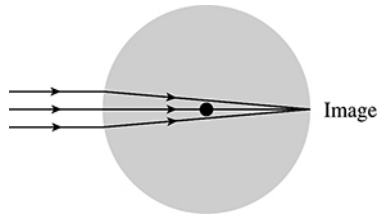**23.60.** **Model:** Use the ray model of light. The surface is a spherically refracting surface.
Visualize:

Solve: Because the rays are parallel, $s = \infty$. The rays come to focus on the rear surface of the sphere, so $s' = 2R$, where R is the radius of curvature of the sphere. Using Equation 23.21,

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Rightarrow \frac{1}{\infty} + \frac{n}{2R} = \frac{n-1}{R} \Rightarrow n = 2.00$$

23.69. **Model:** Assume the lens is a thin lens and the thin-lens formula applies.

Solve: Because we want to form an image of the spider on the wall, the image is real and we need a converging lens. That is, both s' and s are positive. This also implies that the spider's image is inverted, so $M = -s'/s = -\frac{1}{2}$.

Using the thin-lens formula with $s' = \frac{1}{2}s$,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s} + \frac{1}{\frac{1}{2}s} = \frac{1}{f} \Rightarrow \frac{3}{s} = \frac{1}{f} \Rightarrow f = \frac{s}{3}$$

We also know that the spider is 2.0 m from the wall, so

$$s + s' = 2.0 \text{ m} = s + \frac{1}{2}s \Rightarrow s = \frac{1}{3}(4.0 \text{ m}) = 133.3 \text{ cm}$$

Thus, $f = \frac{1}{3}s = 44$ cm and $s' = 2.0 \text{ m} - 1.33 \text{ m} = 0.67 \text{ m} = 67$ cm. We need a 44 cm focal length lens placed 67 cm from the wall.

23.73. **Visualize:** The lens must be a converging lens for this scenario to happen, so we expect f to be positive. In the first case the upright image is virtual ($s_2' < 0$) and the object must be closer to the lens than the focal point. The lens is then moved backward past the focal point and the image becomes real ($s_2' > 0$).

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow f = \frac{ss'}{s + s'}$$

We are given $s_1 = 10\,\text{cm}$ and $m_1 = 2$.

Solve: Since the first image is virtual, $s' < 0$. We are told the first magnification is $m_1 = 2 = -s_1'/s_1 \Rightarrow s_1' = -20$ cm. We can now find the focal length of the lens.

$$f = \frac{s_1 s_1'}{s_1 + s_1'} = \frac{(10\ \text{cm})(-20\ \text{cm})}{10\ \text{cm} - 20\ \text{cm}} = 20\ \text{cm}$$

After the lens is moved, $m_2 = -2 = -s_2'/s_2$. Start with the thin lens equation again.

$$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f}$$

Replace s_2' with $-m_2 s_2$.

$$\frac{1}{s_2} + \frac{1}{-m_2 s_2} = \frac{1}{f}$$

Now solve for s_2.

$$\frac{-m_2 s_2 + s_2}{s_2(-m_2 s_2)} = \frac{1}{f}$$

$$\frac{-m_2 s_2 + s_2}{-m_2 s_2^2} = \frac{1}{f}$$

Cancel one s_2.

$$\frac{m_2 - 1}{m_2 s_2} = \frac{1}{f}$$

Multiply both sides by $f s_2$.

$$s_2 = f\left(\frac{m_2 - 1}{m_2}\right) = (20\ \text{cm})\left(\frac{-2 - 1}{-2}\right) = 30\ \text{cm}$$

The distance the lens moved is $s_2 - s_1 = 30\ \text{cm} - 10\ \text{cm} = 20\ \text{cm}$.

Assess: We knew s_2 needed to be bigger than f; it is, and is in a reasonable range. The final answer for the distance the lens moved also seems reasonable.

24.1. The picture will be underexposed if the shutter speed remains the same. The problem stated that D is not changed. $I \propto \dfrac{D^2}{f^2}$. If f is increased then I is decreased so the picture is underexposed.

24.2. The answer is C. Rays from all parts of the object hit all parts of the lens, so the whole image still appears, but because the area of the aperture is smaller then the image will be less bright. Contrary to choice a, the image actually gets sharper because the off-axis rays contribute to blurriness and a smaller aperture eliminates many of those rays.

24.3. If you try to see underwater with no face mask there is little refraction at the water-cornea boundary because the indices of refraction are so similar. To make up for this loss of refraction (as compared to seeing in air) it would be helpful to have glasses with converging lenses.

24.5. Equation 24.13 gives the width of the spot as $w_{min} \approx \dfrac{2.44\lambda f}{D}$. The minimum spot width is achieved for incoming parallel rays when the lens-to-screen distance is f. (a) Decreasing λ decreases w. (b) Decreasing D increases w. (c) Decreasing f decreases w. (d) Decreasing the lens-to-screen distance means the screen is no longer in the focal plane and the spot is larger than its minimum in the focal plane.

24.6. You would use a lens with a small f-number. The minimum width is given by $w_{min} \approx \dfrac{2.44 \lambda f}{D}$ but since f-number $= f/D$, then $w_{min} \approx 2.44 \lambda$ (f-number).

