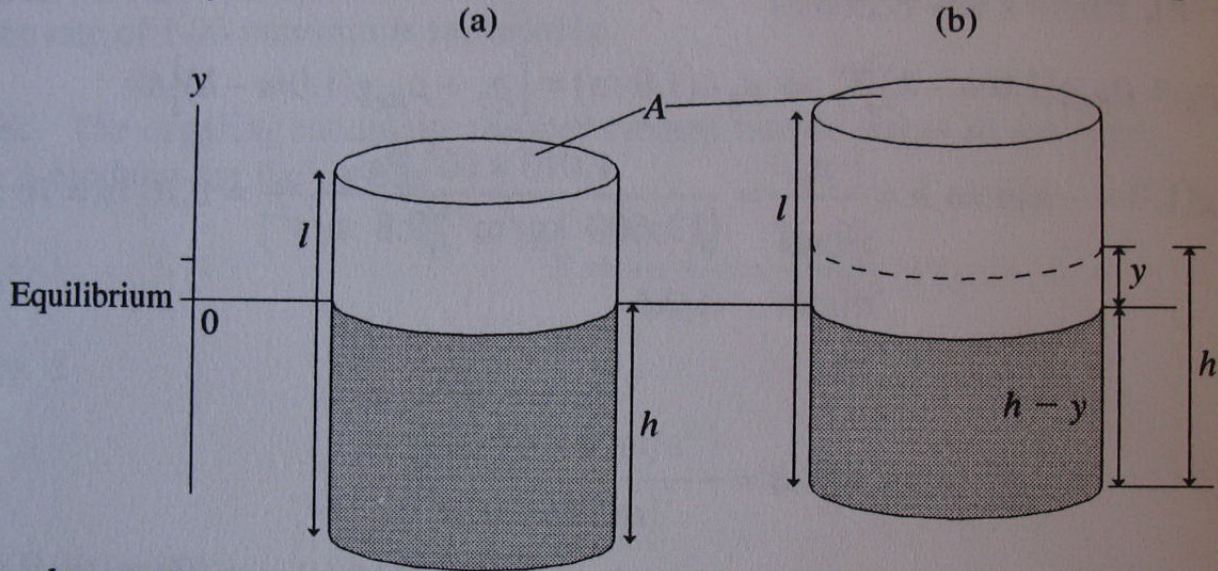**15.75. Model:** The buoyant force on the cylinder is given by Archimedes' principle.

**Visualize:**



(a)          (b)

$A$ is cross-sectional area of the cylinder.

**Solve:** (a) The cylinder's weight is $w = \rho_0(Al)g$ and the weight of the displaced fluid is $F_B = \rho_f(Ah)g$. In static equilibrium, $F_B = w$ and we can write

$$\rho_f Ahg = \rho_0 Alg \Rightarrow h = (\rho_0/\rho_f)l$$

(b) Now the volume of the displaced liquid is $A(h-y)$. Applying Newton's second law in the y-direction,

$$\sum F_y = -w + F_B = -\rho_0 Alg + \rho_f A(h-y)g$$

Using $\rho_f Ahg = \rho_0 Alg$ from part (a), we find

$$(F_{net})_y = -\rho_f Agy$$

(c) The result in part (b) is $F = -ky$, where $k = \rho_f Ag$. This is Hooke's law.

(d) Since the cylinder's equation of motion is determined by Hooke's law, the angular frequency for the resulting simple harmonic motion is $\omega = \sqrt{k/m}$, and the period is

$$T = \frac{2\pi}{\omega} = T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\rho_f Ag}} = 2\pi\sqrt{\frac{\rho_0 Al}{\rho_f Ag}} = 2\pi\sqrt{\frac{h}{g}}$$

where we have used the expression for $h$ from part (a).

