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phy260S08

Homework 8

Due at 11:00pm on Friday, April 4, 2008

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Heat Pumps and Refrigerators

Description: Simple questions to illustrate the concept and basic principles of heat pumps and refrigerators.

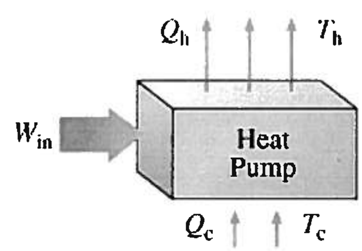
Learning Goal: To understand that a heat engine run backward is a heat pump that can be used as a refrigerator.

By now you should be familiar with heat engines--devices, theoretical or actual, designed to convert heat into work. You should understand the following:

1. Heat engines must be cyclical; that is, they must return to their original state some time after having absorbed some heat and done some work).
2. Heat engines cannot convert heat into work without generating some waste heat in the process.

The second characteristic is a rigorous result even for a perfect engine and follows from thermodynamics. A perfect heat engine is reversible, another result of the laws of thermodynamics.

If a heat engine is run backward (i.e., with every input and output reversed), it becomes a *heat pump* (as pictured schematically). Work W_{in} must be put into a heat pump, and it then pumps heat from a colder temperature T_c to a hotter temperature T_h, that is, against the usual direction of heat flow (which explains why it is called a "heat pump").



The heat coming out the hot side Q_h of a heat pump or the heat going in to the cold side Q_c of a refrigerator is *more than* the work put in; in fact it can be many times larger. For this reason, the ratio of the heat to the work in heat pumps and refrigerators is called the *coefficient of performance*, K. In a refrigerator, this is the ratio of heat removed from the cold side Q_c to work put in:

$$K_{frig} = \frac{Q_c}{W_{in}}.$$

In a heat pump the coefficient of performance is the ratio of heat exiting the hot side Q_h to the work put in:

$$K_{pump} = \frac{Q_h}{W_{in}}.$$

Take Q_h, and Q_c to be the *magnitudes* of the heat emitted and absorbed respectively.

Part A

What is the relationship of W_{in} to the work W done by the system?

Hint A.1 Note the differences in wording

Recall that W is the work done *by* the system; W_{in} is the work done *on* the system.

Express W_{in} in terms of W and other quantities given in the introduction.

ANSWER: $W_{\text{in}} =$ $-W$
$-W$

Part B

Find Q_{h}, the heat pumped out by the ideal heat pump.

Hint B.1 **Conservation of energy and the first law**

Apply conservation of energy. If you think in terms of the first law of thermodynamics, remember the sign conventions for heat and work and note that the internal energy does not change in an engine after one cycle.

Express Q_{h} in terms of Q_{c} and W_{in}.

ANSWER: $Q_{\text{h}} =$ $W_{\text{in}} + Q_c$
$Q_c + W_{in}$

Part C

A heat pump is used to heat a house in winter; the inside radiators are at T_{h} and the outside heat exchanger is at T_{c}. If it is a perfect (i.e., Carnot cycle) heat pump, what is K_{pump}, its coefficient of performance?

Part C.1 **Heat pump efficiency in terms of Q_{h} and Q_{c}**

What is the efficiency of a heat pump K_{pump} in terms of the heats in and out? Use the expression for the efficiency of the heat pump and the expression that you found involving the work done on the system, W_{in}, and the outside heats, Q_{h} and Q_{c}.

Give your answer in terms of Q_{h} and Q_{c}.

ANSWER: $K_{\text{pump}} =$ $\frac{Q_h}{Q_h - Q_c}$
$Q_h/(Q_h - Q_c)$

Hint C.2 **Relation between Q_{h} and Q_{c} in a Carnot cycle**

Recall that in a Carnot cycle,

$$\frac{Q_{\text{h}}}{Q_{\text{c}}} = \frac{T_{\text{h}}}{T_{\text{c}}}.$$

Give your answer in terms of T_{h} and T_{c}.

ANSWER: $K_{\text{pump}} =$ $\frac{T_h}{T_h - T_c}$
$1/(1 - T_c/T_h)$

Part D

The heat pump is designed to move heat. This is only possible if certain relationships between the heats and temperatures at the hot and cold sides hold true. Indicate the statement that must apply for the heat pump to work.

ANSWER:

- ◯ $Q_h < Q_c$ and $T_h < T_c$.
- ◯ $Q_h > Q_c$ and $T_h < T_c$.
- ◯ $Q_h < Q_c$ and $T_h > T_c$.
- ◉ $Q_h > Q_c$ and $T_h > T_c$.

Part E

Assume that you heat your home with a heat pump whose heat exchanger is at $T_c = 2^{\circ}\mathrm{C}$, and which maintains the baseboard radiators at $T_h = 47^{\circ}\mathrm{C}$. If it would cost $\$1000$ to heat the house for one winter with ideal electric heaters (which have a coefficient of performance of 1), how much would it cost if the actual coefficient of performance of the heat pump were 75% of that allowed by thermodynamics?