24.1. **Model:** Each lens is a thin lens. The image of the first lens is the object for the second lens.
Visualize:
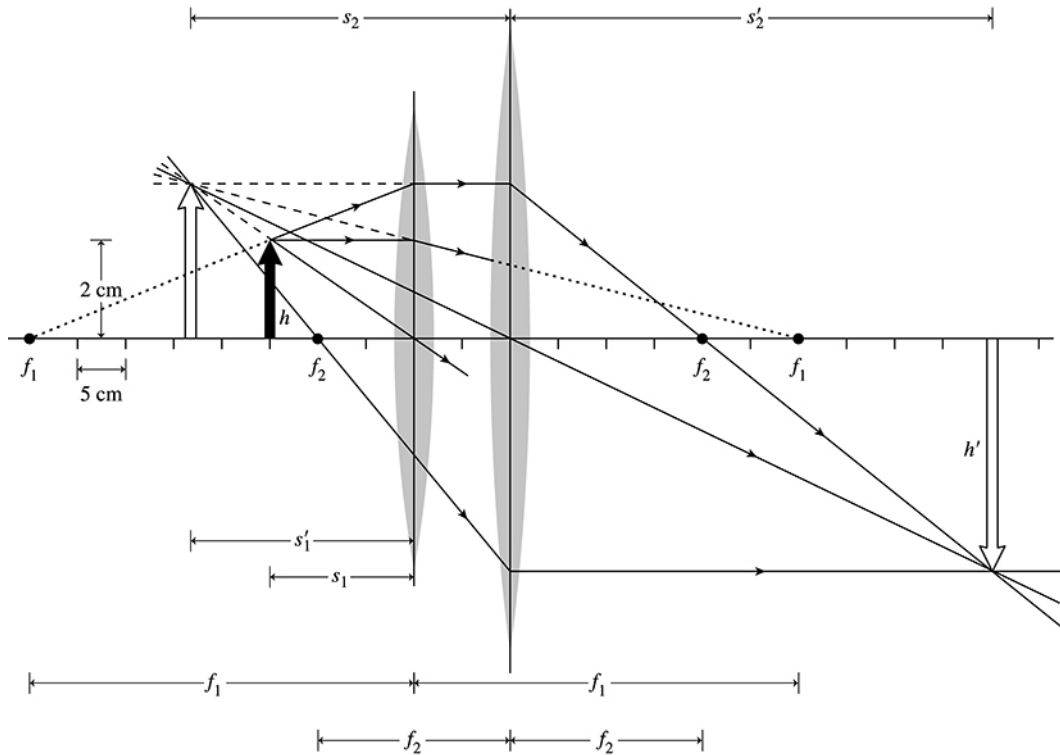
The figure shows the two lenses and a ray-tracing diagram. The ray-tracing shows that the lens combination will produce a real, inverted image behind the second lens.

Solve: **(a)** From the ray-tracing diagram, we find that the image is ≈ 50 cm from the second lens and the height of the final image is 4.5 cm.

(b) $s_1 = 15$ cm is the object distance of the first lens. Its image, which is a virtual image, is found from the thin-lens equation:

$$\frac{1}{s_1'} = \frac{1}{f_1} - \frac{1}{s_1} = \frac{1}{40 \text{ cm}} - \frac{1}{15 \text{ cm}} = -\frac{5}{120 \text{ cm}} \Rightarrow s_1' = -24 \text{ cm}$$

The magnification of the first lens is

$$m_1 = -\frac{s_1'}{s_1} = -\frac{(-24 \text{ cm})}{15 \text{ cm}} = 1.6$$

The image of the first lens is now the object for the second lens. The object distance is $s_2 = 24$ cm $+ 10$ cm $= 34$ cm. A second application of the thin-lens equation yields:

$$\frac{1}{s_2'} = \frac{1}{f_2} - \frac{1}{s_2} = \frac{1}{20 \text{ cm}} - \frac{1}{34 \text{ cm}} \Rightarrow s_2' = \frac{680 \text{ cm}}{14} = 48.6 \text{ cm}$$

The magnification of the second lens is

$$m_2 = -\frac{s_2'}{s_2} = -\frac{48.6 \text{ cm}}{34 \text{ cm}} = -1.429$$

The combined magnification is $m = m_1 m_2 = (1.6)(-1.429) = -2.286$. The height of the final image is $(2.286)(2.0 \text{ cm}) = 4.57$ cm. These calculated values are in agreement with those found in part (a).