(e) The oscillation period for the 100-m-tall iceberg ($\rho_{ice} = 917 \text{ kg/m}^3$) in sea water is

$$T = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{\rho_0 l}{\rho_f g}} = 2\pi\sqrt{\frac{(917 \text{ kg/m}^3)(100 \text{ m})}{(1030 \text{ kg/m}^3)(9.8 \text{ m/s}^2)}} = 18.9 \text{ s}$$

Course PHYSICS260

Assignment 2

Due at 11:00pm on Wednesday, February 13, 2008

## Relating Pressure and Height in a Container

**Description:** Walks the student through a derivation of the law relating height and pressure in a container by analyzing the forces on a thin layer of fluid.

**Learning Goal:** To understand the derivation of the law relating height and pressure in a container.

In this problem, you will derive the law relating pressure to height in a container by analyzing a particular system.

A container of uniform cross-sectional area $A$ is filled with liquid of uniform density $\rho$.
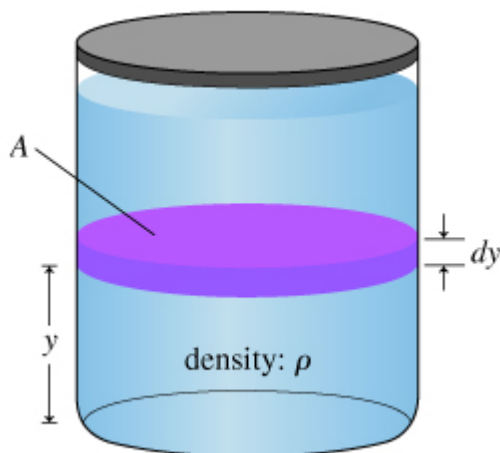


Consider a thin horizontal layer of liquid (thickness $dy$) at a height $y$ as measured from the bottom of the container. Let the pressure exerted upward on the bottom of the layer be $P$ and the pressure exerted downward on the top be $P + dp$. Assume throughout the problem that the system is in equilibrium (the container has not been recently shaken or moved, etc.).

### Part A

What is $F_{up}$, the magnitude of the force exerted upward on the bottom of the liquid?

**Hint A.1     Formula for the force**

Force is equal to pressure times area.

**ANSWER:**     $F_{up} = PA$

## Part B

What is $F_{down}$, the magnitude of the force exerted downward on the top of the liquid?

### Hint B.1    Formula for the force

Force is equal to pressure times area.

**ANSWER:**    $F_{down} = (p + dp)A$

## Part C

What is the weight $W_{layer}$ of the thin layer of liquid?

### Hint C.1  How to approach the problem

The weight of the layer is given by the formula $w = mg$, where $m$ is the mass of the layer and $g$ is the magnitude of the acceleration due to gravity.

### Part C.2  Mass of the layer

Use the definition of density to write the mass $m$ of the layer of liquid in terms of its density and its volume (express volume in terms of physical dimensions given in the problem introduction).

#### Hint C.2.a  Definition of density

The density of an object is equal to its mass divided by its volume.

#### Part C.2.b  Volume of the layer

What is the volume $dV$ of the thin layer of liquid?
Express your answer in terms of quantities given in the problem introduction.

**ANSWER:**    $dV = A\,dy$

Express your answer in terms of quantities given in the problem introduction.

**ANSWER:**    $m = \rho A\,dy$

Express your answer in terms of quantities given in the problem introduction and $g$, the magnitude of the acceleration due to gravity.

**ANSWER:**    $W_{layer} = \rho A\,dy\,g$

## Part D

Since the liquid is in equilibrium, the net force on the thin layer of liquid is zero. Complete the force equation for the sum of the vertical forces acting on the liquid layer described in the problem introduction.

### Hint D.1  How to approach the problem

If you have completed the previous parts, you have already done most of the work needed to answer this part. Just add together the forces that you found in the previous three parts. All three of the forces act along the *y* axis; some are directed

upward, and others are directed downward. Those that act downward should appear with a negative sign.