Hint E.1 **Money, heat, and the efficiency**

The amount of money one has to pay for the heat is directly proportional to the work done to generate the heat. Thus, the more efficient the heat generation the less work needs to be done and the lower the heating bill.

You are given that the cost of Q_h is $\$1000$. You also have an equation for $K_{\rm pump}$ in terms of the temperatures:

$$K_{\rm actual} = 75\% \text{ of } K_{\rm pump} = \frac{3}{4}\frac{T_h}{T_h - T_c}.$$

Set this equal to $Q_h/W_{\rm in}$ and solve for the monetary value of $W_{\rm in}$, the amount of external energy input the pump requires.

You can measure energies in units of currency for this calculation.

Hint E.2 **Units of T_h and T_c**

Keep in mind that when calculating an efficiency of a thermodynamic device you need to use temperature in kelvins. That is, $0^{\circ}\mathrm{C} = 273\mathrm{K}$.

Express the cost in dollars.

ANSWER: Cost = 187.5 dollars

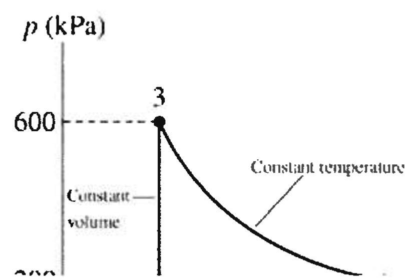
This savings is accompanied by more initial capital costs, both for the heat pump and for the generous area of baseboard heaters needed to transfer enough heat to the house without raising T_h, which would reduce the coefficient of performance. An additional problem is icing of the outside heat exchanger, which is very difficult to avoid if the outside air is humid and not much above zero degrees Celsius. Therefore heat pumps are most useful in temperate climates or where the heat Q_c can be obtained from a groundwater that is abundant or flowing (e.g., an underground stream).

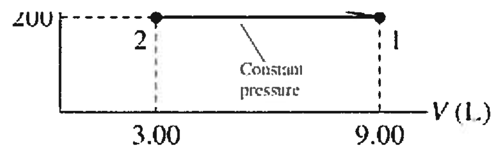
PSS 19.1: Turn Down the Volume!

Description: Compute the efficiency of a three-step (isobaric-isochoric-isothermic) cycle involving a monoatomic gas.(PSS 19.1: Heat-engine problems)

Learning Goal: To practice Problem-Solving Strategy 19.1 for problems involving heat engines.

A heat engine uses the closed cycle shown in the diagram:
. The working substance is n moles of monoatomic ideal gas. Find the efficiency η of such a cycle. Use the values for pressure and volume shown in the diagram and assume that the process between points 1 and 3 is isothermal.





MODEL: Identify each process in the cycle.

VISUALIZE: Draw the pV diagram of the cycle.

SOLVE: There are several steps in the mathematical analysis:

- Use the ideal gas law to complete your knowledge of n, p, V, and T at one point in the cycle.
- Use the ideal gas law and equations for specific gas processes to determine p, V, and T at the beginning and end of each process.
- Calculate Q, W_s, and ΔE_{th} for each process.
- Find W_{out} by adding W_s for each process in the cycle. If the geometry is simple, you can confirm this value by finding the area enclosed within the pV curve.
- Add just the *positive* values of Q to find Q_H.
- Verify that $(\Delta E_{th})_{net} = 0$. This is a self-consistency check to verify that you haven't made any mistakes.
- Calculate the thermal efficiency η and any other quantities you need to complete the solution.

ASSESS: Is $(\Delta E_{th})_{net} = 0$? Do all the signs of W_s and Q make sense? Does η have a reasonable value? Have you answered the question?

Model

First, identify each process in the cycle.

Part A

What are the processes between points 1 and 2 and between points 2 and 3, respectively?

ANSWER:
- ○ isochoric and isobaric
- ● isobaric and isochoric
- ○ isobaric and isothermal
- ○ isothermal and isochoric

Visualize

The pV diagram is already drawn. Copy the diagram and draw the arrows showing the direction of each process. Use your sketch to answer the following question.

Part B

To perform a *positive* amount of work, which cycle must the gas follow?

Hint B.1 How to approach the problem

For the work done by the gas to be positive, the average pressure must be greater during expansion than it is during compression. This condition is not satisfied for one of the two possible cycles.

ANSWER:
- ● $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$
- ○ $1 \rightarrow 3 \rightarrow 2 \rightarrow 1$

Make sure you understand this answer. For the work done by the gas to be positive, the average pressure must be greater during expansion than it is during compression. If the gas cycle were $1 \rightarrow 3 \rightarrow 2$, then the pressure would be greater during compression ($1 \rightarrow 3$) than it is during expansion ($2 \rightarrow 1$).

Solve

Conduct a mathematical analysis of the problem, following the outline given in the problem-solving strategy. Note that good "bookkeeping" will be essential.

Part C

Find the efficiency η of the heat engine.

Hint C.1 **Efficiency of a heat engine**

The efficiency of a heat engine is given by the ratio of the work output to the heat input: $\eta = W_{out}/Q_H$.

Part C.2 **Compute W_{out}**

Compute the work done by the gas, W_s, for each process. Add these to find W_{out}.

