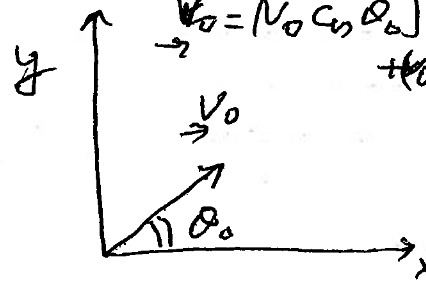
# KINEMATICS – TWO DIMENSIONS – PROJECTILE MOTION

At $t = 0$ a projectile is launched from $x = 0$, $y = 0$ with a velocity of $V_0$ m/sec. at an angle of $\theta_0$ above the horizon (x-axis). What are the equations which describe its motion in the xy-plane

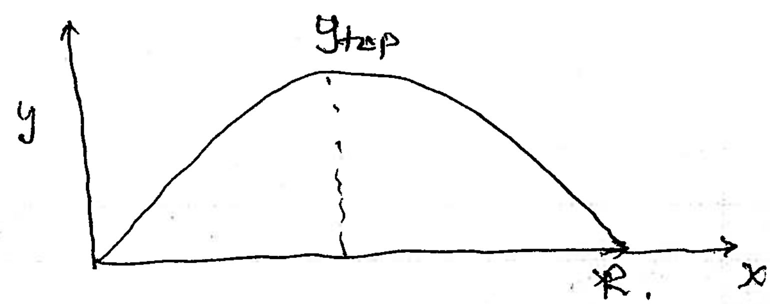
It is best to describe its motion along x, along y and combined.

$\vec{V_0} = (V_0 \cos\theta_0)\hat{x} + (V_0 \sin\theta_0)\hat{y}$




| X-Motion | Y-Motion | XY-Pl. |
|---|---|---|
| $a = 0$ | $\vec{a} = -9.8\,m/s^2\,\hat{y}$ | $\vec{a} = 0\,\hat{x} - 9.8\,m/s^2\,\hat{y}$ |
| $V_x = V_0 \cos\theta_0$ | $\vec{V_y} = V_0 \sin\theta_0 - 9.8t$ | $\vec{V} = (V_0 \cos\theta_0)\hat{x} + [V_0 \sin\theta_0 - 9.8t]\hat{y}$ |
| $x = (V_0 \cos\theta_0)t$ | $y = (V_0 \sin\theta_0)t - 4.9t^2$ | $\vec{r} = x\,\hat{x} + y\,\hat{y}$ $= (V_0 \cos\theta_0)\,t\,\hat{x} + [(V_0 \sin\theta_0)t - 4.9t^2]\,\hat{y}$ |
| | $V_y^2 = (V_0 \sin\theta_0)^2 - 19.6\,y$ | |

**Questions** ① What is its path? It is a parabola. It rises up to $y_{top}$ and returns when $x = R$, the Range.



② why does it stop rising?
Its y-velocity goes to zero.

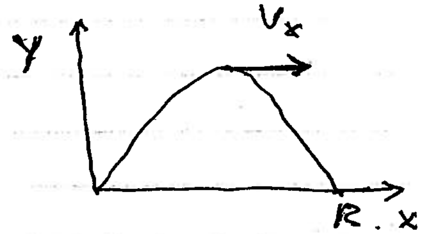
$$y_{top} = (V_0^2 \sin^2\theta_0)/19.6$$

③ what is acceleration at $y_{top}$

$$\underset{\rightarrow}{a} = -9.8\,\mathrm{m/s^2}\,\hat{y}.$$

This is fixed throughout the flight.

④ Velocity at $y_{top}$?




$$\underset{\rightarrow}{V} = 0\,\hat{y} + (V_0 \cos\theta_0)\,\hat{x}$$

⑤ When does it get to $y_{top}$?

$$0 = V_0 \sin\theta_0 - 9.8\,t_{top}$$

$$t_{top} = \frac{V_0 \sin\theta_0}{9.8}.$$

⑥ when does it return to ground?

$$y = 0 = (V_0 \sin\theta_0)t - 4.9\,t^2$$

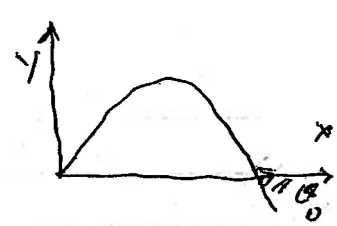
$$t_{gr} = \frac{V_0 \sin\theta_0}{4.9} = 2t_{top}.$$

⑦ What is its velocity just before it hits ground?

