Solutions to hw 13

1) $K = \frac{p^2}{2m}$, $p = \frac{h}{\lambda} \Rightarrow \lambda = \frac{h}{\sqrt{2mK}} = \frac{hc}{\sqrt{2mc^2 K}}$

a) $m_n c^2 = 939.6$ MeV, $\hbar c = 197.3$ eV nm $\Rightarrow \lambda = 2.86 \times 10^{-11}$ m $\left(\frac{K}{eV}\right)^{-1/2}$

b) $K_n = 1.4 \times 10^3$ eV $\Rightarrow \lambda = 764$ fm

2) $\Delta x \Delta p \gtrsim \frac{\hbar}{2} \Rightarrow \Delta x \gtrsim \frac{\hbar}{2m\Delta v}$ $\qquad \hbar = 1.05 \times 10^{-34}$ J s

$\Delta v = 475 \times 10^{-4}$ m/s, $\quad m = 9.11 \times 10^{-31}$ kg $\Rightarrow \Delta x = 1.22$ mm $\quad$ electron

$m = 0.05$ kg $\Rightarrow \Delta x = 2.2 \times 10^{-32}$ m $\quad$ bullet

3) $P = \int_{-a}^{a} dx\, |\psi(x)|^2 = \frac{a}{\pi} \int_{-a}^{a} \frac{dx}{a^2+x^2} = \frac{1}{\pi} \arctan\left(\frac{x}{a}\right)\Big|_{-a}^{a} = \frac{1}{2}$

4) $K_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2 = n^2 K_1$

box width $L = 0.15$ nm, $m_e c^2 = 0.511$ MeV $\Rightarrow K_1 = 16.71$ eV

a) $K_n = n^2 K_1 \Rightarrow K = \{16.71, 66.85, 150.4, 267.4, \ldots\}$ eV

b) $E = \frac{hc}{\lambda} \Rightarrow \lambda_{fi} = \frac{hc}{K_1}\left(n_f^2 - n_i^2\right)^{-1} = 74.2\,\text{nm} \times \left(n_f^2 - n_i^2\right)^{-1}$

for these numbers: $\lambda_{4\to3} = 10.6$ nm, $\lambda_{4\to2} = 6.18$ nm, $\lambda_{4\to1} = 4.95$ nm

$\lambda_{3\to2} = 14.8$ nm, $\lambda_{3\to1} = 9.27$ nm, $\lambda_{2\to1} = 24.7$ nm

5) $E_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2 = n^2 E_1$

$L = 40$ fm, $m_p c^2 = 938.27$ MeV $\Rightarrow E_1 = 0.128$ MeV

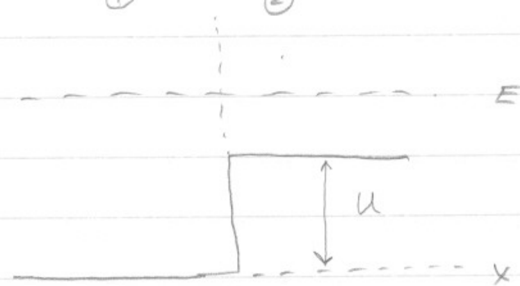
$E_2 - E_1 = 3E_1 = 0.384$ MeV $\Rightarrow \lambda = hc/\Delta E = 3.23 \times 10^3$ fm $= 3.23 \times 10^{-3}$ nm

gamma ray

6) Use the approximate tunneling probability supplied in Eqs (41.17-41.18): $T \approx e^{-2\kappa L}$, $\hbar\kappa = \sqrt{2m(U-E)}$

my numbers: $E = 3.2$ eV, $U = 3.8$ eV, $L = 0.95$ nm $\Rightarrow T = 5.31 \times 10^{-4}$

7) Unfortunately, the hint supplied with this problem is not that helpful. Traveling waves should be represented by complex exponential instead of cosines.



$\psi_1 = Ae^{ik_1x} + Be^{-ik_1x}$ $\quad \hbar k_1 = \sqrt{2mE}$

$\psi_2 = C\,e^{ik_2x}$ $\quad \hbar k_2 = \sqrt{2m(E-U)}$

$\psi_1(0) = \psi_2(0) \Rightarrow A + B = C$

$\psi_1'(0) = \psi_2'(0) \Rightarrow k_1(A-B) = k_2 C$

multiply top by k_2 and subtract $\Rightarrow (k_2-k_1)A + (k_2+k_1)B = 0$

multiply top by k_1 and add $\Rightarrow 2k_1 A = (k_1+k_2)C$

$$\therefore \quad R = \left|\frac{B}{A}\right|^2 = \left(\frac{k_1-k_2}{k_1+k_2}\right)^2$$

$$T = 1 - R = \frac{4k_1k_2}{(k_1+k_2)^2}$$

Note that the transmission probability T is not just $|C/A|^2$ because the particle velocity is smaller in region 2 than in region 1. The particle flux is proportional to k. Thus, $T = \frac{k_2}{k_1}\left|\frac{C}{A}\right|^2$. Also note that for this problem the mass divides out for both R and T.

my numbers: $E = 7.9$ eV, $U = 6.2$ eV $\Rightarrow R = 0.134$, $T = 0.866$

8) $\Delta E\,\Delta t \gtrsim \frac{\hbar}{2}$, $\quad E = mc^2 \Rightarrow \frac{\Delta m}{m} \gtrsim \frac{\hbar}{2mc^2\Delta t}$

my numbers: $mc^2 = 144$ MeV, $\Delta t = 6.30\times10^{-17}$ s $\Rightarrow \frac{\Delta m}{m} \gtrsim 3.6\times10^{-8}$

9a) $\Delta x\,\Delta p \gtrsim \frac{\hbar}{2} \Rightarrow \Delta p \gtrsim \hbar/2r$

b) $\langle K\rangle = \frac{\langle p^2\rangle}{2m} \sim \frac{(\Delta p)^2}{2m} \sim \hbar^2/2mr^2$

$\langle U\rangle = -k_E e^2 \langle \frac{1}{r}\rangle \approx -k_E e^2/r$

$\Bigg\}\; E = \hbar^2/2mr^2 - k_E e^2/r$

c) $\frac{\partial E}{\partial r} = 0 \Rightarrow -\frac{\hbar^2}{mr^3} + \frac{k_E e^2}{r^2} = 0 \Rightarrow r = \hbar^2/mk_E e^2$

$$E = \frac{\hbar^2}{2m}\left(\frac{mk_E e^2}{\hbar^2}\right)^2 - \frac{m(k_E e^2)^2}{\hbar^2} = -\frac{1}{2}\alpha^2 m_e c^2 = -13.6\text{ eV} \qquad \left(\alpha = \frac{k_E e^2}{\hbar c}\right)$$

agrees with Bohr theory