24.2. **Model:** Each lens is a thin lens. The image of the first lens is the object for the second lens.
Visualize:
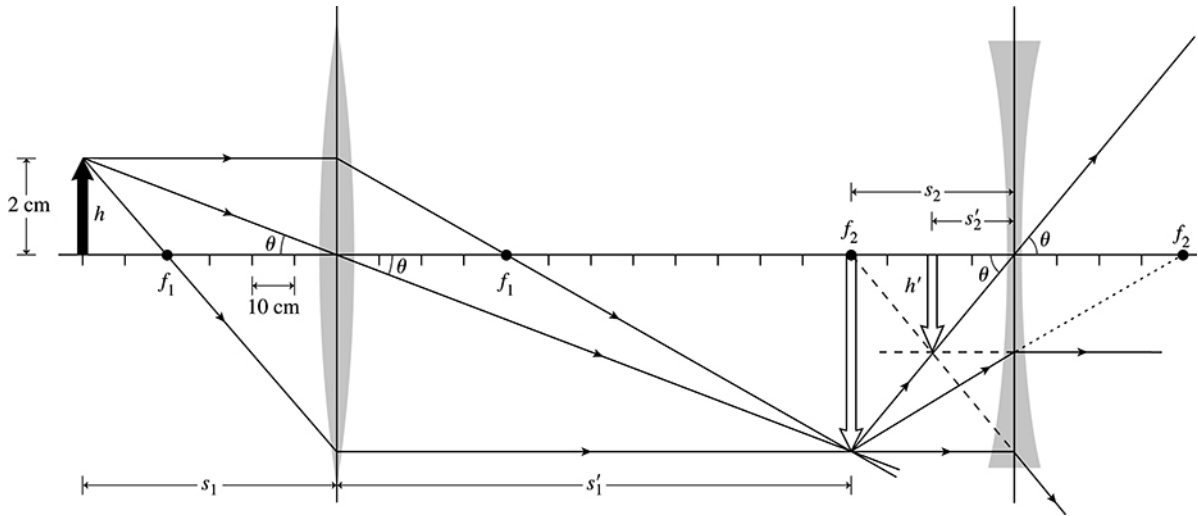
The figure shows the two lenses and a ray-tracing diagram. The ray tracing shows that the lens combination will produce a virtual, inverted image in front of the second lens.
Solve: **(a)** From the ray-tracing diagram, we find that the image is 20 cm in front of the second lens and the height of the final image is 2.0 cm.
(b) $s_1 = 60$ cm is the object distance of the first lens. Its image, which is a real image, is found from the thin-lens equation:

$$\frac{1}{s_1'} = \frac{1}{f_s} - \frac{1}{s_1} = \frac{1}{40 \text{ cm}} - \frac{1}{60 \text{ cm}} = \frac{1}{120 \text{ cm}} \Rightarrow s_1' = 120 \text{ cm}$$

The magnification of the first lens is

$$m_1 = -\frac{s_1'}{s_1} = -\frac{120 \text{ cm}}{60 \text{ cm}} = -2$$

The image of the first lens is now the object for the second lens. The object distance is $s_2 = 160$ cm -120 cm $= 40$ cm. A second application of the thin-lens equation yields:

$$\frac{1}{s_2'} = -\frac{1}{s_2} + \frac{1}{f_2} = \frac{-1}{+40 \text{ cm}} + \frac{1}{-40 \text{ cm}} \Rightarrow s_2' = -20 \text{ cm}$$

The magnification of the second lens is

$$m_2 = -\frac{s_2'}{s_2} = -\frac{-20 \text{ cm}}{40 \text{ cm}} = +0.5$$

The overall magnification is $m = m_1 m_2 = (-2)(0.5) = -1.0$. The height of the final image is $(+1.0)(2.0$ cm$) = 2.0$ cm. The image is inverted because m has a negative sign. These calculated values are in agreement with those found in part (a).

24.5. Model: Each lens is a thin lens. The image of the first lens is the object for the second lens.
Visualize:
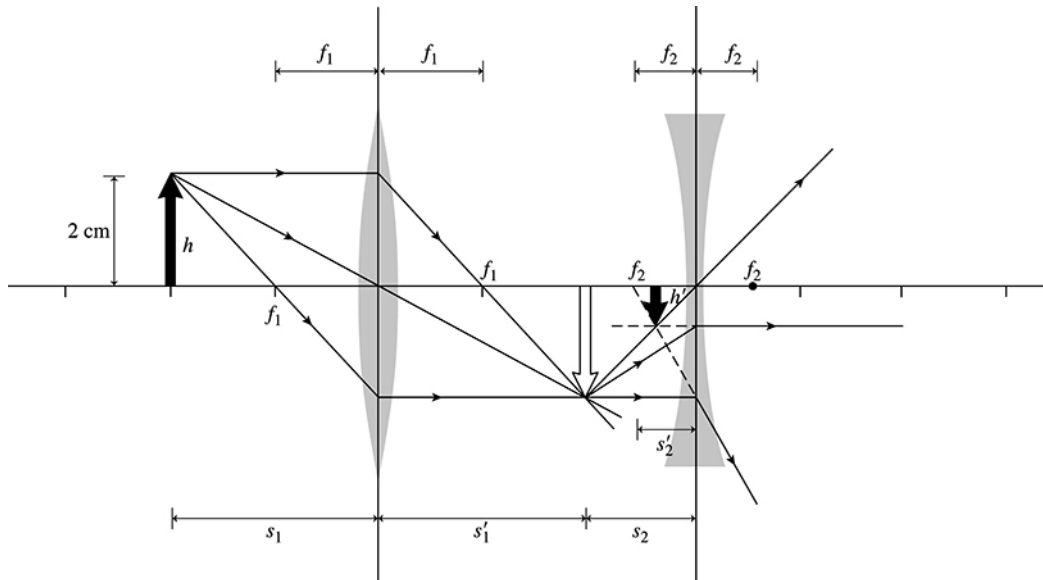
The figure shows the two lenses and a ray-tracing diagram. The ray tracing shows that the lens combination will produce a virtual, inverted image in front of the second lens.
Solve: **(a)** From the ray-tracing diagram, we find that the image is 3.3 cm behind the second lens and the height of the final image is $\approx 0.7\,\text{cm}$.

