Wednesday, September 30, 2009
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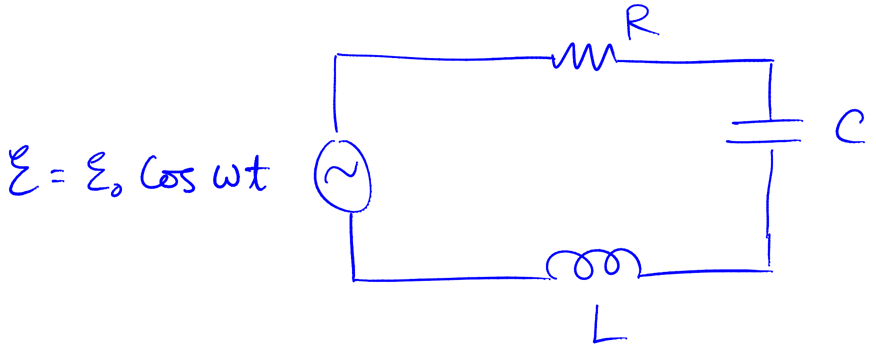
Solution to Problems 5 & 6

Phys 270

Problem 5 :

A series RLC circuit would look like



The Resonance Frequency of the circuit is given as

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

We are given that

$$\omega_0 > \omega$$

The current i is given as

$$i = I \cos(\omega t - \varphi)$$

where φ, which can be positive or negative, is

$$\varphi = \tan^{-1} \frac{X_L - X_C}{R} = \frac{V_L - V_C}{V_R}$$

Now $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$

$$\Rightarrow \varphi = \tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$= \tan^{-1} \frac{\omega L}{R}\left[1 - \frac{1}{(\omega L)(\omega C)}\right]$$

$$= \tan^{-1} \frac{\omega L}{R}\left[1 - \frac{1}{\omega^2 L C}\right]$$

Clearly, since

$\varphi < 0$, $X_L < X_C$

$\Rightarrow I X_L < I X_C$

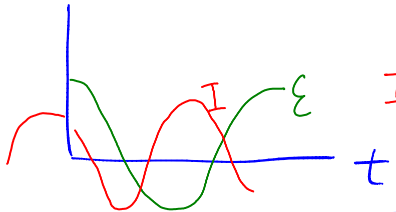
$\Rightarrow V_L < V_C$

$$= \tan^{-1} \frac{\omega L}{R}\left[1 - \frac{(1/\sqrt{LC})^2}{\omega^2}\right]$$

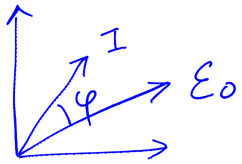
$$= \tan^{-1} \frac{\omega L}{R}\left[1 - \frac{\omega_0^2}{\omega^2}\right]$$

If $\omega_0 > \omega$, $1 - \frac{\omega_0^2}{\omega^2} < 0 \Rightarrow \varphi < 0$

Hence the current leads the EMF



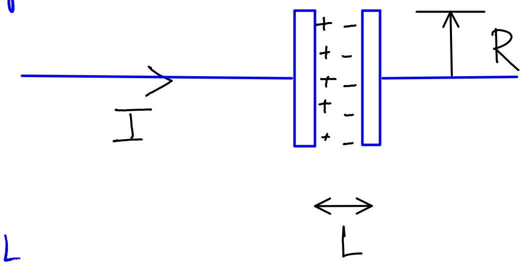
I lag ε



Problem 6 :

The charge on the capacitor is $Q = C \cdot V$

and the Electric-field inside is

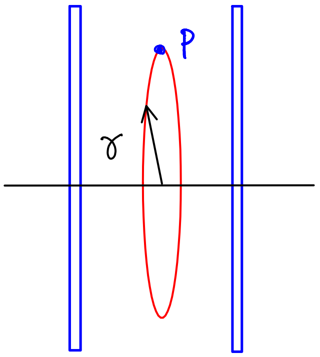


$$E = \frac{V}{L} \Rightarrow V = E \cdot L$$

$$\Rightarrow Q = C \cdot E \cdot L$$

Hence, the current $I = \frac{dQ}{dt} = C \cdot L \cdot \frac{dE}{dt}$ — ①

Now, let's consider a circular loop of radius r, passing thro' point P.



Now, we have

$$\int_{\text{loop}} \vec{B}\cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E^{\text{loop}}}{dt}$$

where Φ_E^{loop} is the flux of the electric field through the loop in picture.

$$\Rightarrow \quad B \cdot 2\pi\gamma = \mu_0\epsilon_0 \cdot \frac{d}{dt} E \cdot \pi r^2$$

$$= \mu_0\epsilon_0 \pi r^2 \frac{dE}{dt}$$

But from ①, $\frac{dE}{dt} = \frac{I}{CL}$

Hence

$$B \cdot 2\pi\gamma = \mu_0\epsilon_0\pi\gamma^2 \cdot \frac{I}{C\cdot L}$$

$$B = \frac{\mu_0\epsilon_0 I}{CL} \cdot \frac{\gamma}{2}$$

Now, for a parallel plate Capacitor,

$$C = \frac{\epsilon_0 A}{L} = \frac{\epsilon_0 \pi R^2}{L}$$

$$\Rightarrow \quad B = \frac{\mu_0 \cancel{\epsilon_0} I}{\frac{\cancel{\epsilon_0}\pi R^2}{\cancel{L}} \cdot \cancel{L}} \cdot \frac{\gamma}{2}$$

$$B = \frac{\mu_0 I}{2\pi} \frac{\gamma}{R^2}$$