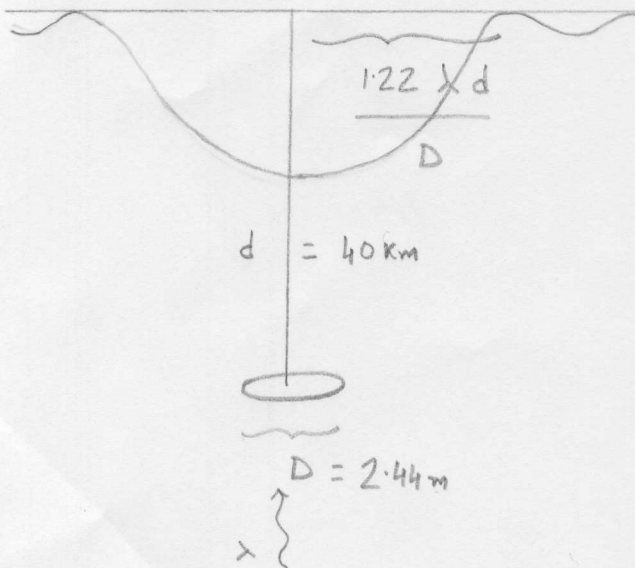1. [10 pts] A radar station for tracking aircraft broadcasts a 12 GHz microwave beam from a 2.44-m-diameter circular radar antenna. From a wave perspective, the antenna is a circular aperture through which the microwaves diffract. What is the diameter of the radar beam at a distance of 40 km?

Note that the speed of light is $c=3.0 \times 10^8$ m/s and 1 GHz = 10^9 Hz.

Hint: derive the equation first and plug the numbers in at the last step to make the numerical calculation easy!

$\frac{1.22 \lambda d}{D}$

$d = 40 \text{ Km}$

$D = 2.44 m$

λ

$f = 12 \times 10^9 \text{ Hz}, \quad c = 3 \times 10^8 \text{ m/s} = f\lambda$

$\rightarrow \lambda = \frac{c}{f} = \frac{1}{40} m$

$\rightarrow$ diameter of the diffracted beam at a distance of $d = 40$ km for the diffracting aperture diameter $D = 2.44m$ is (look at the figure on the left)

$$\frac{2 \cdot 1.22 \times 40 \times 10^3}{2.44 \times 40} m = 10^3 m \text{ or } 1 \text{ Km}.$$