(b) $s_1 = 20$ cm is the object distance of the first lens. Its image, which is real and inverted, is found from the thin-lens equation:

$$\frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f_1} \Rightarrow s_1' = \frac{f_1 s_1}{s_1 - f_1} = \frac{(10\ \text{cm})(20\ \text{cm})}{20\ \text{cm} - 10\ \text{cm}} = 20\ \text{cm}$$

The magnification of the first lens is

$$m_1 = -\frac{s_1'}{s_1} = -\frac{20\ \text{cm}}{20\ \text{cm}} = -1$$

The image of the first lens is now the object for the second lens. The object distance is $s_2 = 30\ \text{cm} - 20\ \text{cm} = 10\ \text{cm}$. A second application of the thin-lens equation yields

$$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2} \Rightarrow s_2' = \frac{f_2 s_2}{s_2 - f_2} = \frac{(-5\ \text{cm})(10\ \text{cm})}{10\ \text{cm} + 5\ \text{cm}} = -3.33\ \text{cm}$$

The magnification of the second lens is

$$m_2 = -\frac{s_2'}{s_2} = -\frac{(-3.3\ \text{cm})}{10\ \text{cm}} = 0.33$$

The combined magnification is $m = m_1 m_2 = (-1)(0.33) = -0.33$. The height of the final image is $(0.33)(2.0\ \text{cm}) = 0.66\ \text{cm}$. The image is inverted because m has a negative sign. These calculated values are in agreement with those found in part (a).
Assess: The thin-lens equation agrees with the ray tracing.

24.7. **Visualize:** Equation 24.2 gives f-number $= f / D.$
Solve:

$$f\text{-number} = \frac{f}{D} = \frac{35 \text{ mm}}{7.0 \text{ mm}} = 5.0$$

Assess: This is in the range of f-numbers for typical camera lenses.

24.9. **Visualize:** First we compute the *f*-number of the first lens and then the diameter of the second.
Solve:

$$f\text{-number} = \frac{f}{D} = \frac{12 \text{ mm}}{4.0 \text{ mm}} = 3.0$$

Now for the new lens.

$$D = \frac{f}{f\text{-number}} = \frac{18 \text{ mm}}{3.0} = 6.0 \text{ mm}$$

Assess: Given the same *f*-number, the longer focal length lens has a larger diameter.

24.11. **Visualize:** We want the same exposure in both cases. The exposure depends on $I\Delta t_{\text{shutter}}$. We'll also use Equation 24.3. The lens is the same lens in both cases, so $f = f'$.

Solve:

$$\text{exposure} = I\Delta t \propto \frac{D^2}{f^2}\Delta t$$

$$\frac{D^2}{f^2}\Delta t = \frac{D'^2}{f'^2}\Delta t'$$

Solve for D'; then simplify.

$$D' = \sqrt{D^2\left(\frac{f'}{f}\right)^2\left(\frac{\Delta t}{\Delta t'}\right)} = D\sqrt{\frac{\Delta t}{\Delta t'}} = (3.0 \text{ mm})\sqrt{\frac{1/125\text{s}}{1/500\text{s}}} = (3.0 \text{ mm})\sqrt{4} = 6.0 \text{ mm}$$

Assess: Since we decreased the shutter speed by a factor of 4 we need to increase the aperture area by a factor of 4, and this means increase the diameter by a factor of 2.

24.12. **Model:** Ignore the small space between the lens and the eye.

Visualize: Refer to Example 24.4, but we want to solve for s', the near point.

Solve:

(a) The power of the lens is positive which means the focal length is positive, so Ramon wears converging lenses. This is the remedy for hyperopia.

(b) We want to know where the image should be for an object $s = 25$ cm given $1/f = 2.0$ m^{-1}.

$$f = \frac{1}{P} = 0.50 \text{ m}$$

$$s' = \frac{fs}{s - f} = \frac{(0.50 \text{ m})(0.25 \text{ m})}{0.25 \text{ m} + 0.50 \text{ m}} = -0.50 \text{ m}$$

So the near point is 50 cm.

Assess: The negative sign on s' is expected because we need the image to be virtual.

24.13. **Model:** Ignore the small space between the lens and the eye.
Visualize: Refer to Example 24.5, but we want to solve for s', the far point.
Solve:
(a) The power of the lens is negative which means the focal length is negative, so Ellen wears diverging lenses. This is the remedy for myopia.
(b) We want to know where the image should be for an object $s = \infty$ m given $1/f = -1.0$ m^{-1}.

$$f = \frac{1}{P} = -1.0 \text{ m}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

When $s = \infty$ m,

$$\frac{1}{\infty \text{ m}} + \frac{1}{s'} = \frac{1}{f} \Rightarrow s' = f = -1.0 \text{ m}$$

So the far point is 100 cm.
Assess: The negative sign on s' is expected because we need the image to be virtual.

24.15. **Model:** The angle subtended by the image is $8\times$ the angle subtended by the object.

Visualize: The angle subtended by the object is h/s.

Solve:

$$\theta = (8\times)\frac{h}{s} = (8\times)\frac{14 \text{ cm}}{1800 \text{ cm}} = 0.0622 \text{ rad} = 3.6°$$

Assess: The binoculars do indeed help.

24.18. **Model:** Assume the thin-lens equation is valid. For part (b) refer to Equation 24.11 and the definition of α.

