NAME / Section #:	Exam I
____________________ / ________	Problem #1
	Phys270

1. [10 pts] Laboratory scientists have created the electric and magnetic fields shown below: the electric field points downward and to the right with magnitude 1.0×10^6 V/m, and the magnetic field is into the page with magnitude 1.0 T. These fields are also observed by scientists that zoom through these fields in a rocket traveling in the +x-direction at a speed of 1.0×10^6 m/s. [For the pedantic reader, assume the rocket is made of insulating material.]

$$\bar{E} = 1.0\times10^6 \frac{V}{m}\left(\hat{i}/\sqrt{2} - \hat{j}/\sqrt{2}\right),\ \bar{B} = 1.0T(-\hat{k}),\ \bar{V} = 10^6 \frac{m}{s}$$

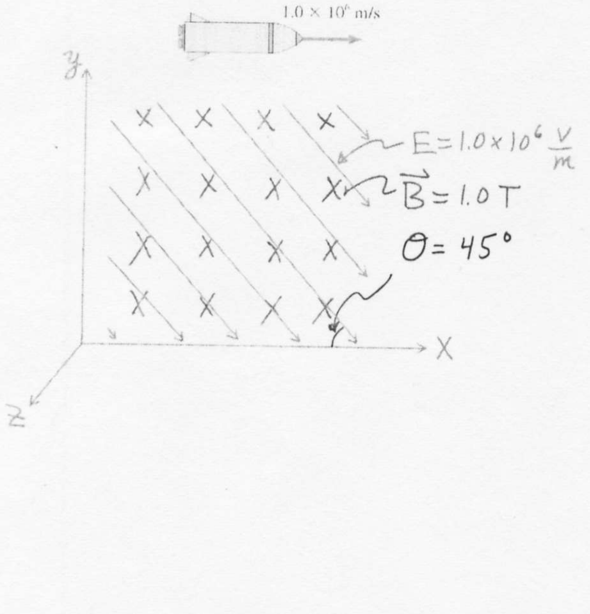
a. [5 pts] According to the scientists inside the rocket, what is the magnetic field expressed as a vector in x, y, and z components?

$$\bar{B}' = \bar{B} - \frac{\bar{V}\times\bar{E}}{c^2} = -1.0T\hat{k} - \frac{1}{c^2}\cdot 10^6\frac{m}{s}\hat{i} \times 10^6\frac{V}{m}\left(\hat{i}/\sqrt{2} - \hat{j}/\sqrt{2}\right)$$

$$= -1.0T\hat{k} + \frac{10^{12}}{c^2}\frac{V}{m}\cdot\hat{k}/\sqrt{2} = -1.0T\hat{k} + \frac{1}{9\sqrt{2}\times10^4}T\hat{k}, \text{ using } c = 3\times10^8$$

$$= -\left(1 - \frac{1}{9\sqrt{2}\times10^4}\right)T\hat{k}.$$

b. [5 pts] According to the scientists inside the rocket, what is the electric field expressed as a vector in x, y, and z components?



$$\bar{E}' = \bar{E} + \bar{V}\times\bar{B}$$

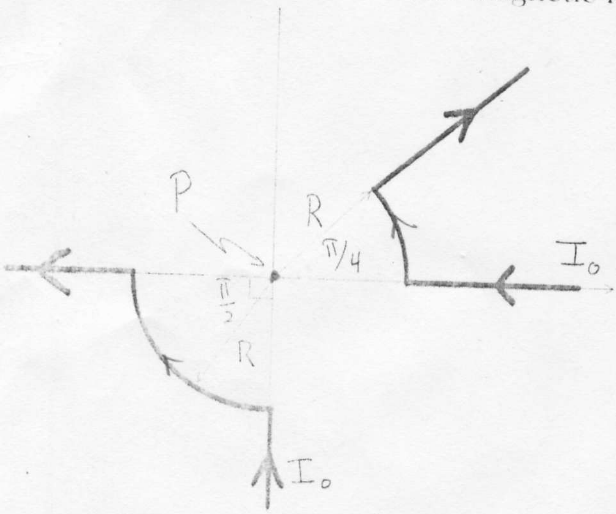
$$= 10^6\frac{V}{m}\left(\hat{i}/\sqrt{2} - \hat{j}/\sqrt{2}\right) + 10^6\frac{m}{s}\hat{i}\times 1.0T(-\hat{k})$$

$$= 10^6\frac{V}{m}\left(\hat{i}/\sqrt{2} - \hat{j}/\sqrt{2}\right) + 10^6\frac{V}{m}\hat{j}$$