Express your answer in terms of quantities given in the problem introduction.

**ANSWER:** $0 = \sum_i F_{y,i} \quad pA - (p + dp)A - \rho A dy g$

## Part E

Solve the sum-of-forces equation just derived,

$$0 = \sum_i F_{y,i} = pA - (p + dp)A - \rho A g\, dy\,,$$

to obtain an expression for $dp$ and thus a differential equation for pressure.

**ANSWER:** $dp = -\rho g\, dy$

## Part F

Integrate both sides of the differential equation you found for $dp$ to obtain an equation for $P$. Your equation should then include a constant that depends on initial conditions. Determine the value of this constant by assuming that the pressure at some reference height $y_0$ is $P_0$.

### Part F.1 Integrate $dp$

What is the expression obtained by integrating the left-hand side of the equation $dp = -\rho g\, dy$? Although the indefinite integral of the left-hand side of the equation should include a constant determined by initial conditions, you can combine it with the constant on the right-hand side. Leave it out of your answer here.

**ANSWER:** $\int dp = p$

### Part F.2 Integrate $\rho g\, dy$

What is the expression obtained by integrating the right-hand side of the equation $dp = -\rho g\, dy$? Here you will need to include the constant determined by initial conditions; call it $C$.

**ANSWER:** $-\int \rho g\, dy = -\rho g y + C$

## Part F.3 Determine the constant of integration

According to the statement of the initial conditions, $P(y_0) = P_0$. Using this fact, find the value of the constant $C$.

Express your answer in terms of $P_0$, $\rho$, and other given quantities.
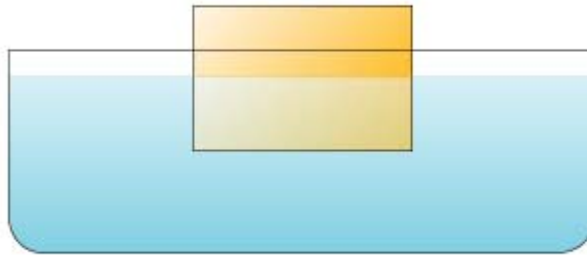
**ANSWER:** $U = P_0 + \rho g h_0$

Express your answer in terms of quantities given in the problem introduction along with $y$ and $P_0$.

**ANSWER:** $P = P_0 + \rho g (h_0 - y)$

# Buoyant Force Conceptual Question

**Description:** Conceptual question on the properties of a floating wooden block.

A rectangular wooden block of weight $W$ floats with exactly one-half of its volume



below the waterline.

**Part A**

What is the buoyant force acting on the block?

**Hint A.1  Archimedes' principle**

The upward buoyant force on a floating (or submerged) object is equal to the weight of the liquid displaced by the object.

Mathematically, the buoyant force $F_{buoyant}$ on a floating (or submerged) object is

$$F_{buoyant} = \rho g V,$$

where $\rho$ is the density of the fluid, $V$ is the submerged volume of the object, and $g$ is the acceleration due to gravity.

**Hint A.2  What happens at equilibrium**

The block is in equilibrium, so the net force acting on it is equal to zero.

**ANSWER:**
- ☐ $2W$
- ⦿ $W$
- ☐ $\frac{1}{2}W$
- ☐ The buoyant force cannot be determined.

## Part B

The density of water is 1.00 $g/cm^3$. What is the density of the block?

### Hint B.1 Applying Archimedes' principle

In Part A, you determined that the buoyant force, and hence the weight of the water displaced, is equal to the weight of the block. Notice, however, that the volume of the water displaced is one-half of the volume of the block.