Visualize: We are given $f_{obj} = 9.0$ mm and

$$m = -\frac{s'}{s} = -40 \quad \Rightarrow \quad s' = 40s$$

Solve: **(a)** Use the thin-lens equation.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f_{obj}}$$

$$\frac{1}{s} + \frac{1}{40s} = \frac{1}{f_{obj}}$$

$$\frac{41}{40s} = \frac{1}{f_{obj}}$$

$$s = \frac{41}{40} f_{obj} = \frac{41}{40}(9.0 \text{ mm}) = 9.2 \text{ mm}$$

(b) We are given $n = 1.00$. Refer to Figure 24.14a to determine α.

$$\alpha = \tan^{-1}(3.0 \text{ mm}/9.0 \text{ mm}) = \tan^{-1}(1/3)$$

$$NA = n \sin \alpha = (1.00) \sin\left(\tan^{-1}\left(\frac{1}{3}\right)\right) = 0.32$$

Assess: We used $s = f$ in this calculation as suggested in the text. If we had used $s = 9.2$ mm (from part (a)) we would get $NA = 0.31$, only a little different. These values of NA are typical for a simple microscope.

24.19. **Visualize:** We are given $NA = 0.90$, $L = 160$ mm, $m_{obj} = -20$, and the book says $n = 1.46$.

Solve: Start with Equation 24.11.

$$NA = n\sin\alpha$$

$$\alpha = \sin^{-1}\left(\frac{NA}{n}\right)$$

$$\tan\alpha = \tan\left(\sin^{-1}\left(\frac{NA}{n}\right)\right)$$

But from Figure 24.14a we also have $\tan\alpha = \left(\frac{1}{2}\right)D/f_{obj}$, where D is the diameter of the lens. So combine those two expressions for $\tan\alpha$ and solve for D.

$$D = 2f_{obj}\tan\left(\sin^{-1}\left(\frac{NA}{n}\right)\right)$$

We need the side calculation using Equation 24.9:

$$m_{obj} = -\frac{L}{f_{obj}} \Rightarrow f_{obj} = -\frac{L}{m_{obj}}$$

Insert this back in the equation for D.

$$D = 2\left(-\frac{L}{m_{obj}}\right)\tan\left(\sin^{-1}\left(\frac{NA}{n}\right)\right)$$

$$D = 2\left(-\frac{160\text{ mm}}{-20}\right)\tan\left(\sin^{-1}\left(\frac{0.90}{1.46}\right)\right) = 13\text{ mm}$$

Assess: The answer seems to be in a reasonable range for objective lens diameter.

24.20. **Visualize:** Figure 24.15 shows from similar triangles that for the eyepiece lens to collect all the light

$$\frac{D_{obj}}{f_{obj}} = \frac{D_{eye}}{f_{eye}}$$

We also see from Equation 24.12 that $M = -f_{obj}/f_{eye}$. We are given $M = -20$ and $D_{obj} = 12$ cm.

Solve:

$$D_{eye} = D_{obj}\frac{f_{eye}}{f_{obj}} = \frac{D_{obj}}{-M} = \frac{12\text{cm}}{20} = 0.60 \text{ cm} = 6.0 \text{ mm}$$

Assess: The answer is almost as wide as a dark-adapted eye.

24.23. **Model:** Two objects are marginally resolvable if the angular separation between the objects, as seen from the lens, is $\alpha = 1.22\lambda / D$.

Solve: Let Δy be the separation between the two light bulbs, and let L be their distance from a telescope. Thus,

$$\alpha = \frac{\Delta y}{L} = \frac{1.22\lambda}{D} \Rightarrow L = \frac{\Delta y \, D}{1.22\lambda} = \frac{(1.0 \text{ m})(4.0\times10^{-2} \text{ m})}{1.22(600\times10^{-9} \text{ m})} = 55 \text{ km}$$

24.27. **Model:** The parallel rays can be considered to come from an object infinitely far away: $s_1 = \infty$. The lens is a diverging lens.

Visualize: If $s_1 = \infty$ the thin lens equation tells us that $s_1' = f_1'$; we are given that $f_1 = -10$ cm. We are also given for the mirror $f_2 = 10$ cm.

Solve: Since $s_1' = -10$ cm the image is virtual 10 cm to the left of the lens. The image from the lens becomes the object for the mirror $\Rightarrow s_2 = 30$ cm; this is three times the mirror's focal length, or $s_2 = 3f_2$.

$$s_2' = \frac{f_2 s_2}{s_2 - f_2} = \frac{f_2(3f_2)}{3f_2 - f_2} = \frac{3}{2} f_2 = 15 \text{ cm}$$

Therefore the initial parallel rays are brought to a focus 15 cm to the left of the mirror, or 5 cm to the right of the lens.

Assess: The answer is reasonable and can be verified by ray tracing.

24.28. **Visualize:** The object is within the focal length of the converging lens, so we expect the image to be upright, virtual, and to the left of the lens. The image of the lens becomes the object for the mirror, and we expect the second image to be upright and virtual behind (to the right of) the mirror.
Solve:

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} = \frac{(10 \text{ cm})(5 \text{ cm})}{5 \text{ cm} - 10 \text{ cm}} = -10 \text{ cm}$$

The image of the lens becomes the object for the mirror $\Rightarrow s_2 = 15 \text{ cm}$, and we expect the second image to be upright and virtual behind (to the right of) the mirror.

$$s_1' = \frac{f_2 s_2}{s_2 - f_2} = \frac{(-30 \text{ cm})(15 \text{ cm})}{15 \text{ cm} - (-30 \text{ cm})} = -10 \text{ cm}$$

$$h' = h \frac{s_1' \, s_2'}{s_1 \, s_2} = (1.0 \text{ cm}) \left(\frac{-10 \text{ cm}}{5.0 \text{ cm}} \right) \left(\frac{-10 \text{ cm}}{15 \text{ cm}} \right) = 1.3 \text{ cm}$$

The final image is 10 cm to the right of the mirror, or 15 cm to the right of the lens. It is upright with a height of 1.3 cm.
Assess: Ray tracing will verify the answer.