Part C.2.a **Processess that involve work output**

Which processes in the cycle involve a nonzero amount of work?

Check all that apply.

ANSWER:
- [x] $1 \rightarrow 2$
- [ ] $2 \rightarrow 3$
- [x] $3 \rightarrow 1$

Part C.2.b **Find the work done during process $1 \rightarrow 2$**

How much work is done by the gas during process $1 \rightarrow 2$?

Enter your answer in joules using the quantities given in the problem introduction. Use three significant figures in your answer.

ANSWER: $W_{12} = -1200$ J

Part C.2.c **Find the work done during process $3 \rightarrow 1$**

How much work is done by the gas during process $3 \rightarrow 1$?

Hint C.2.c.i **How to approach the problem**

To find the work done during an isothermal expansion, you need to integrate $\int_{V_{initial}}^{V_{final}} p\,dV$, holding temperature constant.

Hint C.2.c.ii **How to solve this problem when n and T are not given**

Leave n (the number of moles of gas) and T (the temperature of the gas during the process $1 \leftarrow 3$) as symbols in your calculation. You will find in the end that you need a numerical value for the product: nRT. Use the ideal gas law to compute this product in terms of the pressure and volume at either point 1 or point 3.

Enter your answer in joules using the quantities given in the problem introduction. Use three significant figures in your answer.

ANSWER: $W_{31} =$ 1980 J

1977.5

Express the work in joules using the quantities given in the problem introduction. Use three significant figures in your answer.

ANSWER: W_{out} = 780 / 777.5 J

Part C.3 **Compute Q_H**

Compute Q_H, the amount of heat delivered to the gas during the cycle.

Part C.3.a **Processess that involve heat input**

During which processes in the cycle is heat delivered to the engine?

Check all that apply.

ANSWER:
- [ ] $1 \rightarrow 2$
- [x] $2 \rightarrow 3$
- [x] $3 \rightarrow 1$

Heat is *removed* from the gas during process $1 \rightarrow 2$.

Part C.3.b **Compute Q_{23}**

Calculate the heat input during process $2 \rightarrow 3$.

Part C.3.b.i **Find the appropriate heat capacity**

The heat input for process $2 \rightarrow 3$ is given by an expression of the form $Q_{23} = n \times (\ ) \times \Delta T$, where n is the number of moles of the gas. What quantity should appear within the parentheses?

ANSWER:
- ○ $\frac{R}{2}$
- ◉ $\frac{3R}{2}$
- ○ $\frac{5R}{2}$
- ○ $\frac{7R}{2}$

For monoatomic gases the heat capacities at constant volume and pressure are $c_V = 3R/2$ and $c_p = 5R/2$, respectively. Since proceess $2 \rightarrow 3$ is *isochoric*, you need to use the heat capacity at constant volume.

Part C.3.b.ii **Find a formula for the temperature of the gas**

From the ideal gas law, what is the temperature of the gas?

ANSWER:
- ◉ $\frac{pV}{nR}$
- ○ $\frac{np}{RV}$

- $\frac{nR}{pV}$
- $\frac{nV}{pR}$

Part C.3.b.iii **Find a formula for Q_{23}**

Find the formula for the amount of heat Q_{23} delivered to n moles of monoatomic gas that increases its pressure from p_2 to p_3 while maintaining a constant volume V.

Express your answer in terms of p_2, p_3, and V.

ANSWER: $Q_{23} = \frac{3}{2}(p_3 - p_2)V$

Now use the values given in the problem introduction to solve for Q_{23} numerically.

Enter your answer in joules using the quantities given in the problem introduction. Use three significant figures in your answer.

ANSWER: $Q_{23} =$ 1800 J

Part C.3.c **Compute Q_{31}**

Find the heat input during process $3 \rightarrow 1$.

Part C.3.c.i **Find the relationship between W and Q for an isothermal process**

Which of the following is the correct relationship between the work done by the gas W and the amount of heat delivered to the gas Q for an isothermal process?

ANSWER:
- $Q = W + nRT$
- $Q = W - nRT$
- $W = Q + nRT$
- $W = Q + nRT$
- (selected) $Q = W_s$
- $Q = -W_s$

In an isothermal process, the temperature (and therefore the internal energy of the gas) remains the same. All of the heat input during an isothermal process is converted into work done by the gas. Solve for the work done by the gas during process $3 \rightarrow 1$. The heat input will then be given by $Q_{31} = W_{31}$.

Enter your answer in joules using the quantities given in the problem introduction. Use three significant figures in your answer.

ANSWER: $Q_{31} =$ 1980 / 1977.5 J

Enter your answer in joules using the quantities given in the problem introduction. Use three significant figures in your answer.

ANSWER: Q_H = 3780 3777.5 J

Express the efficiency numerically to three significant figures.

ANSWER: η = 0.206

The efficiency is often quoted as a percentage. In this case you would say that the engine is 20.6% efficient.

Assess

When you work on a problem on your own, without the computer-provided feedback, only you can assess whether your answer seems right. The following questions will help you practice the skills necessary for such an assessment.