### Hint B.2 Density

The density $\rho$ of a material of mass $m$ and volume $V$ is

$$\rho = \frac{m}{V}.$$

$$mg = F_{bouyant} \quad mg = F_{bouyant}$$

$$\rho_{water} g \frac{1}{2} V = \rho_{wood} g V$$

$$\frac{\rho_{water}}{\rho_{wood}} = 2$$

**ANSWER:**
- ☐ 2.00 $g/cm^3$
- ☐ between 1.00 and 2.00 $g/cm^3$
- ☐ 1.00 $g/cm^3$
- ☐ between 0.50 and 1.00 $g/cm^3$
- ⦿ 0.50 $g/cm^3$
- ☐ The density cannot be determined.

## Part C

Masses are stacked on top of the block until the top of the block is level with the waterline. This requires 20 $g$ of mass. What is the mass of the wooden block?

### Hint C.1 How to approach the problem

When you add the extra mass on top of the block, the buoyant force must change. Relating the mass to the new buoyant force will allow you to write an equation to solve for $m$.

## Part C.2  Find the new buoyant force

With the 20-$g$ mass on the block, twice as much of the block is underwater. Therefore, what happens to the buoyant force on the block?

$$F_{bouyant} = \rho_{water} g V_{under} \quad F_{bouyant} = \rho_{water} g V_{under}$$

**ANSWER:**   ◉  The buoyant force doubles.
 ☐  The buoyant force is halved.
 ☐  The buoyant force doesn't change.

## Hint C.3    System mass

Before the 20-$g$ mass is placed on the block, the mass of the system is just the mass of the block ($m$), and this mass is supported by the buoyant force. With the 20-$g$ mass on top, the total mass of the system is now $m + 20\,g$.

## Hint C.4  The mass equation

The original buoyant force was equal to the weight $W$ of the block. If the new buoyant force $F_b$ is twice the old buoyant force, then

$$F_b = 2W = 2mg,$$

where $m$ is the mass of the block.

The new buoyant force must support the weight of the block and the mass, so according to Newton's 2nd law

$$F_b = (m + 20)g.$$

By setting these two expressions for $F_b$ equal to each other, you can solve for $m$.

**ANSWER:**    ☐  40 $g$
 ◉  20 $g$
 ☐  10 $g$

## Part D

The wooden block is removed from the water bath and placed in an unknown liquid. In this liquid, only one-third of the wooden block is submerged. Is the unknown liquid more or less dense than water?

$$mg = \rho_{water} g V_{water} = \rho_{liquid} g V_{liquid}$$

$$\rho_{liquid} = \frac{\rho_{water} V_{water}}{V_{liquid}}$$

**ANSWER:** 
- ☉ more dense
- ☐ less dense

## ⬆☑Part E

What is the density of the unknown liquid $\rho_{unknown}$?

### Hint E.1     Comparing densities

From Part C, the mass of the block is 20 $g$. In water, one-half of the block is submerged, so one-half of the volume of the block times the density of the water must be equivalent to 20 $g$. In the unknown substance, one-third of the block is submerged, so one-third of the volume of the block times the density of the unknown substance must be equivalent to 20 $g$. You can use these observations to write an equation for the density of the unknown substance.

### ⬆☐Hint E.2  Setting up the equation

The buoyant force on the block in water $F_{b,water}$ is the same as the buoyant force in the unknown liquid $F_{b,unknown}$,

$$F_{b,water} = F_{b,unknown},$$

so

$$\rho_{water} g \left(\frac{V}{2}\right) = \rho_{unknown} g \left(\frac{V}{3}\right),$$

where $\rho_{water}$ is the density of water, $\rho_{unknown}$ is the density of the unknown liquid, $V$ is the volume of the entire block, and $g$ is the acceleration due to gravity.

Express your answer numerically in grams per cubic centimeter.

**ANSWER:**   $\rho_{unknown} = $  1.5   $g/cm^3$

# Problem 15.10

**Description:** A 20-cm-diameter circular cover is placed over a 10-cm-diameter hole that leads into an evacuated chamber. The pressure in the chamber is 20 kPa. (a) How much force is required to pull the cover off?