$$V_x = V_0 \cos\theta_0$$

$$V_y = V_0 \sin\theta_0 - \frac{2V_0 \sin\theta_0}{9.8} \times 9.8$$

$$= -V_0 \sin\theta_0$$




$$\underset{\rightarrow}{V} = (V_0 \cos\theta_0)\,\hat{x} - (V_0 \sin\theta_0)\,\hat{y}$$

⑧ what is the range ($R$)?

$$R = V_0 \cos\theta_0 . t_{gr}$$

$$= \frac{2V_0^2 \sin\theta_0 \cos\theta_0}{9.8} = \frac{V_0^2 \sin 2\theta_0}{9.8}$$

⑨ show that the path is a parabola.

$$y = (V_0 \sin\theta_0)t - 4.9t^2$$

But $x = (V_0 \cos\theta_0)t$

so

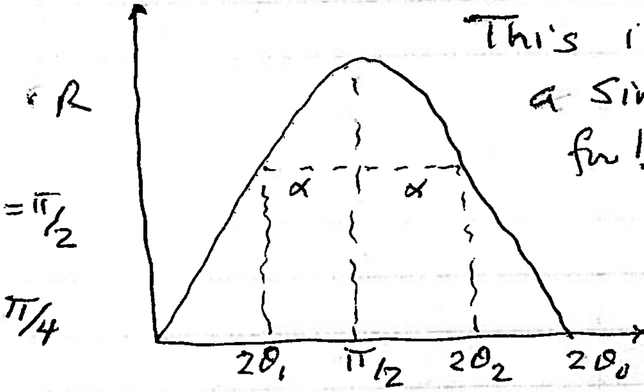
$$y = \frac{(V_0 \sin\theta_0)x}{V_0 \cos\theta_0} - 4.9\left(\frac{x}{V_0 \cos\theta_0}\right)^2$$

$$= x\tan\theta_0 - 4.9\left(\frac{x}{V_0 \cos\theta_0}\right)^2.$$

Note: This is a very useful equ. If the probem gives you $x$, $V_0$, $\theta_0$ you can use it to calculate $y$.

⑩ Galileo's finding. If you keep $V_0$ fixed but vary $\theta_0$ what $\theta_0$ gives you maximum $R$.?

$$R = \frac{V_0^2 \sin 2\theta_0}{9.8}$$




This is a sine fn!

$\sin 2\theta_0 = 1$, for $2\theta_0 = \pi/2$

So Max$^{m.}$ $R$ when $\theta_0 = \pi/4$ or $45^0$

Also note that there are two angles $\theta_1$ & $\theta_2$ for which $R$ is same

$$2\theta_2 = \pi/2 + \alpha.$$

$$2\theta_1 = \frac{\pi}{2} - \alpha.$$

$$\theta_1 + \theta_2 = \pi/2$$

$\theta_1$ & $\theta_2$ are complimentary angles.

(11) What happens if you launch projectile at $x=0$, $y=y_0$, $\vec{v}_0$ as before.

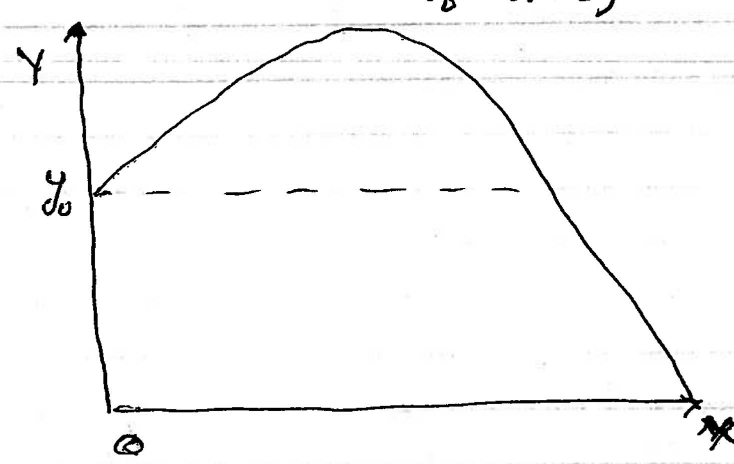
Now $$y_{top} = y_0 + \frac{v_0^2 \sin^2\theta_0}{19.6}$$

and $R$ is obtained by solving the quadratic Eqn:

$$0 = y_0 + (v_0 \sin\theta_0)t_{gr} - 4.9\,t_{gr}^2$$ [Sdn: on p. 6]