24.29. Solve: (a) The image location from the first lens is

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} = \frac{(-2.5 \text{ cm})(2.5 \text{ cm})}{2.5 \text{ cm} - (-2.5 \text{ cm})} = -1.25 \text{ cm}$$
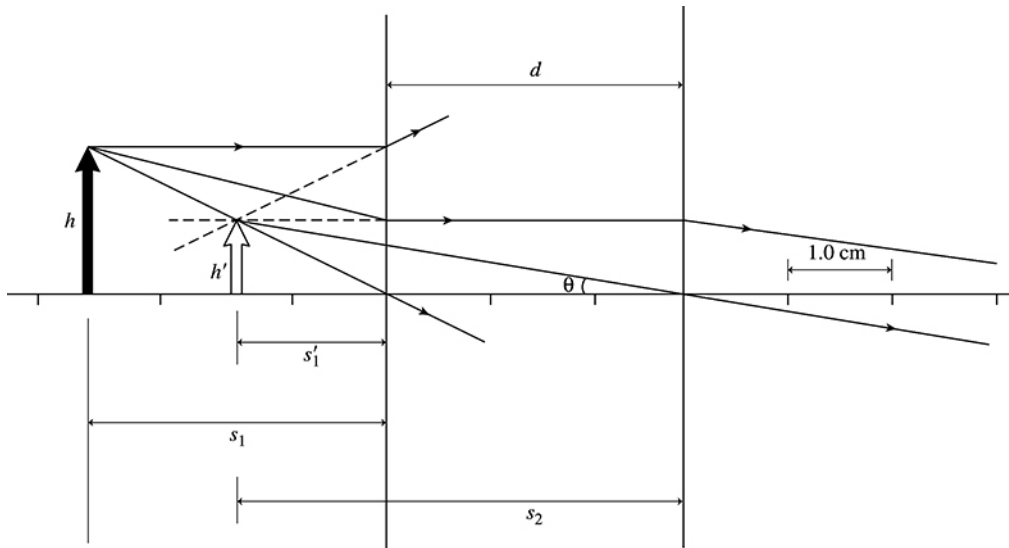
So the image from the first lens is 1.25 cm to the left of the first lens, upright and virtual.
Now, $s_2 = d + 1.25$ cm.
We are told the final image is at infinity: $s_2' = \infty \Rightarrow s_2 = f_2 \Rightarrow f_2 = d + 1.25$ cm

$$d = f_2 - 1.25 \text{cm} = 3.75 \text{ cm}$$

(b)

(c)

$$h' = -h \frac{s_1'}{s_1} = 0.50 \text{ cm}$$

The angular size is

$$\theta = \tan \frac{h'}{f_2} \approx \frac{h'}{f_2} = \frac{0.50 \text{ cm}}{5.0 \text{ cm}} = 0.10 \text{ rad}$$

**(d)** If the object were held at the eye's near point, it would subtend:

$$\theta_{\text{NP}} = \frac{h}{25 \text{ cm}} = \frac{1.0 \text{ cm}}{25 \text{ cm}} = 0.040 \text{ rad}$$

The angular magnification is

$$M = \frac{\theta}{\theta_{\text{NP}}} = \frac{0.10 \text{ rad}}{0.040 \text{ rad}} = 2.5$$

Assess: The numerical answers seem to agree with the drawing.

24.31. **Visualize:** Hard thought shows that if the left focal points for both lenses coincide then the parallel rays before and after the beam splitter are reproduced. The first lens diverges the rays as if they had come from the focal point of the converging lens.
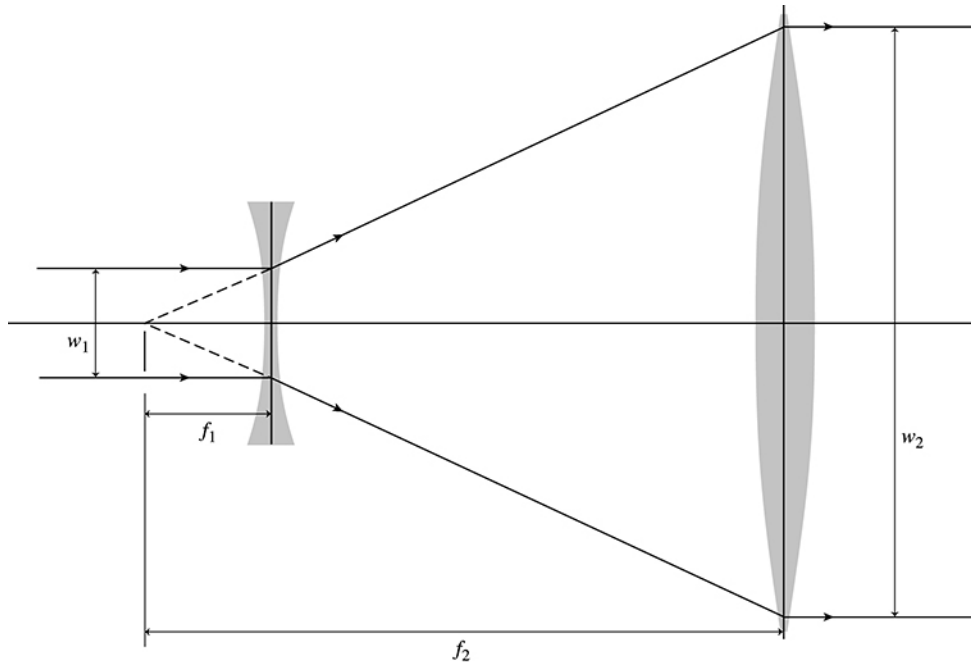
Solve: **(a)**