### Part A

How much force is required to pull the cover off?

$$F + F_{Chamber} = F_{air}$$
$$F + F_{Chamber} = F_{air}$$

$$F = (p_{air} - p_{chamber}) * A_{hole} = (101 - 20)kPa * \pi(\frac{0.1}{2})^2 m^2 = 639N$$

**ANSWER:** 639 $N$

# Problem 15.14

**Description:** (a) How far must a 2.0-cm-diameter piston be pushed down into one cylinder of a hydraulic lift to raise an 8.0-cm-diameter piston by 20 cm?

## Part A

How far must a 2.0-cm-diameter piston be pushed down into one cylinder of a hydraulic lift to raise an 8.0-cm-diameter piston by 20 cm?

$$V_1 = V_2$$

$$\pi \left(\frac{8.0cm}{2}\right)^2 * 20cm = \pi \left(\frac{2.0cm}{2}\right)^2 * S \quad V_1 = V_2$$

$$S = 320cm = 3.20m$$

**ANSWER:** 3.20 $m$

# Problem 15.22

**Description:** A 1.0-cm-diameter pipe widens to 2.0 cm, then narrows to 0.5 cm. Liquid flows through the first segment at a speed of 4.0 m/s. (a) What is the speed in the second segment? (b) What is the speed in the third segment? (c) What is the volume flow rate through the pipe?

## Part A

What is the speed in the second segment?

$$A_1 \Delta s_1 = A_2 \Delta s_2$$

$$A_1 \frac{\Delta s_1}{\Delta t} = A_2 \frac{\Delta s_2}{\Delta t}$$

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\pi(1.0cm/2)^2 * 4.0m/s}{\pi(2.0cm/2)^2} = 1.0m/s$$

**ANSWER:** 1.00 $m/s$

## Part B

What is the speed in the third segment?

$$v_3 = \frac{A_1 v_1}{A_3} = \frac{\pi (1.0cm/2)^2 * 4.0m/s}{\pi (0.5cm/2)^2} = 16.0m/s$$

**ANSWER:**   16.0   m/s

## Part C

What is the volume flow rate through the pipe?

$$\frac{\Delta V}{\Delta t} = Av = \pi (1.0cm/2)^2 * 4.0m/s = 314cm^3/s = 0.314L/s$$

**ANSWER:**   0.314   L/s

# Problem 15.54

**Description:** A cylinder with cross-section area A floats with its long axis vertical in a liquid of density rho.

## Part A

Pressing down on the cylinder pushes it deeper into the liquid. Find an expression for the force needed to push the cylinder distance $x$ deeper into the liquid and hold it there.

$$F + F_g = F_{bouyant1} + F_{bouyant2}$$

$$F_g = F_{bouyant1}$$

$$F = F_{bouyant2} = \rho Axg$$

**ANSWER:**   $\rho Axg$

## Part B

A 4.0-cm-diameter cylinder floats in water. How much work must be done to push the cylinder 10 cm deeper into the water?
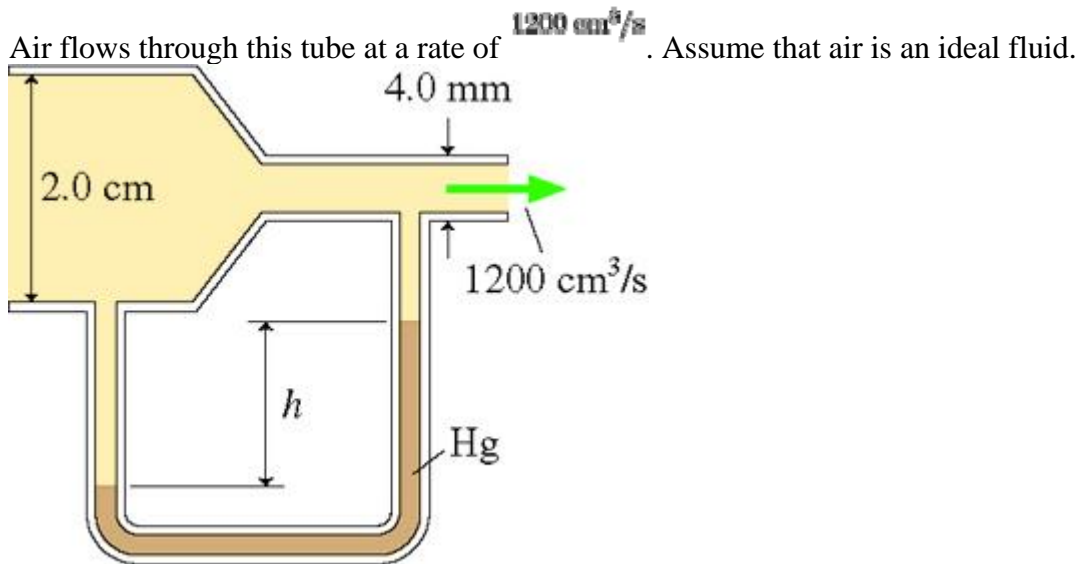
**Hint B.1   Hint**