Part D

Here is a table of energy transfers for this problem, with some entries missing:

Process	W_s (J)	Q (J)	ΔE_{th} (J)
$1 \rightarrow 2$	-1200	?	?
$2 \rightarrow 3$	0	1800	1800
$3 \rightarrow 1$	1980	1980	0

What must be the heat input in process $1 \rightarrow 2$ to satisfy the condition that $(\Delta E_{th})_{net} = 0$?

Express your answer numerically to two significant figures.

ANSWER: Q_{12} = -3000 J

Part E

Which of the following are reasonable efficiencies for a realistic heat engine?

Check all that apply.

ANSWER:
- [x] 9%
- [x] 19%
- [ ] 99%
- [ ] 109%

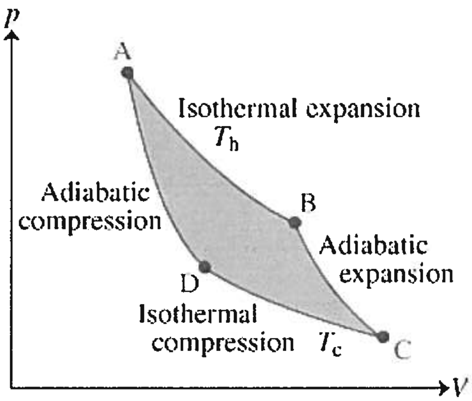
Although a 99% efficient heat engine is theoretically possible, it would be essentially impossible to construct such a device using existing technology.

Carnot Cycle

Description: Derive the efficiency of Carnot cycle: find the heat absorbed and emitted during the isothermal processes; show that Q_h/Q_c=T_h/T_c; then find the efficiency in terms of temperatures of the hot and cold reservoirs.

After Count Rumford (Benjamin Thompson) and James Prescott Joule had shown the equivalence of mechanical energy and heat, it was natural that engineers believed it possible to make a "heat engine" (e.g., a steam engine) that would convert heat completely into mechanical energy. Sadi Carnot considered a hypothetical piston engine that contained n moles of an ideal gas, showing first that it was reversible, and most importantly that—regardless of the specific heat of the gas—it had limited efficiency, defined as $e = W/Q_h$, where W is the net work done by the engine and Q_h is the quantity of heat put into the engine at a (high) temperature T_h. Furthermore, he showed that the engine must necessarily put an amount of heat Q_c back into a heat reservoir at a lower temperature T_c.

The cycle associated with a Carnot engine is known as a *Carnot cycle*. A pV plot of the Carnot cycle is shown in the figure. The working gas first expands isothermally from state A to state B, absorbing heat Q_h from a reservoir at temperature T_h. The gas then expands adiabatically until it reaches a temperature T_c, in state C. The gas is compressed isothermally to state D, giving off heat Q_c. Finally, the gas is adiabatically compressed to state A, its original state.



Part A

Which of the following statements are true?

Hint A.1 **Heat flow in an adiabatic process**

An *adiabatic* process is one in which heat does not flow into or out of the gas.

Check all that apply.

ANSWER:

- [x] For the gas to do positive work, the cycle must be traversed in a clockwise manner.
- [ ] Positive heat is added to the gas as it proceeds from state C to state D.
- [x] The net work done by the gas is proportional to the area inside the closed curve.
- [ ] The heat transferred as the gas proceeds from state B to state C is greater than the heat transferred as the gas proceeds from state D to state A.

Part B

Find the total work W done by the gas after it completes a single Carnot cycle.

Hint B.1 **How to approach the problem**

Find the total amount of heat added during the entire cycle and the change in internal energy of the gas over the entire cycle. Then apply the first law of thermodynamics: $dQ = dU + dW$.

Part B.2 **Compute the change in internal energy**

What is the net change in the gas's internal energy U after one complete cycle?

ANSWER: change in U = 0

Express the work in terms of any or all of the quantities Q_h, T_h, Q_c, and T_c.

ANSWER:

W = $Q_h - Q_c$

$Q_h\left(1 - \frac{T_c}{T_h}\right)$

Part C

Suppose there are n moles of the ideal gas, and the volumes of the gas in states A and B are, respectively, V_A and V_B. Find Q_h, the heat absorbed by the gas as it expands from state A to state B.

Hint C.1 General method of finding Q_h

First, find the work W_{AB} done by the gas as it expands from state A to state B. Then, use the first law of thermodynamics to relate W_{AB} to both Q_h and the change in the gas's internal energy. Finally, recall that this process is *isothermal*; what does this tell you about the change in the gas's internal energy?

Part C.2 Find the work done by the gas

What is W_{AB}, the work done by the gas as it expands from state A to state B?

Hint C.2.a How to find the work done by the gas

To find W_{AB}, the net work done by the gas as it proceeds from state A to state B, you need to integrate $dW = p\,dV$ from state A to state B.

Part C.2.b Express p in terms of V

Use the ideal gas equation of state to find an expression for the pressure p in terms of n, R, T_h, and V.