$$d = f_2 - |f_1|$$

But since we are given $f_1 < 0$, this is equivalent to

$$d = f_2 + f_1$$

(b) Looking at the similar triangles in the diagram shows that

$$\frac{w_1}{|f_1|} = \frac{w_2}{f_2}$$

$$w_2 = \frac{f_2}{|f_1|} w_1$$

Assess: Figure P24.31 says $f_2 > |f_1|$ and our answer then shows that $w_2 > w_1$ which is the goal of a beam expander.

24.32. **Visualize:** We simply need to work backwards. We are given $f_1 = 7.0$ cm and $f_2 = 15$ cm. We are also given $s_2' = -10$ cm. We use this to find s_2.

Solve: **(a)**

$$s_2 = \frac{f_2 s_2'}{s_2' - f_2} = \frac{(15 \text{ cm})(-10 \text{ cm})}{-10 \text{ cm} - 15 \text{ cm}} = 6.0 \text{ cm}$$

So the final image is 6.0 cm to the left of the second lens, or 14 cm to the right of the first lens. That is, the object for the second lens is the image from the first lens, so $s_1' = 20$ cm $= -6.0$ cm $= 14$ cm.

$$s_1 = \frac{f_1 s_1'}{s_1' - f_1} = \frac{(7.0 \text{ cm})(14 \text{ cm})}{14 \text{ cm} - 7.0 \text{ cm}} = 14 \text{ cm}$$

Thus, $L = 14$ cm.

(b) To find the height and orientation we need to look at the magnification.

$$m = m_1 m_2 = \left(-\frac{s_1'}{s_1}\right)\left(-\frac{s_2'}{s_2}\right) = \left(-\frac{14 \text{ cm}}{14 \text{ cm}}\right)\left(-\frac{-10 \text{ cm}}{6.0 \text{ cm}}\right) = -1.7$$

$$h' = hm = (1.0 \text{ cm})(-1.7) = -1.7 \text{ cm}$$

The negative sign indicates that the image is inverted.

Assess: Ray tracing would verify the answers.

24.35. **Model:** Yang has myopia. Normal vision will allow Yang to focus on a very distant object. In measuring distances, we'll ignore the small space between the lens and her eye.

Solve: Because Yang can see objects at 150 cm with a fully relaxed eye, we want a lens that creates a virtual image at $s' = -150$ cm (negative because it's a virtual image) of an object at $s = \infty$ cm. From the thin-lens equation,

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty \text{ m}} + \frac{1}{-1.5 \text{ m}} = -0.67 \text{ D}$$

So Yang gets a prescription for a -0.67 D lens which has $f = -150$ cm.

Since Yang can accommodate to see things as close as 20 cm we need to create a virtual image at 20 cm of objects that are at $s =$ new near point. That is, we want to solve the thin-lens equation for s when $s' = -20$ cm and $f = -150$ cm.

$$s = \frac{fs'}{s' - f} = \frac{(-150 \text{ cm})(-20 \text{ cm})}{-20 \text{ cm} - (-150 \text{ cm})} = 23 \text{ cm}$$

Assess: Diverging lenses are always used to correct myopia.

24.36. **Visualize:** Use Equation 23.21:

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

where $n_1 = 1.00$ for air and $n_2 = 1.34$ for aqueous humor. If we think of incoming parallel rays coming to a focus in the humor then we have $s = \infty$ and $s' = f$.

Solve:

$$\frac{1.0}{\infty} + \frac{n_2}{f} = \frac{n_2 - n_1}{R}$$

Solve for R.

$$R = f\frac{n_2 - n_1}{n_2} = (3.0 \text{ cm})\frac{1.34 - 1.00}{1.34} = 0.76 \text{ cm}$$

Assess: If you think about the dimensions of an eye, this answer seems physically possible.

24.37. **Visualize:** Use Equation 23.27, the lens makers' equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For a symmetric lens $R_1 = R_2$ and

$$f = \frac{R}{2(n-1)} \quad \text{and} \quad R = 2(n-1)f$$

Also needed will be the magnification of a telescope: $M = -f_{obj}/f_{eye} \Rightarrow f_{eye} = -f_{obj}/M$ (but we will drop the negative sign).

We are given $R_{obj} = 100$ cm and $M = 20$.

Solve:

$$R_{eye} = 2(n-1)f_{eye} = 2(n-1)\frac{f_{obj}}{M} = 2(n-1)\frac{\dfrac{R_{obj}}{2(n-1)}}{M} = \frac{R_{obj}}{M} = \frac{100 \text{ cm}}{20} = 5.0 \text{ cm}$$

Assess: We expect a short focal length and small radius of curvature for telescope eyepieces.

24.38. **Model:** Assume that each lens is a simple magnifier with $M = 25 \text{ cm}/f$.

