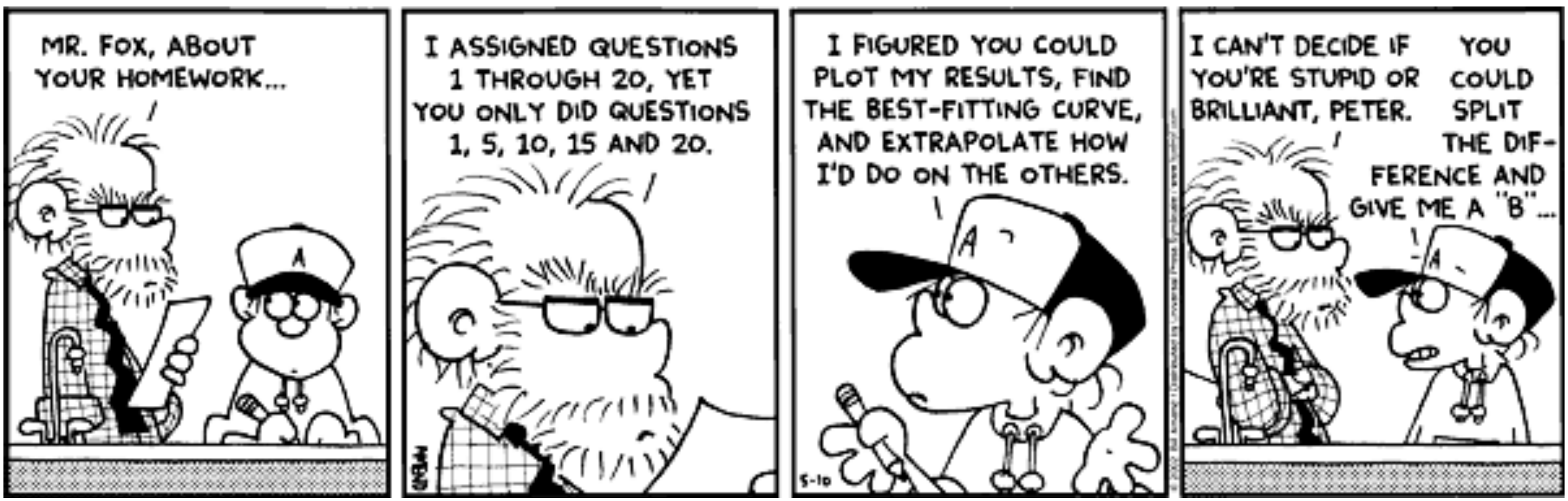
- **Theme Music:** **Information Society**

  Pure Energy

- **Cartoon:** **Bill Amend**

  FoxTrot



Outline

- Quiz 6 (Momentum)
- The Work-Energy Theorem along a line
- The dot product
- The general Work-Energy Theorem
- Potential Energy
- Gravity

Kinetic Energy and Work

- Consider an object moving along a line feeling a single force, *F*. When is moves a distance Δx, how much does its speed change?

$$a = F^{net} / m$$

$$\frac{\Delta v}{\Delta t} = \frac{F^{net}}{m}$$

$$\frac{\Delta v}{\Delta t}\Delta x = \frac{F^{net}}{m}\Delta x$$

$$\Delta v \frac{\Delta x}{\Delta t} = \frac{F^{net}\Delta x}{m}$$

$$\Delta v \frac{\Delta x}{\Delta t} = \frac{F^{net}\Delta x}{m}$$

$$\langle v \rangle \Delta v = \frac{F^{net}\Delta x}{m}$$

$$\frac{v_i + v_f}{2}(v_f - v_i) = \frac{F^{net}\Delta x}{m}$$

$$\tfrac{1}{2}(v_f^2 - v_i^2) = \frac{F^{net}\Delta x}{m}$$

$$\tfrac{1}{2}m(v_f^2 - v_i^2) = F^{net}\Delta x$$

Definitions:

Kinetic energy = $\tfrac{1}{2}mv^2$

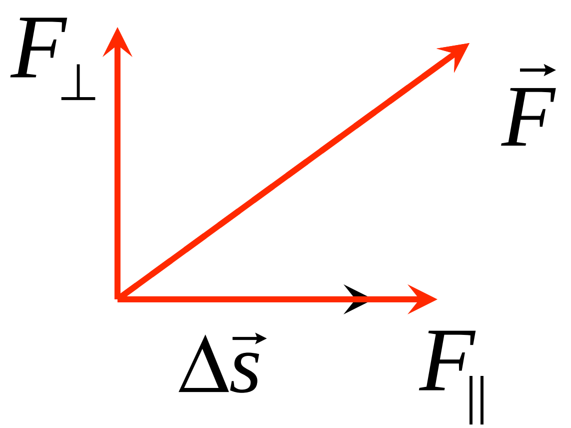
Work done by a force F = $F\ \Delta x$

Result

$$\Delta(\tfrac{1}{2}mv^2) = F^{net}\ \Delta x$$

Work in another direction: The dot product

- Suppose we are moving along a line, but the force we are interested in in pointed in another direction? (How can this happen?)
- Only the part of the force in the direction of the motion counts to change the speed (energy).



$$\text{Work} = F_{||}\,\Delta s = F\cos\theta\,\Delta s \equiv \vec{F}\cdot\Delta\vec{s}$$