$$= 10^6\frac{V}{m}\left(\hat{i}/\sqrt{2} + (1 - 1/\sqrt{2})\hat{j}\right).$$

NAME / Section #:	Exam I
SOLUTION / ______	Problem #2 Phys270

2. [20 pts] A current Io is present in the two circularly arced wire segments of radius R depicted below as thick black lines. One are subtends an angle of $\pi/2$, and the other subtends and angle of $\pi/4$. The lead wires point radially outward from point P, the location of the center of the two arced wire segments. Pay special attention to the direction of the current in the two arcs. Derive from the Biot-Savart law the magnitude and direction of the magnetic field at the point P.



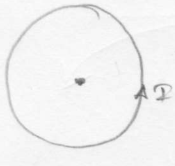
(2)

we can break each current carrying conductor into the curved part and the straight (radially outward) segment

(4) If we extend the radial portion of the wire then P lies on it, hence $\bar{B}$ due to the straight segment will be nil.

we need to find $\vec{B}$ due to curved part.

Using Biot Savart law $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{r}}{|\vec{r}|^3}$ (2)



current at the center of the circle of radius R will be (4)

$$\int dB = \frac{\mu_0 I}{4\pi} \frac{1}{R^2} \int dl = \frac{\mu_0 I}{2R}$$

Due to $\pi/4$ segment $B_{\pi/4} = \frac{\pi/4}{2\pi} \times \frac{\mu_0 I}{2r} = \frac{1}{8} \frac{\mu_0 I}{2r}$

Directed upward out of the paper (2)

Due to $\pi/2$ segment $B_{\pi/2} = \frac{\pi/2}{2\pi} \times \frac{\mu_0 I}{2r} = \frac{1}{4} \frac{\mu_0 I}{2r}$

Directed downward into the paper. (2)

Hence net ~~ctr~~ $\vec{B}$ is $\left(\frac{1}{4} - \frac{1}{8}\right) \frac{\mu_0 I}{2r} = \frac{\mu_0 I}{16r}$

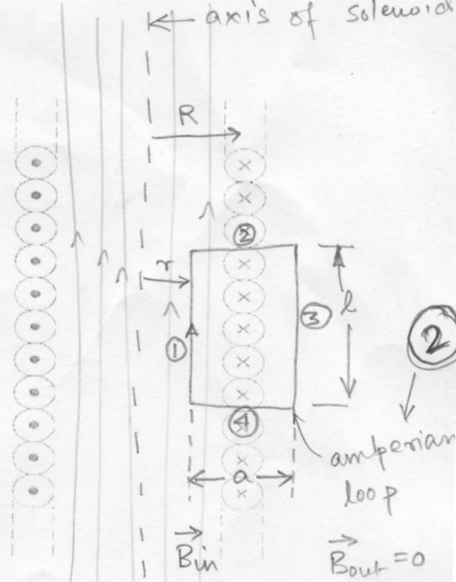
Direction is $\otimes$ into the plane of the paper (2)

Ans $\frac{\mu_0 I}{16r}$ (2)

NAME / Section #:	Exam I
SOLUTION. /	Problem #3
	Phys270

3. [30 pts] A solenoid with N turns, length L, and radius R has a current $I = I_0\,(1 + t/\tau)$ where τ and I_0 are both positive constants and t is time.

a. [8 pts] Derive an expression for the magnetic field inside the solenoid at a radius of r where r<R at time t=0. Make sure you draw your Amperian loop on the diagram (which depicts a cross sectional view of a solenoid with the current represented as in and out of the page) and describe in detail the integration around the loop to derive the B-field.



The amperian loop is shown in the diagram.

Let B inside the solenoid be $\vec{B}_{in} \neq 0$ since $I \neq 0$ and is uniform inside the solenoid $\parallel$ to the axis of the solenoid. $\vec{B}_{out} = 0$

Using amperes law

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{net} \quad \text{— (2)}$$

now $\vec{B}_{out} = 0$ and $\vec{B}_{in}$ is $\parallel$ to segment (1) but $\perp$ to (2) and (3)

(2) $\therefore \oint \vec{B}\cdot d\vec{s} = \int_{1} \vec{B}\cdot d\vec{s} + \int_{2} \overset{0}{\cancel{\vec{B}\cdot d\vec{s}}} + \int_{3} \overset{0}{\cancel{\vec{B}\cdot d\vec{s}}} + \int_{4} \overset{0}{\cancel{\vec{B}\cdot d\vec{s}}}$

$= Bl$

$I_{net} = l \times \frac{N}{L} I \qquad \therefore \quad B = \frac{\mu_0 N I_0}{L}$ at $t=0$ (2)

b. [2 pts] Draw the direction of the magnetic field inside the solenoid at time t=0 on the diagram above which depicts a cross sectional view of the solenoid.

