NAME / Section #: SOLUTION / _____	Exam III Problem #2 Phys270

2. [25 pts] A planet is 500 light years away from earth as measured in the earth's reference frame. An astronaut makes the one-way trip in 20 years as measured in the rocket's frame. The rocket has a rest mass of 10,000 kg. Assume the rocket quickly accelerates and decelerates so that the time required to do so may be ignored.

(a) [9 pts] How fast is the rocket moving as measured in earth's reference frame?

Let the velocity of rocket w.r.t Earth frame be v

Time taken by the rocket to travel to the planet, $\Delta t = \frac{500\ ly}{v}$ in Earth frame

$\therefore$ Time taken in rocket frame $\Delta t' = \frac{\Delta t}{\gamma} = \sqrt{1-\frac{v^2}{c^2}}\left(\frac{500\ ly}{v}\right)$

Substituting $\Delta t' = 20\ yrs$

we have $\left[\frac{(20\ yr)\ c}{500\ ly}\right]^2 = \frac{c^2-v^2}{v^2}$

$v = 0.9992c$ (Ans)

(b) [8 pts] How long does the trip take as measured by people in the earth's reference frame?

$\Delta t = \frac{500\ ly}{v} = \frac{500\ ly}{0.9992c} = 500.4$ years (Ans)

applying time dilation we would obtain the same value.

$\Delta t = \frac{20\ y}{\left[1-\frac{v^2}{c^2}\right]^{1/2}} = 500.4$ years (Ans)

(c) [8 pts] How much energy is required to accelerate the rocket to this speed (as found in part a)?

Lets assume that the rocket is at rest initially

$\therefore$ $E_{initial} = mc^2$ [m = rest mass of rocket = 10^4 kg]

$E_{final} = \gamma mc^2$

$\therefore$ Energy required to accelerate the rocket is,

$\Delta E = E_{final} - E_{initial} = (\gamma - 1)mc^2 = 24mc^2 = 2.16 \times 10^{22}$ Joules