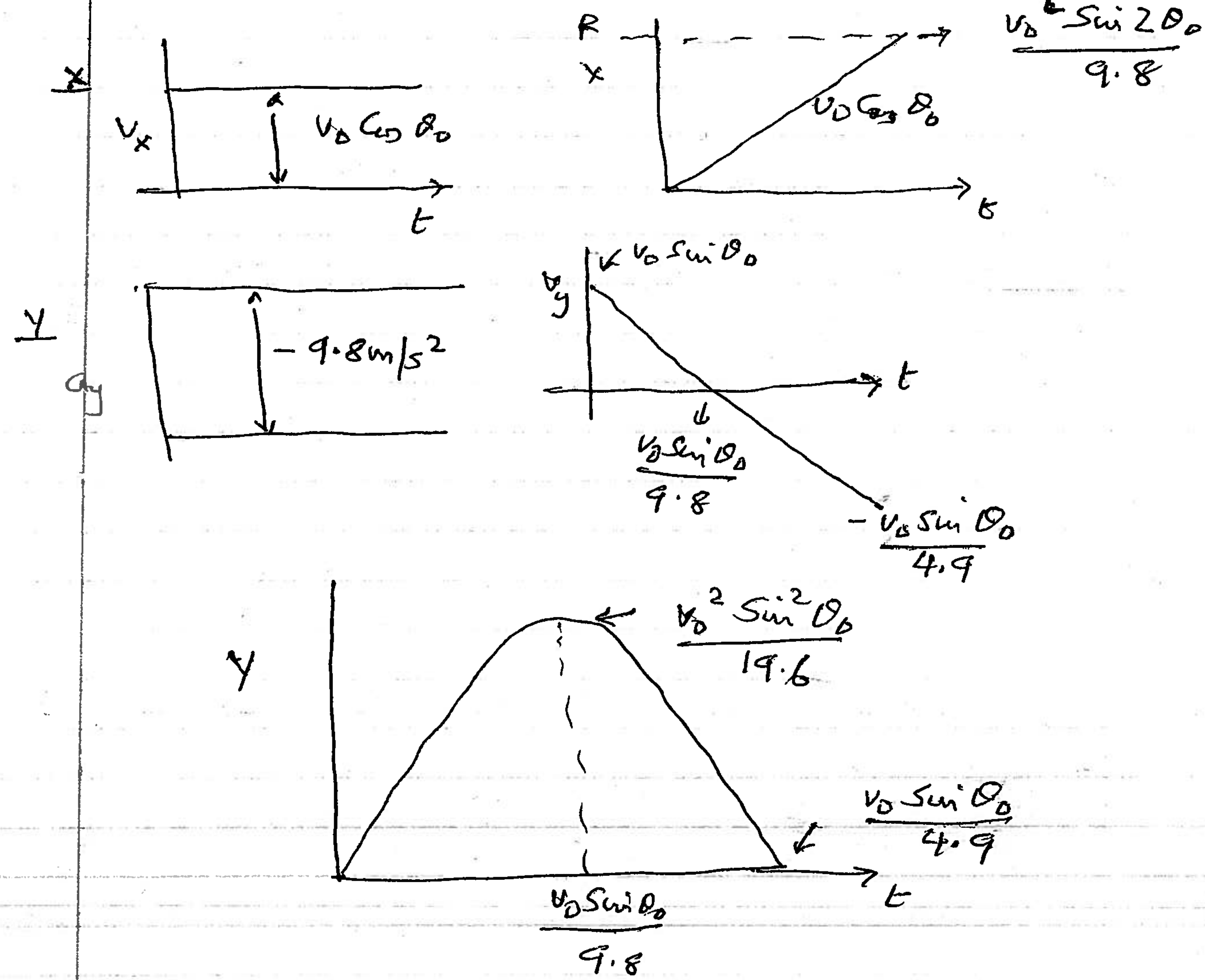
or $$0 = y_0 + R\tan\theta_0 - 4.9\left(\frac{R^2}{v_0^2\cos^2\theta_0}\right)$$

Not surprisingly the projectile travels farther before returning to ground. This is what led Newton to suggest that if one goes



high and throws a ball with a large enough velocity, it will go around the Earth. He had thought of a satellite many centuries ago.

(12) Draw Pictures to illustrate the Results.



$$R = \frac{V_0^2 \cos^2\theta_0 \tan\theta_0 \pm \sqrt{(V_0^2 \cos^2\theta_0 \tan\theta_0)^2 + 19.6 y_0}}{9.8}$$

Put $R = R_0$ when $y = 0$, $R_0 = \frac{V_0^2 \sin 2\theta_0}{9.8}$

$$R = \frac{R_0}{2} + \sqrt{\left(\frac{R_0}{2}\right)^2 + \frac{2 y_0}{9.8}}$$