ANSWER: $p(V) = \frac{nRT_h}{V}$

Part C.2.c Express the integral over dV

Given that $p(V) \propto 1/V$ and that

$$W_{AB} = \int_{V_A}^{V_B} p(V)\,dV,$$

what is the integral $\int_{V_A}^{V_B} dV/V$?

Express your answer in terms of V_A and V_B.

ANSWER: $\int_{V_A}^{V_B} \frac{dV}{V} = \ln\left(\frac{V_B}{V_A}\right)$

Express the work in terms of n, V_A, V_B, the temperature of the hot reservoir T_h, and the gas constant R.

ANSWER: $W_{AB} = nRT_h \ln\left(\frac{V_B}{V_A}\right)$

Hint C.3 Relation between Q_h and W_{AB}

Because the process is isothermal, the internal energy of the gas does not change. Therefore, the work done by the gas will equal the net heat flow into the gas: $Q_h = W_{AB}$.

Express the heat absorbed by the gas in terms of n, V_A, V_B, the temperature of the hot reservoir, T_h, and the gas constant R.

ANSWER: $Q_{\rm h} = nRT_h \ln\left(\frac{V_B}{V_A}\right)$

Part D

The volume of the gas in state C is V_C, and its volume in state D is V_D. Find Q_c, the magnitude of the heat that flows out of the gas as it proceeds from state C to state D.

Hint D.1 How to approach the problem

First, find the work $W_{\rm CD}$ done by the gas as it expands from state C to state D. Then, use the first law of thermodynamics to relate $W_{\rm CD}$ to both Q_c and the change in the gas's internal energy. Finally, recall that this process is *isothermal*; what does this tell you about the change in the gas's internal energy?

Express your answer in terms of n, V_C, V_D, T_c (the temperature of the cold reservoir), and R.

ANSWER: $Q_c = nRT_c \ln\left(\frac{V_C}{V_D}\right)$

Observe that the three parts together imply that $W_{\rm BC} + W_{\rm DA} = 0$. This is because BC and DA are adiabatic processes. So using the first law, $W_{\rm BC} = -\Delta U_{\rm BC} = -nC_V(T_{\rm h} - T_c)$, whereas $W_{\rm DA} = -\Delta U_{\rm DA} = -nC_V(T_c - T_{\rm h})$. So $W_{\rm BC} = -W_{\rm DA}$, or $W_{\rm BC} + W_{\rm DA} = 0$. This is a general result: Any two adiabatic processes operating between the same two temperatures result in the same amount of work, regardless of the pressure and volume differences.

Part E

Now, by considering the adiabatic processes (from B to C and from D to A), find the ratio $V_{\rm C}/V_{\rm D}$ in terms of V_A and V_B.

Hint E.1 How to approach the problem

Suppose the ratio of the gas's specific heats is denoted by $\gamma = C_p/C_v$. Along adiabatic curves, you know that $pV^\gamma = C$, where C is some constant. Rewrite this equation in terms of T and V instead of p and V. Then use this new equation to relate the temperature and volume at the end points of the two adiabatic legs of the Carnot cycle. This will give you two equations that you can solve for $V_{\rm C}/V_{\rm D}$.

Part E.2 Rewrite pV^γ in terms of T and V

Use the ideal gas equation of state to eliminate p from the expression pV^γ.

Express your answer in terms of γ, n, R, and the temperature T.

ANSWER: $pV^\gamma = nRTV^{(\gamma-1)}$

Note that, since n is fixed and R is a constant, if $pV^\gamma = C$, then

$$TV^{(\gamma-1)} = C_2,$$

where $C_2 = C/(nR)$ is constant for an adiabatic process.

Part E.3 Express $T_{\rm h}$ and V_B in terms of T_c and V_C

States B and C are connected by an adiabatic expansion. Use the result found in the previous hint to find an expression for $T_{\rm h}V_{\rm B}^{(\gamma-1)}$.

Express your answer in terms of T_c, V_C, and γ.

ANSWER: $T_h V_B^{(\gamma-1)} = T_c V_C^{(\gamma-1)}$

Part E.4 **Express T_h and V_A in terms of T_c and V_D**

States D and A are connected by an adiabatic expansion. Use the result found in Hint 2 to find an expression for $T_h V_A^{(\gamma-1)}$.

Express your answer in terms of T_c, V_D, and γ.

ANSWER: $T_h V_A^{(\gamma-1)} = T_c V_D^{(\gamma-1)}$

Hint E.5 **Solving for V_C/V_D in terms of V_A and V_B**

Combine the results of the previous two hints, eliminating the temperatures T_h and T_c. (One way to do this is to divide one equation by the other.) This should allow you to solve for V_C/V_D in terms of V_A and V_B.

ANSWER: $V_C/V_D = \frac{V_B}{V_A}$

Part F

Using your expressions for Q_h and Q_c (found in Parts C and D), and your result from Part E, find a *simplified* expression for Q_c/Q_h.

No volume variables should appear in your expression, nor should any constants (e.g., n or R).

ANSWER: $Q_c/Q_h = \frac{T_c}{T_h}$

Part G

The efficiency of any engine is, by definition, $e = W/Q_h$. Carnot proved that no engine can have an efficiency greater than that of a Carnot engine. Find the efficiency e_{Carnot} of a Carnot engine.

