April 22, 2011 Physics 122 Prof. E. F. Redish

Theme Music: **Kraftwerk**

Ohm Sweet Ohm

- **Cartoon:** **Bob Thaves**

Frank & Ernest

The math of it

- To make our discussion of electric current quantitative, we have to
  - define what we mean by **current.**
  - describe the mechanism of the resistance (a **viscous drag**)
  - describe the force pushing the charges through the drag (an **electric field**).

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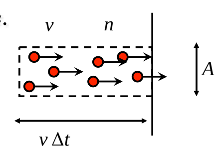
Quantifying current

- Consider a wire containing movable current carriers (electrons).
- Define the electric current as rate at which charge moves past a surface.

$$I = \frac{\Delta q}{\Delta t}$$



- The unit of current is the Ampere (= 1C/s)

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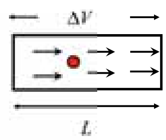
Pushing through the drag

- Consider an electron moving in a conductor with a uniform electric field
- By Newton 2, a constant flow (= constant v) means the forces balance.

$$ma = F^{net}$$
$$0 = qE - bv$$
$$qE = bv$$



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Building the current equation

- I =(charge) (number per unit volume) (Volume)/(time) = $qn(A\Delta x)/(\Delta t) = qnAv$
- Substituting into $v = (q/b)\,E$
  - (basically says that if the field in the wire is more, then the charges move faster)
- $I = (q^2/b)n\;A\;E$

material props. size How much push

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Rearrange

- Express E in terms of ΔV (easier to measure)
- Express v in terms of I (ditto).

$$\Delta V = EL \quad \Rightarrow \quad E = \frac{\Delta V}{L}$$

$$I = qnAv \quad \Rightarrow \quad v = \frac{I}{qnA}$$

$$\text{So} \quad qE = bv \quad \Rightarrow \quad \frac{q\Delta V}{L} = \frac{bI}{qnA}$$

$$\Delta V = I\left(\frac{bL}{q^2 nA}\right) \equiv IR$$

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What?! (Ohm's Law)

- Current proportional to velocity
- Due to resistance,
  Electric force proportional to velocity.
- Force proportional to "electric pressure drop" = "electric PE"
- Therefore, current proportional to "electric PE"

Like force down nail board proportional to *mgh*

$$\Delta V = IR$$

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For biologists only

- One way of reading Ohm's law is to say that the push divided by the resistance gives the current. ($\Delta V/R = I$)
- Instead of resistance, biologists often like to talk about how a material *facilitates* rather than *resists* current flow.
- They introduce the **conductance** $G = 1/R$. Then Ohm's law becomes $G\Delta V = I$.

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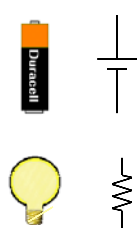
Electric circuit elements

- *Batteries* —devices that maintain a constant electrical pressure difference across their terminals (like a water pump that raises water to a certain height).
- *Resistance* —devices that have significant drag and oppose current. Pressure will drop across them.
- *Wires* — have very little resistance. We can ignore the drag in them (mostly – as long as there are other resistances present).



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The current rule

- The most useful result that carries over from the water flow analogy to the flow of electric current is:
- Kirchoff's current rule:
  - The total amount of current flowing into any point in a network equals the amount flowing out (there is no significant build-up of charge anywhere).

The Potential Rule

- Both the water flow and nail board analogies use gravity instead of electric force.They have the following property:
- Gravity potential rule:
  - Whenever you walk around a loop, however far you went up is equal to however far you went down. (You wind up at the same place.)
- Electric potential for electric forces is analogous to height (times g) for gravitational forces.
- Kirchoff's potential rule:
  - Around any loop the sum of the potential drops = the sum of the potential rises.

Foothold ideas (Kirchoff's Rules)

- Flow Rule
  - The total amount of current flowing into any point in a network equals the amount flowing out (no significant build-up of charge anywhere).
- Potential Rule
  - Following around any loop in an electrical network the potential has to come back to the same value (sum of drops = sum of rises).
- Ohm's Rule
  - When a current I passes through a resistance R, there is a voltage drop across the resistor of an amount $\Delta V = IR$

Very useful heuristic

- The Constant Potential Trick (CPT)
  - Along any part of a circuit with 0 resistance, then $\Delta V = 0$, i.e., the voltage is constant.

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Electric Power Dissipation

- We can figure out the energy needed to push the electrons through the material against the resistance using the W-E theorem.

$$P = \text{rate of doing work (using energy)} = \frac{\Delta W}{\Delta t}$$

$$= (\text{number of charges moved}) \times (\text{force}) \times (\text{distance moved in a time } \Delta t)$$

$$P = \frac{(nAL)(qE)(v\Delta t)}{\Delta t} = (nAL)qv\frac{\Delta V}{L} = (nAqv)\Delta V = I\Delta V$$

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$$P = I\,\Delta V$$

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Units of power

- Since the units of work (energy) is the Joule, the unit of power is the Joule/second.
  - 1 Watt = 1 Joule/second (definition)
- Our analysis shows that current x voltage = power.
- 1 Watt = 1 Ampere x 1 Volt

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