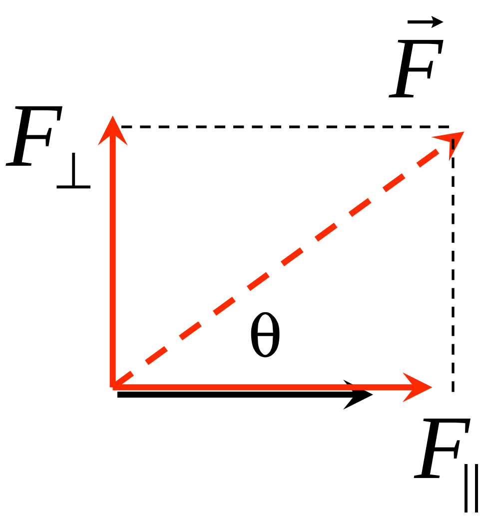
Calculating dot products

$$F_{\parallel}\,\Delta s \equiv \vec{F}\cdot\Delta\vec{s} \qquad \vec{F}\cdot\Delta\vec{s} = F\cos\theta\,\Delta s$$

In general, for any two vectors that have an angle q between them, the dot product is defined to be

$$\vec{a}\cdot\vec{b} = ab\cos\theta$$

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y$$



The dot product is a scalar.
Its value does not depend on the coordinate system we select.

The Work-Energy Theorem

- If we want to calculate how much forces on its object change its speed, we transform N2 into the Work-Energy theorem.

$$m\vec{a} = \vec{F}^{net}$$

$$\Delta\left(\tfrac{1}{2}mv^2\right) = \vec{F}^{net} \cdot \Delta\vec{r}$$

$$\Delta KE = W$$

- The free-body diagram we used to decide what forces were acting (N0) is useful for figuring out what work is done.

Potential Energy

- For some forces (gravity, springs) the amount of work done only depends of the change in position.
- Such forces are called <u>conservative</u>.
- For these forces the work done by them is written

$$\vec{F} \cdot \Delta\vec{r} = -\Delta U$$

- *U* is called a *potential energy*.

Example: Gravity

- Consider our work-energy result for a particular case: free-fall
(the only force is gravity).

$$\Delta(\tfrac{1}{2}mv^2) = \vec{F}^{net} \cdot \Delta\vec{r}$$
$$= m\vec{g} \cdot \Delta\vec{r}$$
$$= -mg\,\Delta h$$

Potential Energy: Gravity

- Since the work term also looks like a change, we can bring it to the left and get a *conservation law*.

$$\Delta(\tfrac{1}{2}mv^2 + mgh) = 0$$

$$U_g = mgh$$

- We interpret the quantity *mgh* as a new kind of energy — *gravitational potential energy*.

Mechanical

~~An~~ Energy Conservation Theorem

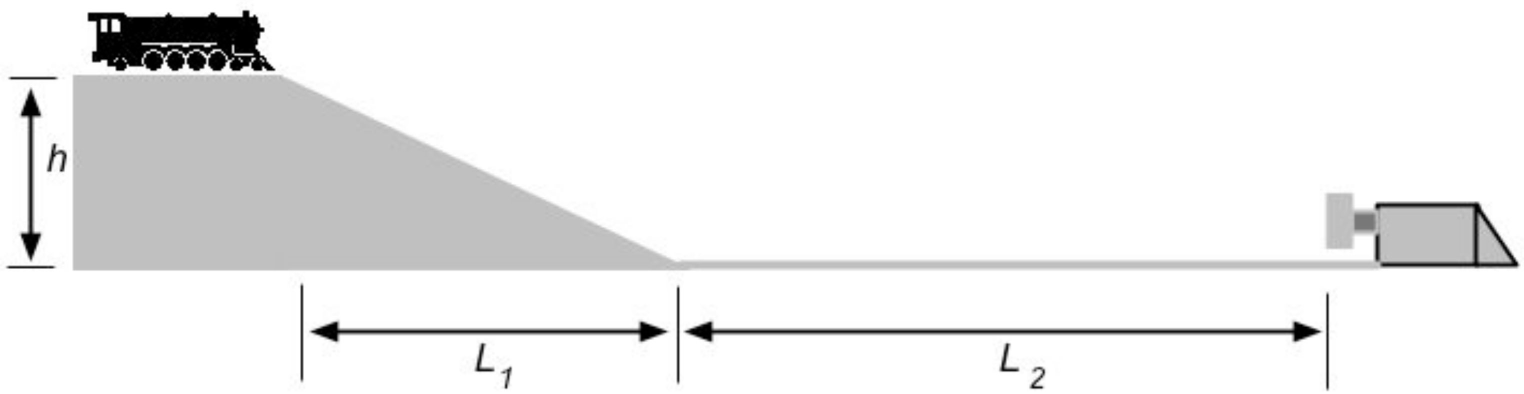
- Suppose the only force that has a component along the direction of motion is gravity.
  - The only force that changes the object's speed is gravity.
  - Other forces (normal forces) can change direction.
  - Friction must be negligible.
- Examples:
  - free fall
  - object rolling on a track.

$$\Delta(\tfrac{1}{2}mv^2 + mgh) = 0$$

$$\tfrac{1}{2}mv_i^2 + mgh_i = \tfrac{1}{2}mv_f^2 + mgh_f$$

Stopping a train

A toy train of mass *m* comes off a hill traveling at a velocity v_0, rolls down an incline of height *h*, rolls a short distance on a straight track, and strikes a bumper containing a spring of spring constant *k*. The distances are as indicated on the figure below. The train is just rolling, not powered.



(a) Assuming that friction and the rotational energy of the wheels can be ignored, describe the changes in the forms of energy of the system starting with the instant the train begins down the hill until it comes to a stop at the bumper.

(b) What is the speed of the train, *v,* when it is on the straight piece of track? Express your answer in terms of the symbols given above.