Part G.1 **Express the efficiency in terms of Q_h and Q_c**

Using your result from Part B, find the efficiency e (of any engine) in terms of the engine's heat input and output.

Express your answer in terms of Q_h and Q_c.

ANSWER: $e = 1 - \frac{Q_c}{Q_h}$

Express the efficiency in terms of T_h and T_c.

ANSWER: $e_{Carnot} = 1 - \frac{T_c}{T_h}$

Because T_c is generally fixed (e.g., the cold reservoir for power plants is often a river or a lake), engineers, trying to increase efficiency, have always sought to raise the upper temperature T_h. This explains why (historically) there were some spectacular explosions of boilers used for steam power.

The Carnot Icemaker

Description: If a carnot engine creates some ice, calculate the heat rejected to the surrounding room and the energy that must be supplied.

An ice-making machine inside a refrigerator operates in a Carnot cycle. It takes heat from liquid water at 0.0 $°C$ and rejects heat to a room at a temperature of 24.7 $°C$. Suppose that liquid water with a mass of 70.4 kg at 0.0 $°C$ is converted to ice at the same temperature.

Take the heat of fusion for water to be $L_f = 3.34 \times 10^5 \ J/kg$.

Part A

How much heat $|Q_H|$ is rejected to the room?

Hint A.1 **How to approach the problem**

Calculate the heat absorbed by the refrigerator when the water freezes, and use this to calculate the heat rejected to the room using the equation for heat transfer in a Carnot engine. Remember to convert your temperatures into kelvins.

Part A.2 **Calculate the heat absorbed**

Calculate the heat $|Q_C|$ absorbed by the Carnot engine when the ice freezes.

Hint A.2.a **Equation for freezing**

If a liquid freezes into a solid, the magnitude of the heat lost is given by $|Q| = mL_f$.

Express your answer in joules to four significant figures.

ANSWER: $|Q_C| = mL_f$ J

Hint A.3 **Equation for heat transfer in a Carnot engine**

For a Carnot engine, even when used as a refrigerator, $|Q_C|/|Q_H| = T_C/T_H$, where $|Q_C|$ is the magnitude of the heat absorbed from or rejected to a cold room of temperature T_C, and $|Q_H|$ is the magnitude of the heat rejected to or absorbed from a hot room of temperature T_H.

Express your answer in joules to four significant figures.

ANSWER: $|Q_H| = \frac{mL_f(T)}{273.15}$ J

Part B

How much energy E must be supplied to the device?

Hint B.1 **How to approach the problem**

Since more heat is rejected into the room than is absorbed from the water freezing, the energy supplied to the device must account for the difference; otherwise energy would not be conserved.

Express your answer in joules.

19.8. Model: Assume that the car engine follows a closed cycle.
Solve: (a) Since 2400 rpm is 40 cycles per second, the work output of the car engine per cycle is

$$W_{\text{out}} = 500\ \frac{\text{kJ}}{\text{s}} \times \frac{1\ \text{s}}{40\ \text{cycles}} = 12.5\ \frac{\text{kJ}}{\text{cycle}}$$

(b) The heat input per cycle is calculated as follows:

$$\eta = \frac{W_{\text{out}}}{Q_H} \Rightarrow Q_H = \frac{12.5\ \text{kJ}}{0.20} = 62.5\ \text{kJ}$$

The heat exhausted per cycle is

$$Q_C = Q_H - W_{\text{in}} = 62.5\ \text{kJ} - 12.5\ \text{kJ} = 50\ \text{kJ}$$

19, 29

19.29. Model: The minimum possible value of T_C occurs with a Carnot refrigerator.
Solve: **(a)** For the refrigerator, the coefficient of performance is

$$K = \frac{Q_C}{W_{in}} \Rightarrow Q_C = KW_{in} = (5.0)(10\text{ J}) = 50\text{ J}$$

The heat energy exhausted per cycle is

$$Q_H = Q_C + W_{in} = 50\text{ J} + 10\text{ J} = 60\text{ J}$$

(b) If the hot-reservoir temperature is 27°C = 300 K, the lowest possible temperature of the cold reservoir can be obtained as follows:

$$K_{\text{Carnot}} = \frac{T_C}{T_H - T_C} \Rightarrow 5.0 = \frac{T_C}{300\text{ K} - T_C} \Rightarrow T_C = 250\text{ K} = -23°\text{C}$$

where $r_p = p_{max}/p_{min} = 10$.

19.59. Model: Process $1 \rightarrow 2$ of the cycle is isochoric, process $2 \rightarrow 3$ is isothermal, and process $3 \rightarrow 1$ is isobaric. For a monatomic gas, $C_V = \frac{3}{2}R$ and $C_P = \frac{5}{2}$ R.

Visualize: Please refer to Figure P19.59.

