

■ **Theme Music: Antonio Jobim**

Wave

■ **Cartoon: Bob Thaves**

Frank & Ernest



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Outline

- Propagation of a pulse: What controls the speed of a pulse?
- Propagation of a pulse: What controls the width of a pulse in time?
- Superposition: How do waves combine?
- Waves: Foothold principles

Speed of a wave on a string

- The masses (m) in a string of beads of length (L) are pulled by the tensions (T) of the springs.
- The speed of the pulse must depend on these – and only these parameters.
- Can we create a velocity from these using dimensional analysis?

Dimensional analysis

- Square brackets are used to indicate a quantities dimensions
 - mass (\mathcal{M}), length (\mathcal{L}), or time (\mathcal{T})

- $[m] = \mathcal{M}$

- $[L] = \mathcal{L}$

- $[t] = \mathcal{T}$

- $[F] = \mathcal{M}\mathcal{L}/\mathcal{T}^2$



- Build a velocity using mass (m), length (L), and tension (T) of the string:

- $[v] = \mathcal{L}/\mathcal{T}$

- $[T] = \mathcal{M}\mathcal{L}/\mathcal{T}^2$

- $[T/m] = \mathcal{L}/\mathcal{T}^2$

- $[TL/m] = \mathcal{L}^2/\mathcal{T}^2$

$$v_0^2 = \frac{TL}{m}$$

or, using $\mu = m/L$ $v_0 = \sqrt{\frac{T}{\mu}}$

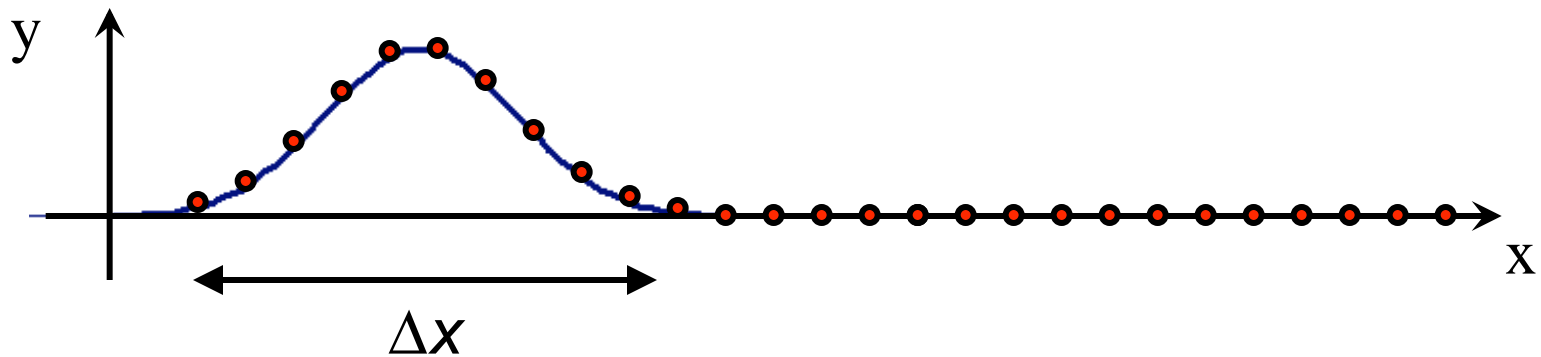
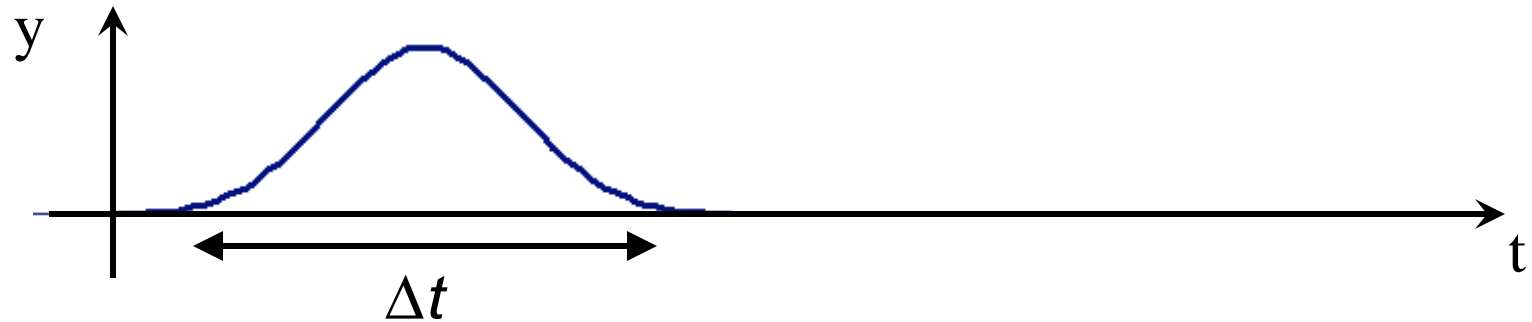
Motion of a wave pulse

