$\vec{E}$, $\vec{D}$, $\vec{B}$ & $\vec{H}$: What do they all mean?

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POLARIZATION OF DIELECTRIC MEDIA

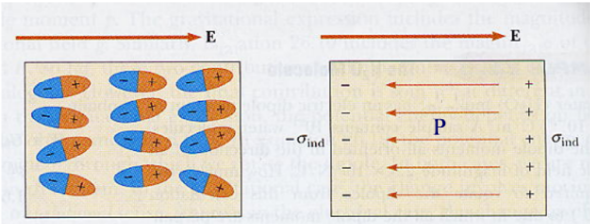

FIG. 1: Dielectric materials, which have no free electrons, can be polarized (internal dipole moments are induced) by an external electric field $\vec{E}$ resulting in a polarization field, $\vec{P}$ (the sum of all the induced dipoles). At internal locations, the positive and negative charges cancel leaving a net positive surface charge on the right and net negative surface charge on the left. The resultant polarization field, which points in the opposite direction to $\vec{E}$, is normalized by the volume of the material so that $\vec{P}$ has units of electric dipole-moment per unit volume. Figure after Fig. 26.23 of Serway and Beichner, 5^{th} edition.

When the applied electric field is weak and the medium is isotropic, $\vec{E}$ depends linearly on $\vec{P}$ and are parallel. Thus, we can write

$$\vec{P} = \chi_E(\vec{E})\vec{E}, \tag{1}$$

where $\chi_E(\vec{E})$ is a scalar, in this case, and is called the electric susceptibility. When the medium is not isotropic, such as a crystal, $\vec{P}$ and $\vec{E}$ are not parallel necessarily, and χ_E will be represented by a matrix (a second rank tensor). In the case were $\vec{E}$ is large, $\vec{P}$ can have a non-linear dependence on $\vec{E}$,

$$\vec{P} = \chi_E^{(1)}\vec{E} + \chi_E^{(2)}\vec{E}\vec{E} + \cdots, \tag{2}$$

where $\chi_E^{(1)}$ is the linear susceptibility from Eq. 1 and $\chi_E^{(2)}$ is the second-order nonlinear susceptibility. Depending on the strength of the field there could be higher terms as well. In what follows, we will look at the simplest case of an isotropic medium that responds linearly to the applied field.

THE ELECTRIC DISPLACEMENT, $\vec{D}$

The total field due to the polarization is given by adding $\vec{P}$ to the applied field $\vec{E}$. However, we cannot simply add $\vec{E}$ and $\vec{P}$ because they do not have the same units in MKS or the SI system of units. The polarization has units of C-m/m^3 $\equiv$ C/m^2, which means χ_E must have units of C^2/N-m^2. Since ϵ_o, the vacuum permittivity, also has units of C^2/N-m^2, we multiply $\vec{E}$ by ϵ_o and add the result to $\vec{P}$ to give a new quantity that bears the name the electric displacement,

$$\begin{aligned} \vec{D} &= \epsilon_o\vec{E} + \vec{P} & (3)\\ &= (\epsilon_o + \chi_E)\vec{E} & (4)\\ &= \epsilon\vec{E}, & (5) \end{aligned}$$

where ϵ is the permittivity of the medium. With this definition, $\chi_E \to 0$ and $\epsilon \to \epsilon_o$ in vacuum.

It is important to recognize that $\vec{E}$ is the fundamental field! If we write $\vec{E}$ as follows,

$$\vec{E} = \frac{1}{\epsilon_o}(\vec{D} - \vec{P}), \tag{6}$$

it becomes helpful to relate $\vec{D}$ with free charges, in the sense of Gauss' Law in electrostatics,

$$\oint \epsilon_o\vec{E}\cdot dA = \oint \vec{D}\cdot dA = Q, \tag{7}$$

where Q is the charge enclosed. The polarization, $\vec{P}$, is then related to the bound charges in the medium associated with the dipoles in Fig. 1.

DIELECTRIC CONSTANT

For media that responds linearly to applied fields, we usually do not use χ_E but a dimensionless quantity κ, the dielectric constant, which is related to the permittivity,[1]

$$\epsilon = \epsilon_o\kappa. \tag{8}$$

This leads to

$$\vec{D} = \kappa\epsilon_o\vec{E} = \epsilon\vec{E}, \tag{9}$$

where, for most media, $\kappa > 1$.

