Physics 374 Project

A project is a major part of a student's responsibilities in Physics 374. In this project you will be required to study a nontrivial problem using both analytic and numerical methods.
I recommend that you study the undamped physical pendulum in the unbounded regime where (the energy is greater than maximum value of the potential energy). You may do a project studying a different system if you wish but you must clear the topic with me. *You should work independently on this project.* If you get stuck it is perfectly legitimate to seek help from me, from fellow students, or from other knowledgeable people provided that they do not do the work for you. You may also consult the literature. You must however acknowledge any outside help in the paper and cite the literature.

Topic Selection: If you wish to work on a topic other than the physical pendulum you must let me get my approval by **October 13.**

Progress Report/ : Rough Draft You are required to submit a progress report of approximately 1-2 pages by **November 10.** *Some* progress is required by this date. Minimally, you must indicate exactly what calculations you plan to do ultimately and indicate what progress has been on these calculations. I strongly recommend that you complete all your calculations by this date and write up a rough draft of the final report. If you do this I will make comments on it to allow you to strengthen the project and get it back to you by **November 17** .

Final Paper: The final report is due **November 24.**

Paper write up: You should write this up as though it were a scientific article for a scientific journal. You should include an abstract, an introductory section, a description of the analysis in one or more sections and a conclusion. You may also include appendices in which you include gory mathematical details and/or Mathematica code. It is often useful to segregate the grungy details into an appendix to make the paper more readable. The paper should also cite any references used and acknowledge any outside help.

The paper should be written at a level accessible to students in this class. The paper should be well written with all of the physical and mathematical issues clearly explained. The paper should be typed. Equations can either be handwritten in or include using a word processor. I expect the final paper to be somewhere between 10-20 pages (excluding any Mathematica code included) although this is only a rough guideline.

Grading: The paper will be graded both for the scientific content and for the clarity of explanation. The grading will be modeled in part on Olympic diving scoring---I will include the "degree of difficulty" of the project.

Recommended Project:

I recommend that you study the physical pendulum for unbounded motion (E > Umax).

The equation of motion is: $\ddot{\theta} = -\,\omega_0^2\, \sin(\theta)$

The problem is quite rich and has a number of different regimes you might choose to study using approximate techniques.

1) High-energy regime---assume that the total energy is much larger than the maximum potential energy. Thus, $\dot{\theta}$ would expected to be essentially constant and naively, this would seem to imply that $-\,\omega_0^2\, \sin(\theta)$ could be treated as a constant. This would naturally lead to the same expansion of the following form $\ddot{\theta} = -\,\lambda\, \omega_0^2\, \sin(\theta)$ with $\theta = \theta_0 + \lambda\theta_1 + \lambda^2\,\theta_2 + \ldots$. Note that in order to implement this expansion you must Taylor expand $\sin(\,\theta_0 + \lambda\theta_1 + \lambda^2\,\theta_2 + \ldots)$ as a series in λ.

   One issue need to address is the periodic nature of the motion. It is possible to prove that in this unbounded regime $\theta(t) = \theta_0 + \frac{2\pi t}{\tau} + f(t)$ where $f(t)$ is periodic in time, $f(t+\tau) = f(t)$ where τ is the period for one revolution. It is necessary to first prove this and then to show that solutions to your perturbative expansion can be cast into this form. In practice, this will mean choosing initial conditions on your differential equations that imply that "secular" terms, *i.e.* terms which grow with time in a nonperiodic manner are zero. This in turn means imposing boundary conditions for which the period is determined at lowest order and is fixed by boundary conditions at higher orders rather than the initial velocity being held fixed.

2) Alternative treatment of high energy regime----expand the integral expression based on energy conservation to find an approximate relation between initial velocity and period.

3) Low energy end of the unbound regime---assume that the energy is near the minimum for unbounded motion. Find an approximate expression for the period.

4) Small time regime--- assume that effect of the torque on the motion is small. This is clearly valid for small times. Thus organize the perturbative calculation as follows: $\ddot{\theta} = -\lambda\,\omega_0^2\,\sin(\theta)$ with $\theta = \theta_0 + \lambda\theta_1 + \lambda^2\,\theta_2 + \ldots$ which is formally identical to the expansion in part 1). However here you should impose boundary conditions that the velocity is held fixed as you work to higher orders. Thus secular terms are not eliminated. This should work for any energy provided the time is small enough. Issue to be addressed---what sets the scale for "small enough" time?

5) Alternative small time expansion---treat θ as a Taylor series in time $\theta = c_0 + \lambda c_1\,t + \lambda^2\,c_2\,t^2 + \ldots$. Issue---is this expansion better or worse than the one in 4)?

You may study one or more of these regimes using approximate methods. Whatever you do decide to study analytically, you should also solve the problem numerically either by direct numerical solution of the differential equation or by integrating an expression based on energy conservation (or better yet both) and test how well your approximation methods work.