Visualize:

$$M_{\text{obj}} = \frac{25 \text{ cm}}{f_{\text{obj}}} \Rightarrow f_{\text{obj}} = \frac{25 \text{ cm}}{M_{\text{obj}}}$$

$$M_{\text{eye}} = \frac{25 \text{ cm}}{f_{eye}} \Rightarrow f_{\text{eye}} = \frac{25 \text{ cm}}{M_{\text{eye}}}$$

Solve: **(a)** The magnification of a telescope is

$$M = -\frac{f_{\text{obj}}}{f_{\text{eye}}} = -\frac{\dfrac{25 \text{ cm}}{M_{\text{obj}}}}{\dfrac{25 \text{ cm}}{M_{\text{eye}}}} = -\frac{M_{\text{eye}}}{M_{\text{obj}}}$$

The way to maximize the magnitude of this is to have $M_{\text{eye}} > M_{\text{obj}}$.

$$M = -\frac{5.0}{2.0} = -2.5$$

The magnification is usually given without the negative sign, so it is $2.5\times$.

(b) To achieve this we used the $2.0\times$ lens as the objective, which coincides with the text which says the objective should have a long focal length and the eyepiece a short focal length.

(c)

$$L = f_{\text{obj}} + f_{\text{eye}} = \frac{25 \text{ cm}}{M_{\text{obj}}} + \frac{25 \text{ cm}}{M_{\text{eye}}} = \frac{25 \text{ cm}}{2.0} + \frac{25 \text{ cm}}{5.0} = 17.5 \text{ cm}$$

Assess: This is not a very powerful telescope.

24.40. **Model:** Assume thin lenses and treat each as a simple magnifier with $M = 25\,\text{cm}/f$.

Visualize: Equation 24.10 gives the magnification of a microscope.

$$M = m_{obj}M_{eye} = -\frac{L}{f_{obj}}\frac{25\,\text{cm}}{f_{eye}}$$

Solve: **(a)** The more powerful lens $(4\times)$ with the shorter focal length should be used as the objective.

(b) Solve the equation above for L (drop the negative sign).

$$L = \frac{Mf_{obj}f_{eye}}{25\,\text{cm}} = \frac{(12)(\frac{25\,\text{cm}}{4})(\frac{25\,\text{cm}}{2})}{25\,\text{cm}} = 37.5\ \text{cm}$$

Assess: This is a long microscope tube.

24.43. **Model:** The width of the central maximum that accounts for a significant amount of diffracted light intensity is inversely proportional to the size of the aperture. The lens is an aperture that focuses light.

Solve: To focus a laser beam, which consists of parallel rays from $s = \infty$, the focal length needs to match the distance to the target: $f = L = 5.0$ cm. The minimum spot size to which a lens can focus is

$$w = \frac{2.44\lambda f}{D} \Rightarrow 5.0 \times 10^{-6} \text{ m } = \frac{2.44\left(1.06 \times 10^{-6} \text{ m}\right)\left(5.0 \times 10^{-2} \text{ m}\right)}{D} \Rightarrow D = 2.6 \text{ cm.}$$

24.45. **Visualize:** The angle subtended at the eye due to a circle of diameter d at a near point of 25 cm is $\alpha = d/25$ cm. The angle to the first dark minimum in a circular diffraction pattern is $\theta_1 = 1.22\lambda/D$, where $\lambda = \lambda_{air}/n$ is the wavelength of the light in the eye and D is the pupil diameter. To just barely see the circle as a circle the condition $\alpha = \theta_1$ must be met.

We are given $D = 2.0 \times 10^{-3}$ m and $\lambda = \lambda_{air}/n = 600$ nm$/1.33 = 450$ mm.

Solve: Equating α and θ_1 we have $d/25$ cm $= 1.22\lambda/D$, which may be solved for the diameter of the circle.

$$d = \frac{(25 \text{ cm})(1.22\lambda)}{D} = \frac{(25 \text{ cm})(1.22)(450 \times 10^{-9} \text{m})}{2.0 \times 10^{-3} \text{ m}} = 6.9 \times 10^{-5} \text{ m} = 0.069 \text{ mm}$$

Assess: The above number is about the size of a needle point.

24.48. **Visualize:** We'll start with Equation 24.15 and substitute in Equation 24.11 and an expression for α from Figure 24.14a.

We are given $f_{obj} = 1.6$ mm, $d_{min} = 400$ nm, $n = 1.0$, and $\lambda = 550$ nm.

Solve: Solve for D.

$$d_{min} = \frac{0.61\lambda}{\text{NA}} = \frac{0.61\lambda}{n\sin\alpha} = \frac{0.61\lambda}{n\sin\left(\tan^{-1}\dfrac{D/2}{f_{obj}}\right)} =$$

$$\sin\left(\tan^{-1}\frac{D/2}{f_{obj}}\right) = \frac{0.61\lambda}{n\,d_{min}}$$

$$\tan^{-1}\frac{D/2}{f_{obj}} = \sin^{-1}\frac{0.61\lambda}{n\,d_{min}}$$

$$\frac{D/2}{f_{obj}} = \tan\left(\sin^{-1}\frac{0.61\lambda}{n\,d_{min}}\right)$$

$$D = 2f_{obj}\tan\left(\sin^{-1}\frac{0.61\lambda}{n\,d_{min}}\right) = 2(1.6\text{ mm})\tan\left(\sin^{-1}\frac{0.61(550\text{ nm})}{(1.0)(400\text{ nm})}\right) = 4.9\text{ mm}$$

The diameter of the lens must exceed 4.9 mm.

Assess: A half centimeter is in the ballpark for microscope lens diameters.