- Whatever shape we make, it moves down the spring without changing its shape with a speed

$$v_0 = \sqrt{\frac{T}{\mu}}$$

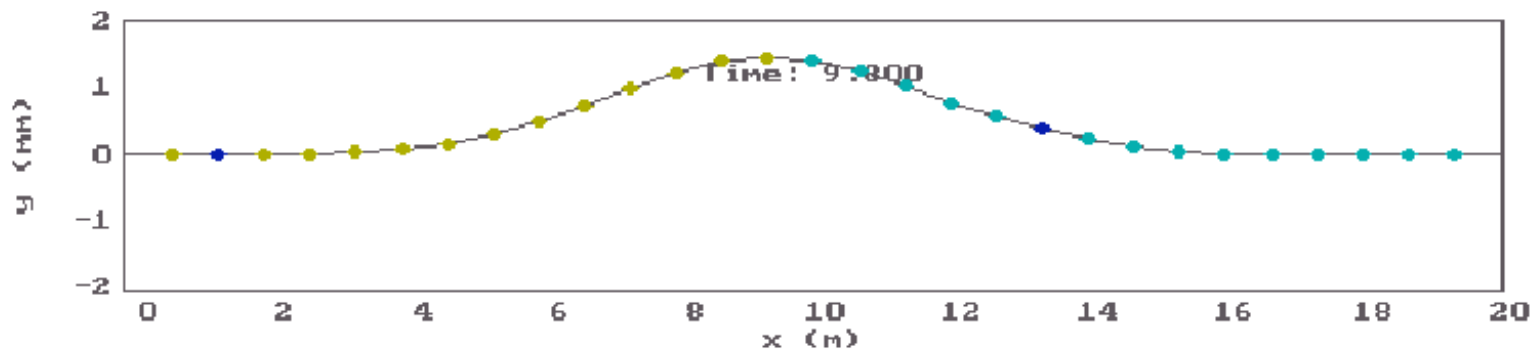
v_0 = speed of pulse
 T = tension of spring
 μ = mass density of spring (mass/length)

What controls the widths of the pulses in time and space?



Simulation

- Does propagation time depend on amplitude?
- Does propagation time depend on the properties of the string (masses, tension)?



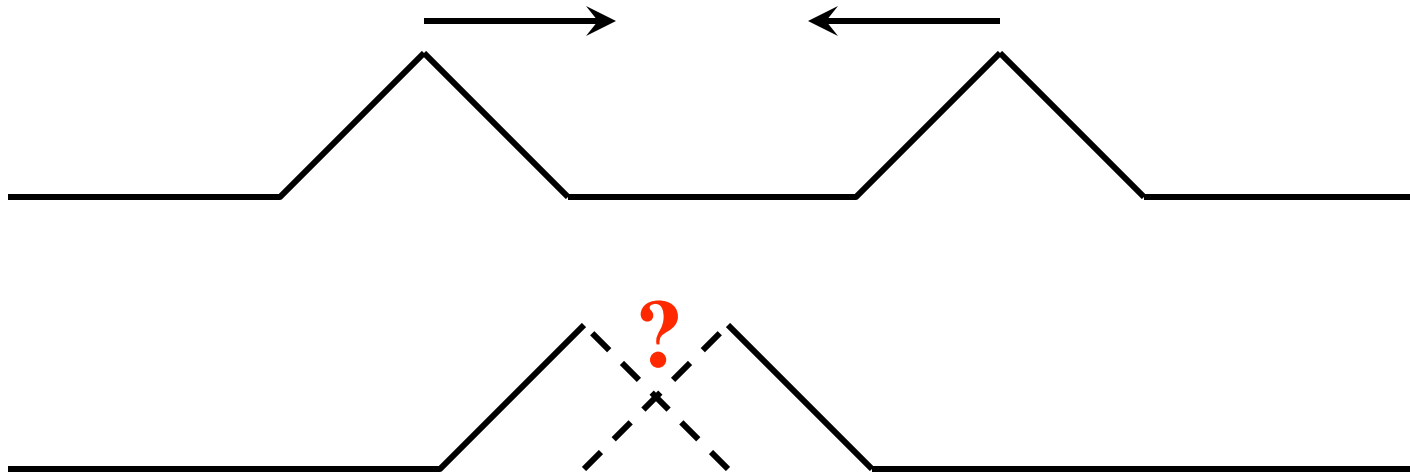
Propagating a pulse

- The amount of time the demonstrator's hand was displaced up and down determines the time width of the t-pulse, Δt .
- The speed of the signal propagation on the string controls the width of the x-pulse, ΔL .
 - The leading edge takes off with some speed, v_0 .
 - The pulse is over when the trailing edge is done.
 - The width is determined by “how far the leading edge got to” before the displacement was over.

$$\Delta L = v_0 \Delta t$$

How do waves combine?

- We know how one wave moves.
What happens when we get two waves on top of each other?



Superposition:

A way of understanding the answer

- A pulse or a wave passing by a particular point on a string gives the string an instruction.
- If we have a pulse $y = f(x, t)$ it says to the bit of string labeled by x “displace the amount y at the time t ”.
- If more than one pulse is at a given point the result is the “sum of the messages.”
- This rule is called *superposition*. It means “the wave pulses add – at each point.”

Hypothesis: Superposition

- Suppose we use the math to add two waves together:

$$y(x, t) = f(x, t) + g(x, t)$$

- What does this say?
- At a given time t for a bit of the string labeled “ x ” the displacement is the arithmetic sum of the displacements it would have from each piece of the wave.

Foothold principles: Mechanical waves



- *Key concept*: We have to distinguish the motion of the bits of matter and the motion of the pattern.
- *Pattern speed*: a disturbance moves into a medium with a speed that depends on the properties of the medium (but not on the shape of the disturbance)
- *Matter speed*: the speed of the bits of matter depend on both the size and shape of the pulse and on the pattern speed.
- *Superposition*: when one or more disturbances overlap, the result is that each point displaces by the sum of the displacements it would have from the individual pulses. (signs matter)
- *Mechanism*: the pulse propagates by each bit of string pulling on the next.