance is a *nonlocal* quantity, and therefore the appropriate symmetries to consider are those involving the total resistance R.

To demonstrate reciprocity in the data, one determines the symmetric and antisymmetric parts of the resistance fluctuations.

$$R_S(H) = \tfrac{1}{2}\{R_{14,23}(H) + R_{23,14}(H)\}$$
$$R_A(H) = \tfrac{1}{2}\{R_{14,23}(H) - R_{23,14}(H)\}$$

Figure 6b shows this decomposition of the data. To within the noise level of the experiment, R_S is perfectly symmetric with respect to magnetic field direction and R_A is perfectly antisymmetric. Applying this procedure to $R_{13,24}$ and $R_{24,13}$ results in a similar R_S but a different R_A. R_A is not simply due to classical Hall resistance. The antisymmetric part fluctuates randomly with an amplitude similar to the symmetric fluctuations. Once again the conductance fluctuation is of order e^2/h. The amplitude of the antisymmetric component is nearly constant, independent of L/L_ϕ, because it arises from nonlocal effects occurring within a distance L_ϕ of the junctions of the voltage probes. Excursions of electrons into the voltage probes account for all of the fluctuations in R_A.

Perhaps the clearest experimental evidence of nonlocal resistance is shown in figure 7. Two identical four-probe wires were fabricated at IBM, but on the second sample a small ring was attached 0.2 μm from the classical current path. Figure 7 shows the conductance of both wires. Both exhibit random conductance fluctuations with the same characteristic field scale, but the second sample shows additional high-frequency "noise." The Fourier transforms of both data sets clearly show that this high-frequency noise is in fact an h/e oscillation with the characteristic flux period equal to what is expected from the area of the ring.[25] For this effect to be observable, some large fraction of the electrons must have encircled the ring coherently.

The magnitude of the h/e oscillations decays exponentially with distance from the classical current path, but the characteristic length scale of this decay is not the geometric dimension, but rather the quantum coherence length L_ϕ.[24] Thus the quantum interference measured in any small sample will have nonlocal contributions, and the distance between voltage probes will not define the sample length.

The future

We have tried to review some of the recent and unexpected experimental consequences of long-range phase coherence in disordered systems. Although many of the observations were unexpected, they make sense in hindsight once we remember that the electron is a wave. Many of the aspects of quantum transport we measure are similar to the physics of electromagnetic radiation in waveguides and light interference in disordered media. The concept of a coherence length thousands of times longer than the wavelength is familiar in these fields.

Long-range phase coherence in disordered systems may make it possible to observe Bloch oscillations and persistent currents. Our study of these systems has already begun to teach us what it means to make a measurement on a quantum system. Further progress will come from extending the measurements to smaller sample sizes and higher frequencies. The trend in technology has been to make smaller devices, pack them closer together, and operate them at lower temperatures and lower voltages. In the not-too-distant future the engineers will have to allow for quantum interference effects in their efforts to optimize these circuits.

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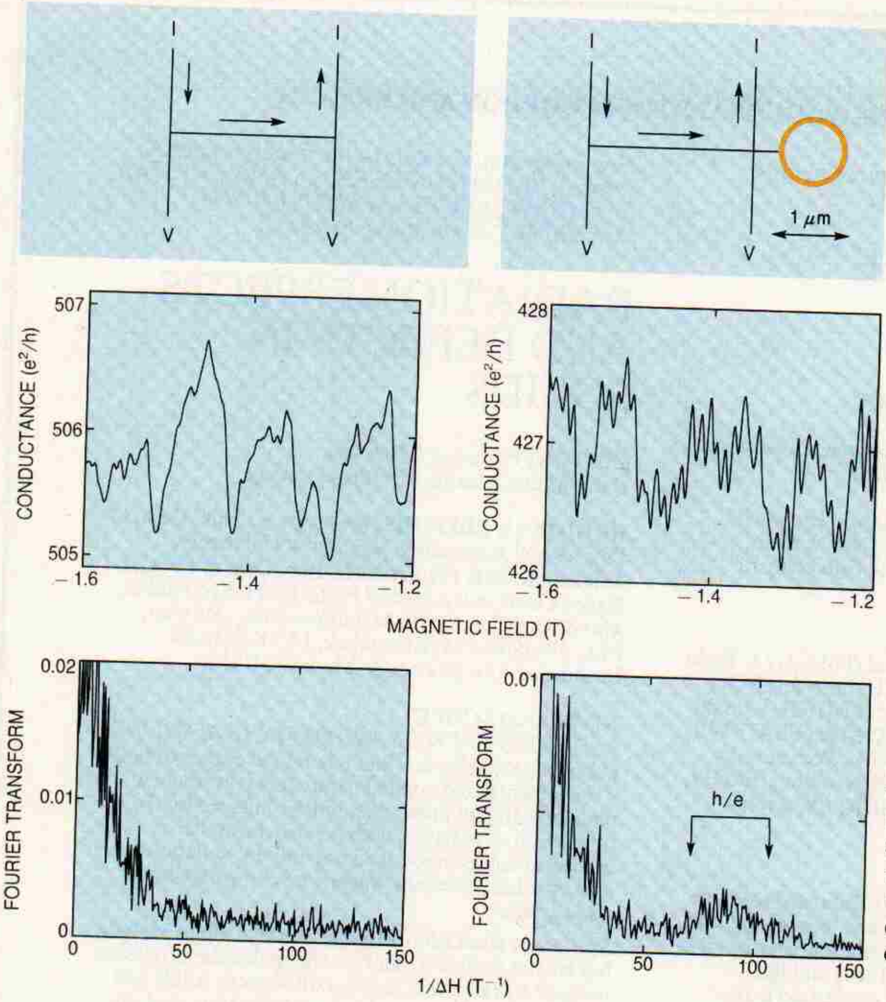

Magnetoresistance of two similar wires (schematics on top), one of which has a ring dangling outside the classical path, where it "should" have no effect. In fact the Fourier transforms of the conductance curves (bottom) show an additional peak for the circuit with the ring, indicating *h*/*e* Aharonov–Bohm oscillations corresponding to the area enclosed by the ring. **Figure 7**

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