Show the direction. (2)

NAME / Section #:	Exam I
SOLUTION /	Problem #3 (continued) Phys270

Problem #3 Continued ----

c. [15 pts] What is the magnitude of the electric field at radius r<R inside the same solenoid at some arbitrary time t>0?

Let us imagine that we had an imaginary circular wire of radius r and ⊥ to the solenoid axis [see figure below].

5 → There is a time varying ⊗ $\vec{B}$ field passing normally through the cross section of the wire.

for writing the maxwells equation. or stating the approach.

∴ The flux change inside the wire induces a voltage V_{in} in the wire which implies that there is a field $\vec{E}_{in}$ along the tangent of the wire such that $E_{in} \times 2\pi r = V$ since $|E| = \left|\frac{dV}{dl}\right|$ and by symmetry E_{in} is same at all points of the wire.

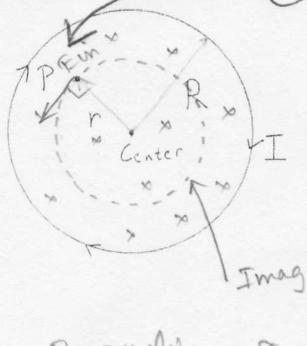
now using Faradays law $|V_{in}| = \left|-\frac{d\Phi_B}{dt}\right| = -\frac{d}{dt} B(t)\pi r^2 = \frac{d}{dt}\pi r^2 I_0 \left(1+\frac{t}{\tau}\right)\mu_0\frac{N}{L}$

or $2\pi r|E_{in}| = \pi r^2 I_0 \frac{1}{\tau}\frac{\mu_0 N}{L} \Rightarrow |E_{in}| = \frac{r I_0}{2\tau}\mu_0\frac{N}{L} = \mu_0 \frac{N}{L} r\frac{I_0}{2\tau}$

10

d. [5 pts] Draw the direction of the electric field at point P (at radius r) on the diagram below depicting a bottom view of a solenoid with the current moving clockwise. Explain your reasoning.

3



2 → If we had a loop of radius r then since $\vec{B}$ is increasing with t hence an anticlockwise current would be induced, since E_{in} drives current I_{in} hence $\vec{E}_{in}$ would be tangential and anticlockwise directing as shown in the diagram.

NAME / Section #:	Exam I
______________________________ / ________	Problem #4
	Phys270

4. [15 pts] An electromagnetic plane wave with wavelength λ and period T travels in the +z-direction in vacuum where the electric field is polarized in the x-direction and has an amplitude Eo.

a. [3 pts] Write a vector expression of the electric field for arbitrary time t and position z.

$$\bar{E}(x,y,z,t) = E_0 \hat{x} \cos\left(\frac{2\pi}{\lambda} z - \frac{2\pi}{T} t + \phi_0\right)$$

b. [3 pts] Write a vector expression of the magnetic field for arbitrary time t and position z.

$$\bar{B}(x,y,z,t) = E_0/c \; \hat{y} \; \cos\left(\frac{2\pi}{\lambda} z - \frac{2\pi}{T} t + \phi_0\right)$$

c. [4 pts] What is the magnitude and direction of the electric field vector at position x=0.3 λ, y=0.9 λ, and z= 0.1 λ at time t=0.3 T?

Assuming $\phi_0 = 0$, $E_0 \hat{x} \cos\left(2\pi \cdot 0.1 - 2\pi \, 0.3\right) = E_0 \hat{x} \cos(-.4\pi)$

$$= E_0 \hat{x} \cos(0.4\pi)$$

d. [5 pts] Assume that the above wave is incident on a perfectly absorbing circular plate of radius R oriented parallel to the x-y plane. Derive an expression for the total energy absorbed over a time interval equal to 10 periods, t=10.0 T.

$I = \frac{P}{A} = \frac{1}{2} \epsilon_0 E_0^2 c$ where $A = \pi R^2$ here

→ Energy absorbed in time 10T is $P \cdot 10T$

$$= I A \cdot 10T$$

$$= 5\pi\epsilon_0 c R^2 E_0^2 T$$