PHYS 373 (Fall 2015): Mathematical Methods for Physics II
Final exam: Monday, December 14, 8.00 am.-10.00 am.

Read the instructions below and do *not* flip to next page till you are told to do so.

Name:

Student ID:

Useful formulae:

$f(x) = \frac{a_0}{2} + \sum_1^\infty \left(a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}\right)$	$f(x) = \sum_{l=0}^\infty c_l P_l(x)$	$f(x) = \sum_{-\infty}^\infty c_n e^{in\pi x/l}$
$N_p(x) = \frac{\cos(\pi p) J_p(x) - J_{-p}(x)}{\sin \pi p}$	$a_n = \frac{1}{l}\int_{-l}^{l} f(x) \cos\frac{n\pi x}{l} dx$	$P_3(x) = \frac{1}{2}(5x^3 - 3x)$
$\int_0^1 x J_p(\alpha x) J_p(\beta x) = 0 \ (\alpha \neq \beta)$	$b_n = \frac{1}{l}\int_{-l}^{l} f(x) \sin\frac{n\pi x}{l} dx$	$P_1(x) = x$
$c_m = \frac{2m+1}{2}\int_{-1}^{1} f(x) P_m(x) dx$	$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \ \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$	$P_0(x) = 1$
$f(z) = f(x+iy) = u(x,y) + iv(x,y)$	$J_{p-1}(x) - J_{p+1}(x) = 2J_p'(x)$	$J_0(0) = 1$
$\frac{d}{dx}\left[x^{-p} J_p(x)\right] = -x^{-p} J_{p+1}(x)$	$N_n(0) = \infty$	$J_{n\neq 0}(0) = 0$
$\frac{\partial^2}{\partial x^2} T(x,y) + \frac{\partial^2}{\partial y^2} T(x,y) = 0$	$T = \left\{ \begin{array}{c} e^{kx} \\ e^{-kx} \end{array} \right\} \left\{ \begin{array}{c} \sin ky \\ \cos ky \end{array} \right\}$	or $\left\{ \begin{array}{c} \sin kx \\ \cos kx \end{array} \right\} \left\{ \begin{array}{c} e^{ky} \\ e^{-ky} \end{array} \right\}$
$R(z_0) = \lim_{z \to z_0} (z - z_0) f(z)$	$g(\alpha) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x) e^{-i\alpha x} dx$	$J_n(\infty) = 0$
$\int f(z) dz$ (around C) $=$	$2\pi i$.sum of residues of $f(z)$	inside C
$-$ (residue of $f(Z)$at $Z = \infty$) $=$	(residue of $\frac{1}{z^2} f\left(\frac{1}{z}\right)$ at $z = 0$)	
$\int_{-\infty}^{\infty} \frac{P(x)}{Q(x)} e^{imx} dx \ (m > 0) =$	$2\pi i$.sum of residues of $\frac{P}{Q} e^{imz}$	in upper half-plane
$J_p(x) = \sum_{n=0}^\infty \frac{(-1)^n}{\Gamma(n+1)\Gamma(n+1+p)}$	$P_l(x) = \frac{1}{2^l l!}\frac{d^l}{dx^l}(x^2-1)^l$	$\int f(z) dz = 0,$ around C
$ax^2 + bx + c = 0 \rightarrow$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$R(z_0) = \frac{g(z_0)}{h'(z_0)},$ if $f = \frac{g}{h}$

It is necessary to show the details of the derivation and not just the final answer for *all* problems.

This is a closed book exam: crib sheets are not allowed.

In case they are needed, more blank paper and stapes are provided.

Please write clearly and if you use the backside of a page (or continue working on a problem on a later one), then please indicate so.

Check that there are total of 6 (printed) *pages* (containing 6 problems).

Note that some problems have *multiple* parts; so, please read the statement of each problem carefully.

Remember the honor pledge that you signed at the start of the semester.