Solve: (a) At point 1: The pressure $p_1 = 1.0$ atm $= 1.013 \times 10^5$ Pa and the volume $V_1 = 1000 \times 10^{-6}$ m$^3 = 1.0 \times 10^{-3}$ m^3. The number of moles is

$$n = \frac{0.120 \text{ g}}{4 \text{ g/mol}} = 0.030 \text{ mol}$$

Using the ideal-gas law,

$$T_1 = \frac{p_1 V_1}{nR} = \frac{(1.013 \times 10^5 \text{ Pa})(1.0 \times 10^{-3} \text{ m}^3)}{(0.030 \text{ mol})(8.31 \text{ J/mol K})} = 406 \text{ K}$$

At point 2: The pressure $p_2 = 5.0$ atm $= 5.06 \times 10^5$ Pa and $V_2 = 1.0 \times 10^{-3}$ m^3. The temperature is

$$T_2 = \frac{p_2 V_2}{nR} = \frac{(5.06 \times 10^5 \text{ Pa})(1.0 \times 10^{-3} \text{ m}^3)}{(0.030 \text{ mol})(8.31 \text{ J/mol K})} = 2030 \text{ K}$$

At point 3: The pressure is $p_3 = 1.0$ atm $= 1.013 \times 10^5$ Pa and the temperature is $T_3 = T_2 = 2030$ K. The volume is

$$V_3 = V_2 \frac{p_2}{p_3} = (1.0 \times 10^{-3} \text{ m}^3)\left(\frac{5 \text{ atm}}{1 \text{ atm}}\right) = 5.0 \times 10^{-3} \text{ m}^3$$

(b) For isochoric process $1 \rightarrow 2$, $W_{1\rightarrow 2} = 0$ J and

$$Q_{1\rightarrow 2} = nC_V \Delta T = (0.030 \text{ mol})(\tfrac{3}{2}R)(2030 \text{ K} - 406 \text{ K}) = 607 \text{ J}$$

For isothermal process $2 \rightarrow 3$, $\Delta E_{th\ 2\rightarrow 3} = 0$ J and

$$Q_{2\rightarrow 3} = W_{2\rightarrow 3} = nRT_2 \ln\frac{V_3}{V_2} = (0.030 \text{ mol})(8.31 \text{ J/mol K})(2030 \text{ K})\ln\left(\frac{5.0 \times 10^{-3} \text{ m}^3}{1.0 \times 10^{-3} \text{ m}^3}\right) = 815 \text{ J}$$

For isobaric process $3 \rightarrow 1$,

$$W_{3\rightarrow 1} = p_3 \Delta V = (1.013 \times 10^5 \text{ Pa})(1.0 \times 10^{-3} \text{ m}^3 - 5.0 \times 10^{-3} \text{ m}^3) = -405 \text{ J}$$

$$Q_{3\rightarrow 1} = nC_P \Delta T = (0.030 \text{ mol})(\tfrac{5}{2})(8.31 \text{ J/mol K})(406 \text{ K} - 2030 \text{ K}) = -1012 \text{ J}$$

The total work done is $W_{net} = W_{1\rightarrow 2} + W_{2\rightarrow 3} + W_{3\rightarrow 1} = 410$ J. The total heat input is $Q_H = Q_{1\rightarrow 2} + Q_{2\rightarrow 3} = 1422$ J. The efficiency of the engine is

$$\eta = \frac{W_{net}}{Q_H} = \frac{410 \text{ J}}{1422 \text{ J}} = 28.8\%$$

(c) The maximum possible efficiency of a heat engine that operates between T_{max} and T_{min} is

$$\eta_{max} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{406 \text{ K}}{2030 \text{ K}} = 80\%$$

Assess: The actual efficiency of an engine is less than the maximum possible efficiency.

19.61. Model: The closed cycle in this heat engine includes adiabatic process $1 \to 2$, isobaric process $2 \to 3$, and isochoric process $3 \to 1$. For a diatomic gas, $C_V = \frac{5}{2}R$, $C_P = \frac{7}{2}R$, and $\gamma = \frac{7}{5} = 1.4$.

Visualize: Please refer to Figure P19.61.

Solve: (a) We can find the temperature T_2 from the ideal-gas equation as follows:

$$T_2 = \frac{p_2 V_2}{nR} = \frac{(4.0\times10^5\ \text{Pa})(1.0\times10^{-3}\ \text{m}^3)}{(0.020\ \text{mol})(8.31\ \text{J/mol K})} = 2407\ \text{K}$$

We can use the equation $p_2 V_2^\gamma = p_1 V_1^\gamma$ to find V_1,

$$V_1 = V_2\left(\frac{p_2}{p_1}\right)^{1/\gamma} = (1.0\times10^{-3}\ \text{m}^3)\left(\frac{4.0\times10^5\ \text{Pa}}{1.0\times10^5\ \text{Pa}}\right)^{1/1.4} = 2.692\times10^{-3}\ \text{m}^3$$

The ideal-gas equation can now be used to find T_1,

$$T_1 = \frac{p_1 V_1}{nR} = \frac{(1.0\times10^5\ \text{Pa})(2.692\times10^{-3}\ \text{m}^3)}{(0.020\ \text{mol})(8.31\ \text{J/mol K})} = 1620\ \text{K}$$