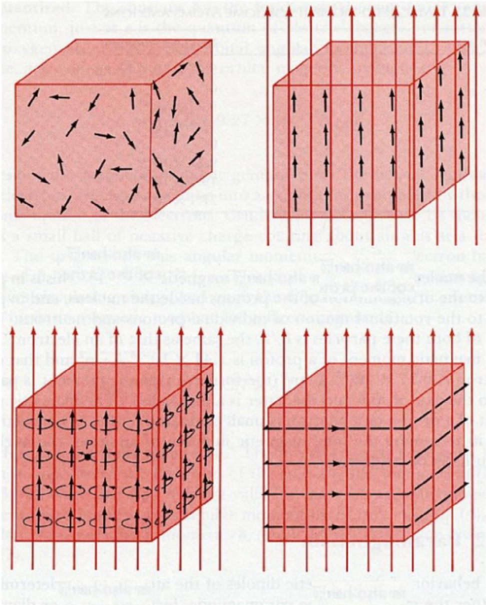

FIG. 2: The magnetization produced by an external field, $\vec{H}$, on a material with permanent dipole moments, upper images. This is an example of paramagnetic material where the magnetic susceptibility is positive and the magnetization, which has units of magnetic-dipole per unit volume, is aligned with the applied field. The lower left image shows the microscopic currents producing the permanent magnetic dipoles. As in dielectric material, the internal currents cancel leaving a net surface current producing the magnetization. Figure after Figs. 33.3 (top) and 33.4 of Ohanian Vol 2.

MAGNETIZATION OF MAGNETIC MEDIA

Magnetic media is in general more complicated than electronic media. We have three different conditions: ferromagnetism, paramagnetism and diamagnetism, in order of decreasing strength. Ferromagnetism and paramagnetism lead to a magnetization of the material upon the application of an external field due to the alignment of permanent dipoles. Diamagnetism, on the other hand, is similar to polarization in dielectric material in that the a magnetization is induced in the medium. Whereas paramagnetism and diamagnetism can be treated with the magnetic analog to Eq. 1,

$$\vec{M} = \chi_M(\vec{H})\vec{H}, \tag{10}$$

it is more appropriate to describe the $\vec{M}$ for ferromagnetic material as a strong function of $\vec{H}$. In Eq. 10, χ_M is the magnetic susceptibility. In contrast to χ_E, however, χ_M is dimensionless and can be both positive and negative.

THE FIELDS $\vec{H}$ AND $\vec{B}$

As in the electric case, we have two fields in the magnetic case. The quantity $\vec{H}$ plays the role of $\vec{D}$ for the magnetic case and is related to currents through Ampere's Law

$$\oint \frac{1}{\mu_o}\vec{B}\cdot d\vec{l} = \oint \vec{H}\cdot d\vec{l} = I + \epsilon_o \frac{d\Phi_E}{dt}, \tag{11}$$

where we see that $\vec{H}$ and $\vec{B}$ are related through the magnetic permeability. To be consistent with the electric case, $\vec{H}$ should be defined as μ times $\vec{B}$. However, the definition is

$$\vec{H} = \frac{\vec{B}}{\mu}, \tag{12}$$

where $\mu > 1$. In analogy to Eq. 6, $\vec{H}$ can also be written in terms of $\vec{M}$, the magnetization of the medium, and μ_o,

$$\vec{H} = \frac{1}{\mu_o}\vec{B} - \vec{M}. \tag{13}$$

We can see that the two terms on the right have the same units by doing a little dimensional analysis. From Table I we see that $\vec{B}$ has units of N/A-m while $\vec{M}$ has units of A-m^2/m^2 = A/m. Since μ_o has units of N/A^2, $\vec{M}$ and $\vec{B}/\mu_o$ have the same units.

Putting this all together we can write

$$\vec{B} = \mu_o(\vec{H} + \chi_M\vec{H}) \tag{14}$$
$$= \mu_o(1+\chi_M)\vec{H}. \tag{15}$$

It is clear when we compare Eqs. 12 and 14 that

$$\mu = \mu_o(1+\chi_M), \tag{16}$$

where paramagnetic materials have positive χ and diamagnetic material have negative χ. Thus, $\vec{M}$ points in the opposite direction in the two cases.

TABLE I: The names and units of the six electromagnetic fields: $\vec{E}$, $\vec{D}$, $\vec{P}$, $\vec{B}$ $\vec{H}$ and $\vec{M}$.

Symbol	Name	Units
$\vec{E}$	Electric Field	V/m = N/C
$\vec{P}$	Polarization	C/m^2
$\vec{D}$	Electric Displacement	C/m^2
$\vec{B}$	Magnetic Induction	N/A-m
$\vec{M}$	Magnetization	A/m
$\vec{H}$	Magnetic Intensity	A/m

SUMMARY

The six fields along with their names and units are summarized in Table I. The literature can be a little sloppy in referring to $\vec{B}$ and $\vec{H}$. Often $\vec{H}$ is called the magnetic field but it is not uncommon to see that name used for $\vec{B}$ as well. Regardless of the names, just as $\vec{E}$

is the true mean field, it should be understood that $\vec{B}$ is the true mean field!

[1] The quantity κ is related to the index of refraction of a material and will be used later in the semester when we talk about electromagnetic radiation and optics.