An integration is required.

$$W = \int_0^{0.1} dxF = \int_0^{0.1} dx\rho Axg = 1.0*10^3 kg/m^3 \pi \left(\frac{0.04m}{2}\right)^2 9.8kg \cdot m^{-1} \cdot s^{-2} \int_0^{0.1} xdx = 0.616J$$

**ANSWER:**   0.616   J

# Problem 15.64

**Description:**

Air flows through this tube at a rate of $1200\ cm^3/s$ . Assume that air is an ideal fluid.



4.0 mm

2.0 cm

$h$

1200 cm³/s

Hg

## Part A

What is the height $h$ of mercury in the right side of the U-tube?

Bernoulli's Equation $p_1 + \dfrac{1}{2}\rho v_1^{\,2} = p_2 + \dfrac{1}{2}\rho v_2^{\,2}$

$A_1 v_1 = A_2 v_2$

$v_1 = \dfrac{A_2 v_2}{A_1} = \dfrac{1200 cm^3/s}{\pi\left(2.0cm/2\right)^2} = 3.82 m/s$

$v_2 = \dfrac{1200 cm^3/s}{A_2} = \dfrac{1200 cm^3/s}{\pi\left(4mm/2\right)^2} = 95.5 m/s$

$p_1 = p_2 + \rho_{Hg} g h$

$h = \dfrac{p_1 - p_2}{\rho_{Hg} g} = \dfrac{1/2\rho_{Hg}\left(v_2^{\,2}-v_1^{\,2}\right)}{\rho_{Hg} g} = \dfrac{1/2*1.2 kg/m^3 *\left((95.5 m/s)^2 - (3.82 m/s)^2\right)}{13.5*10^3 kg/m^3 *9.8 kg\cdot m\cdot s^{-1}} = 4.37 cm$

**ANSWER:** 4.37 cm

# Problem 15.63

**Description:** A hurricane wind blows across a a * b flat roof at a speed of v. (a) What is the pressure difference? (b) How much force is exerted on the roof?

A hurricane wind blows across a 6.10 m × 15.2 m flat roof at a speed of 190 km/hr .

## Part A

What is the pressure difference?

Bernoulli's Equation $p_1 + \dfrac{1}{2}\rho v_1^{\ 2} = p_2$

$\Delta p = \dfrac{1}{2}\rho v^2 = \dfrac{1}{2} * 1.28 kg/m^3 v^2 m^2/s^2 = \dfrac{1.28}{2}v^2 Pa$

**ANSWER:** $\dfrac{1.28v^2}{2}$ $Pa$

## Part B

How much force is exerted on the roof?

$F = \Delta p \cdot A = \dfrac{1.28}{2}v^2 ab \, N$

**ANSWER:** $\dfrac{1.28v^2 ab}{2}$ $N$