At point 3, $V_3 = V_1$ so we have

$$T_3 = \frac{p_3 V_3}{nR} = \frac{(4\times10^5\ \text{Pa})(2.692\times10^{-3}\ \text{m}^3)}{(0.020\ \text{mol})(8.31\ \text{J/mol K})} = 6479\ \text{K}$$

(b) For adiabatic process $1 \to 2$, $Q = 0$ J, $\Delta E_{\text{th}} = -W_s$, and

$$W_s = \frac{p_2V_2 - p_1V_1}{1-\gamma} = \frac{nR(T_2 - T_1)}{1-\gamma} = \frac{(0.020\ \text{mol})(8.31\ \text{J/mol K})(2407\ \text{K} - 1620\ \text{K})}{(1-1.4)} = -327.0\ \text{J}$$

For isobaric process $2 \to 3$,

$$Q = nC_P\Delta T = n(\tfrac{7}{2}R)(\Delta T) = (0.020\ \text{mol})\tfrac{7}{2}(8.31\ \text{J/mol K})(6479\ \text{K} - 2407\ \text{K}) = 2369\ \text{J}$$

$$\Delta E_{\text{th}} = nC_V\Delta T = n(\tfrac{5}{2}R)\Delta T = 1692\ \text{J}$$

The work done is the area under the p-versus-V graph. Hence,

$$W_s = (4.0\times10^5\ \text{Pa})(2.692\times10^{-3}\ \text{m}^3 - 1.0\times10^{-3}\ \text{m}^3) = 677\ \text{J}$$

For isochoric process $3 \to 1$, $W_s = 0$ J and

$$\Delta E_{\text{th}} = Q = nC_V\Delta T = (0.020\ \text{mol})(\tfrac{5}{2})(8.31\ \text{J/mol K})(1620\ \text{K} - 6479\ \text{K}) = -2019\ \text{J}$$

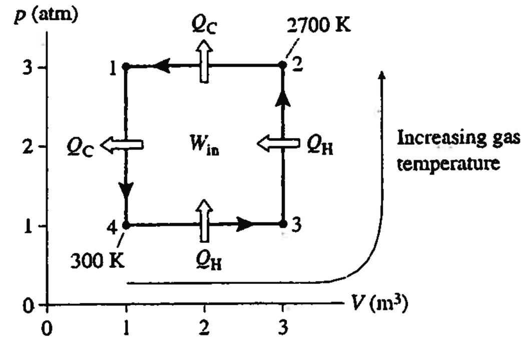
	ΔE_{th} (J)	W_s (J)	Q (J)
$1 \to 2$	327	−327	0
$2 \to 3$	1692	677	2369
$3 \to 1$	−2019	0	−2019
Net	0	350	350

(c) The engine's thermal efficiency is

$$\eta = \frac{350\ \text{J}}{2369\ \text{J}} = 0.148 = 14.8\%$$

19.64. Model: Processes 2 → 1 and 4 → 3 are isobaric. Processes 3 → 2 and 1 → 4 are isobaric.

Visualize:



Solve: **(a)** Except in an adiabatic process, heat must be transferred into the gas to raise its temperature. Thus heat transferred in during processes 4 → 3 and 3 → 2. This is the reverse of the heat engine in Example 19.2.

(b) Heat flows from hot to cold. Since heat energy is transferred into the gas during processes 4 → 3 and 3 → 2, which end with the gas at temperature 2700 K, the reservoir temperature must be $T > 2700$ K. This is the hot reservoir, so the heat transferred is Q_H. Similarly, heat energy is transferred out of the gas during processes 2 → 1 and 1 → 4. This requires that the reservoir temperature be $T < 300$ K. This is the cold reservoir, and the energy transferred during these two processes is Q_C.

(c) The heat energies were calculated in Example 19.2, but now they have the opposite signs.

$$Q_H = Q_{43} + Q_{32} = 7.09 \times 10^5 \text{ J} + 15.19 \times 10^5 \text{ J} = 22.28 \times 10^5 \text{ J}$$

$$Q_C = Q_{21} + Q_{14} = 21.27 \times 10^5 \text{ J} + 5.06 \times 10^5 \text{ J} = 26.33 \times 10^5 \text{ J}$$

(d) For a counterclockwise cycle in the pV diagram, the work is W_{in}. Its value is the area inside the curve, which is $W_{in} = (\Delta p)(\Delta V) = (2 \times 101{,}300 \text{ Pa})(2 \text{ m}^3) = 4.05 \times 10^5$ J. Note that $W_{in} = Q_C - Q_H$, as expected from energy conservation.

(e) Since $Q_C > Q_H$, more heat is exhausted to the cold reservoir than is extracted from the hot reservoir. In this device, work is used to transfer energy "downhill," from hot to cold. The exhaust energy is $Q_C = Q_H + W_{in} > Q_H$. This is the energy-transfer diagram of Figure 19.19.

(f) No. A refrigerator uses work input to transfer heat energy from the cold reservoir to the hot reservoir. This device uses work input to transfer heat energy from the hot reservoir to the cold reservoir.